

Regular Generalized Fuzzy b-Irresolute in Fuzzy Topology

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ABSTRACT

The aim of this paper is to introduce a new form of generalized mapping that is rgfb-irresolute, Some of their characteristics and attributes have been verified.

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1. INTRODUCTION

In fuzzy topological space various forms of continuous mapping, irresolute mapping, generalized continuous mapping and generalized irresolute mapping were introduced by several authors. Balasubramanian et.al studied some generalization of continuous mapping. Benchalli *et al.*, studied fuzzy b-continuous mapping, irresolute mapping, fb-closed mapping. In this chapter, the notion of regular generalized fuzzy b-irresolute mapping was introduced and their properties studied. Characterizations of these mappings are obtained. Composition of these mappings is also studied.

2. PRELIMINARIES

$(X_1, \tau), (X_2, \sigma), (X_3, \gamma)$ (or simply $X_1, X_2, X_3,$) states fuzzy topological spaces in this article. $Cl(\alpha)$ states closure of α and $Int(\alpha)$ states interior of α . Where α is fuzzy set of X_1 ,

Definition 2.1: In fts X_1 , α be fuzzy set.

- (i) If $\alpha = IntCl(\alpha)$ then α is fuzzy regular open (in short, fr-open).
- (ii) If $\alpha = ClInt(\alpha)$ then α is fuzzy regular closed (in short, fr-closed).

Definition 2.2[3]: In fts X_1 , α be a fuzzy set.

- (i) If $\alpha \leq (\text{IntCl}\alpha) \vee (\text{ClInt}\alpha)$ then α is f b-open set(in brief, fbOS) .
- (ii) If $\alpha \geq (\text{IntCl}\alpha) \wedge (\text{ClInt}\alpha)$ then α is f b-closed set(in brief, fbCS).

Definition 2.3[3]: In a fuzzy topological space X_1 , α be fuzzy set then

- (i) $1 - \alpha$ is a fbCS when α is a fbOS.
- (ii) $1 - \alpha$ is a fbOS when α is a fbCS.

Remark 2.4 [1] : The implication given below are in a fuzzy topological space are true.



Definition 2.5 [3]: Let α be a fuzzy set in a fts X_1 . Then

- (i) $\text{bCl}\alpha = \wedge \{ \lambda : \lambda \text{ is a fbCS}(X_1), \geq \alpha \}$.
- (ii) $\text{bInt}\alpha = \vee \{ \delta : \delta \text{ is a fbOS}(X_1), \leq \alpha \}$.

Definition 2.6[5]: In a fts X_1 , if $\text{bCl}(\alpha) \leq \beta$, at any time when $\alpha \leq \beta$, then fuzzy set α is named as regular generalized fuzzy b-closed set (rgfbCS).Where β is fr open.

Remark 2.7[5] : In a fts X_1 , if $1 - \alpha$ is rgfbCS(X_1) then fuzzy set α is rgfbOS.

Definition 2.8: The regular generalized fuzzy b-closure is denoted and defined by, $\text{rgfbCl}(\alpha) = \wedge \{ \lambda : \lambda \text{ is a rgfbCS}(X_1), \geq \alpha \}$. Where α be fuzzy set in X_1 .

Definition 2.9: The regular generalized fuzzy b-interior is denoted and defined by, $\text{rgfbInt}(\alpha) = \vee \{ \delta : \delta \text{ is a rgfbOS}(X_1), \leq \alpha \}$. Where α be fuzzy set in X_1 .

Definition 2.10: (X_1, τ) is known as regular generalized fuzzy $\text{bT}_{1/2}$ -space (in short, rgfb $\text{T}_{1/2}$ -space), if each rgfbCS(X_1) is fbCS.

Definition 2.11: (X_1, τ) is known as regular generalized fuzzy $\text{bT}_{1/2}^*$ -space (in short, rgfb $\text{T}_{1/2}^*$ -space), if each rgfbCS(X_1) is closed fuzzy set.

Definition 2.12: Let (X_1, τ) , (X_2, σ) be two fuzzy topological spaces. Let $f : X_1 \rightarrow X_2$ be mapping,

- (i) if $f^1(\alpha)$ is open fuzzy in X_1 , for each open fuzzy set α of X_2 , then f is fuzzy -continuous (f-continuous)².
- (ii) if $f^1(\alpha)$ is fbCS(X_1), for each closed fuzzy set α of X_2 , then f is fuzzy b- continuous (fb-continuous)⁴.

