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A Note on Power Whole Multiset of A Multiset and Multiset Topologies

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ABSTRACT

It is pointed in[1] that the empty multiset is a whole submultiset of any multiset and the power whole multiset of any multiset is an M-topology. Also, in[2], it ispointed out that the cardinality of a power whole multiset of a multiset is 2^n where n is the cardinality of its root set. In this note, we disagree with these assertions by counterexamples.

Keywords: Submultiset, whole submultiset, power whole multiset, M-topology, root set.

INTRODUCTION

The theory of multisets (bags) as a natural extension of the set theory was introduced by Cerf *et al.*[7], Peterson[6], Yager[5] and Blizard[4] etc. Many results have been established by these authors. Singh and Singh[2] derived the cardinalities of the power multiset, power full multiset and power whole multiset of a multiset. Their results indicates that the cardinality of a power whole multiset of a multiset is 2^n where n is the cardinality of its root set. This result have been verified in[1] with the claim that a null (empty) is a whole submultiset and hence a power whole multiset of a multiset an M-Topology. In this paper, we give the related concept and a counterexample to disagree with these assertions and to suggest a rethink on the cardinality of a power whole multiset of a multiset derived in[2].

Definition 1 [1]. A multiset (an mset for short) M drawn from the set X is represented by a function *Count* M denoted C_M defined: $C_M : X \rightarrow \mathbb{N}$ where $\mathbb{N}$ represents the set of non negative integers. Here, $C_M(x)$ is the number of occurrences of the element x in the mset M. For example if $M = \{a,a,a,c,d,e,e,e\}$, we

have:
$C_M(a)=3, C_M(c=1), C_M(d)=1, C_M(e)=3$.

Definition 2 [1]. An mset M over the set X is called null or empty denoted $\varnothing$ if $C_\varnothing(x)=0$ for all $x \in X$

Definition 3 [1]. The root set of an mset M over the set X denoted M^* is a subset of X defined: $M^*=\{x \in X \mid C_M(x)>0\}$.
For example $M=\{a,a,a,c,d,e,e,e\}$, then

$M^*=\{a,c,d,e\}$.

Note [4]: $x \in M^* \leftrightarrow x \in M$

Definition 4 [1]. An mset M over the set X is a submultiset (submset for short) of an mset N over the set X denoted $M \subseteq N$ if $C_M(x) \le C_N(x)$ for all $x \in X$

Definition 5 [1]. A submset M over the set X of N over X is a whole submset if $C_M(x)=C_N(x)$ for all $x \in M$

Proposition 1 . Let M,N be msets over the set X . Then
$C_M(x) \le C_N(x) \ \ \forall x \in X \leftrightarrow C_M(x) \le C_N(x)$ for all $x \in M$

Proof:
Let $C_M(x) \le C_N(x)$ for all $x \in X$.
Since $x \in M^* \leftrightarrow x \in M \wedge M^* \subseteq X$, we have $C_M(x) \le C_N(x)$ for all $x \in M$
That is, $C_M(x) \le C_N(x) \ \ \forall x \in X \rightarrow C_M(x) \le C_N(x) \ \ \forall x \in M$ (1)

Conversely,
Let $C_M(x) \le C_N(x)$ for all $x \in M$.
Clearly, $C_M(x) \le C_N(x)$ for all $x \in M^*$ (since $x \in M^* \leftrightarrow x \in M$)
But $C_M(x)=0$ for all $x \in X - M^*$. In particular, $C_M(x) \le C_N(x)$ for all $x \in X - M^* = (M^*)^c$
Since $(M^*)^c \cup M^* = X$, we have $C_M(x) \le C_N(x)$ for all $x \in X$
Thus, $C_M(x) \le C_N(x) \ \ \forall x \in M \rightarrow C_M(x) \le C_N(x) \ \ \forall x \in X$ (2)
Comparing (1) and (2) above, we have

$$C_M(x) \le C_N(x) \ \ \forall x \in X \leftrightarrow C_M(x) \le C_N(x) \text{ for all } x \in M$$

Corollary 2 . Let M,N be msets over the set X . Then

$$C_M(x) = C_N(x) \ \ \forall x \in X \leftrightarrow C_M(x) = C_N(x) \text{ for all } x \in M$$

Note [1]. Empty mset $\varnothing$ is a whole submset of every mset

The following example demonstrate the incorrectness of the above remark

Example 1: Let $M = \{a, a, b, c, c, c\}$ be an mset over English alphabets.