- (iii) if $f^{-1}(\alpha)$ is fbCS(X_1), for each fbCS α of X_2 , then f is fuzzy b^* -continuous (fb^* -continuous)⁴.
- (iv) if $f^{-1}(\alpha)$ is rgfbCS(X_1), for each closed fuzzy set α in X_2 , then f is said to be regular generalized fuzzy b -continuous (briefly rgfb-continuous).¹¹
- (v) if $f^{-1}(\alpha)$ is open fuzzy in X_1 , for each rgfbOS α in X_2 , then f is called strongly rgfb-continuous.¹¹
- (vi) if $f(\alpha)$ is fbCS(X_2), for each closed fuzzy set α in X_1 , then f is fb-open mapping.⁴
- (vii) if $f(\alpha)$ is fbCS(X_2), for each fbCS α in X_1 , then f is fb^* -closed mapping.⁴

3. REGULAR GENERALIZED FUZZY b-IRRESOLUTE MAP

Definition 3.1: A mapping $f: X_1 \rightarrow X_2$ is said to be regular generalized fuzzy b -irresolute (briefly, rgfb-irresolute), if $f^{-1}(\alpha)$ is rgfbCS in X_1 , for each rgfbCS α in X_2 .

Theorem 3.2: A mapping $f: X_1 \rightarrow X_2$ is rgfb-irresolute mapping if and only if the $f^{-1}(\alpha)$ is rgfbOS in X_1 , for each rgfbOS set α in X_2 .

Proof: It follows from the definition.

Theorem 3.3: In fts X_1 , each rgfb-irresolute mapping is rgfb-continuous.

Proof: Assume $f: X_1 \rightarrow X_2$ is rgfb-irresolute. Let α be a closed fuzzy set in X_2 , it follows that α is rgfbCS fuzzy set in X_2 . While f is rgfb-irresolute, then the inverse image of α is rgfbCS fuzzy set in X_1 . Hence f is rgfb-continuous.

The converse of above theorem is in correct as shown in the following example.

Example 3.4: Let $X_1 = X_2 = \{a, b\}$ and the fuzzy sets A_1, A_2, A_3, A_4 and A_5 be defined as follows. $A_1 = \{(a, 0.5), (b, 0.2)\}$, $A_2 = \{(a, 0.4), (b, 0.3)\}$, $A_3 = \{(a, 0.5), (b, 0.3)\}$, $A_4 = \{(a, 0.4), (b, 0.2)\}$, $A_5 = \{(a, 0.6), (b, 0.8)\}$. Consider $\tau = \{0, 1, A_1, A_2, A_3, A_4\}$ and $\sigma = \{0, 1, A_5\}$. Define $f: X_1 \rightarrow X_2$ by $f(a)=b, f(b)=a$. Then f is rgfb-continuous but not rgfb-irresolute as the fuzzy set A_5 is not rgfbCS in X_2 , but $f^{-1}(A_5)$ is rgfbCS in X_1 .

Theorem 3.5: Let $f: X_1 \rightarrow X_2, g: X_2 \rightarrow X_3$ be two functions. Then

- (1) $g.f: X_1 \rightarrow X_3$ is rgfb-continuous, if f is rgfb-continuous and g is fuzzy-continuous.
- (2) $g.f: X_1 \rightarrow X_3$ is rgfb-irresolute, if f and g are rgfb irresolute functions.
- (3) $g.f: X_1 \rightarrow X_3$ is rgfb-continuous if f is rgfb-irresolute and g is rgfb-continuous.

Proof: (1) Assume C be closed fuzzy subset of X_3 . While $g: X_2 \rightarrow X_3$ is f -continuous and $f: X_1 \rightarrow X_2$ is rgfb continuous then by definition $f^{-1}\{g^{-1}(C)\} = (g.f)^{-1}C$ is rgfb closed in X_1 . Hence $g.f: X_1 \rightarrow X_3$ is rgfb-continuous.

(2) $g: X_2 \rightarrow X_3$ is rgfb-irresolute and C be rgfbCS subset of X_3 . As g is rgfb-irresolute by definition, $g^{-1}(C)$ is rgfb closed of X_2 . Also $f: X_1 \rightarrow X_2$ is rgfb-irresolute therefore $f^{-1}\{g^{-1}(C)\} = (g.f)^{-1}(C)$ is rgfbCS. Hence $g.f: X_1 \rightarrow X_3$ is rgfb irresolute.

(3) Assume C be fuzzy-closed subset of X_3 . While $g : X_2 \rightarrow X_3$ is rgfb continuous, $g^{-1}(C)$ is rgfbCS subset of X_2 . Also $f : X_1 \rightarrow X_2$ is rgfb-irresolute. Therefore $f^{-1}\{g^{-1}(C)\} = (g \cdot f)^{-1}(C)$ is rgfbCS of X_1 . Thus $g \cdot f : X_1 \rightarrow X_3$ is rgfb-continuous.

Theorem 3.6: Let $f : X_1 \rightarrow X_2$ and $g : X_2 \rightarrow X_3$ be functions. If f is strongly rgfb-continuous any g is rgfb-continuous, then $g \cdot f : X_1 \rightarrow X_3$ is fuzzy continuous.

Proof: Assume α be closed fuzzy in X_3 . While $g : X_2 \rightarrow X_3$ is rgfb-continuous by definition $g^{-1}(\alpha)$ is rgfbCS in X_2 . Also $f : X_1 \rightarrow X_2$ is strongly rgfb-continuous therefore $f^{-1}\{g^{-1}(\alpha)\} = (g \cdot f)^{-1}(\alpha)$ is closed fuzzy in X_1 . Hence $g \cdot f : X_1 \rightarrow X_3$ is fuzzy continuous.