Clearly, $C_{\varnothing}(x) \neq C_M(x)$ for some alphabets x since $C_{\varnothing}(c) = 0 \neq C_M(c) = 3$

Definition 6 [1]. Let M be an mset over a set X.

The power mset of M denoted $P(M)$ is the set of all submsets of M.

We have $N \in P(M)$ if and only if $N \subseteq M$

Definition 7 [1]. Let M be an mset over a set X.

The power whole mset of M denoted by $PW(M)$ is defined as the set of all whole submsets of M.

Definition 8 [1]. Let M and N be msets drawn from the set X.

(i) the mset union of M and N denoted $M \bigcup N$ is an mset A such that

$$\mathrm{C_A}(x) = C_{M \bigcup N}(x) = \max\{C_M(x), C_N(x)\} \ \forall x \in X.$$

(ii) the mset intersection of M and N denoted $M \bigcap N$ is an mset A such that

$$\mathrm{C_A}(x) = C_{M \bigcap N}(x) = \min\{C_M(x), C_N(x)\} \ \forall x \in X.$$

Definition 9 [1]. Let M be an mset over a set X and $\tau \subseteq P(M)$.

Then τ is called an mset topology of M denoted $M-$topology if τ satisfies the following properties.

1. The mset M and the empty mset $\varnothing$ are in τ,
2. The mset union of the elements of any subcollection of τ is in τ,
3. The mset intersection of the elements of any finite subcollection of τ is in τ,

Example 2 [1]. Let M be an mset over a set X.

The collection $PW(M)$ is an $M-$topology

This is incorrect since $\varnothing \notin PW(M)$ from the counter example1

Theorem 3 (Singh and Singh, 2003). Let M be an mset over a set X.

The cardinality of the power whole mset of M denoted $|PW(M)|$ is given by

$|PW(M)| = 2^n$ where $n = |M^*|$, the cardinality of its root set.

The above is incorrect. For instance, given $M = \{x, x, y, y, y\}$, we have $PW(M) = \{\{x, x\}, \{y, y, y\}, M\}$ and $|PW(M)| = 3 \neq 2^2 = 4$

Thus, $|PW(M)| = 2^n$ where $n = |M^*|$, the cardinality of the root set of an mset M does not hold.

REFERENCES

1. K.P. Girish and Sunil Jacob John, On Multiset Topologies, *Theory and applications of Mathematics and Computer Science* 2(1), 37-52 (2012).
2. Singh, D. and J. N. Singh, Some Combinatorics of Multisets*, International Journal of Mathematical Education in Science and Technology* 34(4), 489-499 (2003).
3. S. P. Jena, S. K. Ghosh and B. K. Tripathy, On the theory of bags and lists, *Information Science* 132, 241-254 (2001).
4. Wayne D. Blizard, Multiset theory, *Notre Dame Journal of formal logic,* 30(1), 36-66 (1989).
5. R.R. Yager, On the theory of bags, *Int. J. General Syst.* 13, 23-37 (1986).
6. J. Peterson, Computation sequence sets, *J. Comput. Syst. Sci.* 13 (1), 1-24 (1976).
7. V. Cerf, E. Fernandez, K. Gostelow, S. Volausky, Formal control and low properties of a model of computation, *Report ENG 7178, Computer Science Department,* University of California, Los Angeles, CA, p. 81 December, (1971).