Theorem 3.7: Let $f : X_1 \rightarrow X_2$ and $g : X_2 \rightarrow X_3$ be functions. If f is fuzzy continuous and g is strongly rgfb-continuous, then $g \cdot f : X_1 \rightarrow X_3$ is strongly rgfb-continuous.

Proof: Assume α be rgfbCS in X_3 . While $g : X_2 \rightarrow X_3$ is strongly rgfb-continuous then $g^{-1}(\alpha)$ is closed fuzzy in X_2 . Also $f : X_1 \rightarrow X_2$ is fuzzy-continuous hence $f^{-1}\{g^{-1}(\alpha)\}$ is fuzzy-closed in X_1 . That is $(g \cdot f)^{-1}(\alpha)$ is closed fuzzy in X_1 . So that $g \cdot f : X_1 \rightarrow X_3$ is strongly rgfb-continuous.

Theorem 3.8: Let $f : X_1 \rightarrow X_2$ and $g : X_2 \rightarrow X_3$ be functions. If f is strongly rgfb-continuous and g is rgf-continuous, then $g \cdot f : X_1 \rightarrow X_3$ is fuzzy continuous.

Proof: Assume α be fuzzy-closed in X_3 . While g is rgf-continuous then $g^{-1}(\alpha)$ is rgfCS set in X_2 . While each rgfCS set is rgfbCS. Therefore $g^{-1}(\alpha)$ is rgfbCS in X_2 , also f is strongly rgfb-continuous then $f^{-1}\{g^{-1}(\alpha)\} = (g \cdot f)^{-1}(\alpha)$ is closed fuzzy in X_1 . Hence $g \cdot f : X_1 \rightarrow X_3$ is fuzzy continuous function.

Theorem 3.9: Let $f : X_1 \rightarrow X_2$ and $g : X_2 \rightarrow X_3$ be functions. If f is rgfb-continuous and g is strongly rgfb-continuous, then $g \cdot f : X_1 \rightarrow X_3$ is rgfb irresolute.

Proof: Assume α be rgfbCS in X_3 . While $g : X_2 \rightarrow X_3$ be strongly rgfb-continuous then $g^{-1}(\alpha)$ is fuzzy-closed in X_2 . Also $f : X_1 \rightarrow X_2$ be rgfb-continuous then $f^{-1}\{g^{-1}(\alpha)\} = (g \cdot f)^{-1}(\alpha)$ is rgfbCS in X_1 . Hence $g \cdot f : X_1 \rightarrow X_3$ is rgfb-irresolute.

Theorem 3.10: Let $f : X_1 \rightarrow X_2$ be fb*-continuous and $g : X_2 \rightarrow X_3$ be rgfb-continuous then $g \cdot f : X_1 \rightarrow X_3$ is fb continuous if X_2 is rgfb $T_{1/2}$ -space.

Proof: Assume α be closed fuzzy set in X_3 . While $g : X_2 \rightarrow X_3$ is rgfb-continuous then $g^{-1}(\alpha)$ is rgfbCS in X_2 . While X_2 is rgfb $T_{1/2}$ - space therefore $g^{-1}(\alpha)$ is fbCS in X_2 . Also $f : X_1 \rightarrow X_2$ is fb*-continuous, $f^{-1}\{g^{-1}(\alpha)\} = (g \cdot f)^{-1}(\alpha)$ is fbCS in X_1 . Hence $g \cdot f : X_1 \rightarrow X_3$ fb-continuous.

Theorem 3.11: Let $f : X_1 \rightarrow X_2$ be rgfb-continuous and X_1 is rgfb $T_{1/2}$ - space then f is fb continuous.

Proof: Assume α be closed fuzzy in X_2 . While f is rgfb-continuous then $f^{-1}(\alpha)$ is rgfbCS in X_1 . Hence $f^{-1}(\alpha)$ is fbCS in X_1 , because X_1 is rgfb $T_{1/2}$. So that f is fb-continuous.

Theorem 3.12: Let $f : X_1 \rightarrow X_2$ be rgfb-continuous and X_1 is rgfb $T_{1/2}^*$ -space then f is fuzzy continuous.

Proof: Assume α be closed fuzzy in X_2 . While f is rgfb-continuous then $f^{-1}(\alpha)$ is rgfbCS in X_1 . While X_1 is rgfb- $T_{1/2}^*$, $f^{-1}(\alpha)$ is closed fuzzy in X_1 . Therefore f is fuzzy continuous.

Theorem 3.13: Let $f : X_1 \rightarrow X_2$ be onto, rgfb-irresolute and fb*-closed. X_2 is rgfb $T_{1/2}$ -space, when X_1 is rgfb $T_{1/2}$ -space.

Proof: Assume α be rgfbCS in X_2 . While f is rgfb-irresolute, $f^{-1}(\alpha)$ is rgfbCS in X_1 . Where X_1 is rgfb $T_{1/2}$ -space by definition $f^{-1}(\alpha)$ is fbCS in X_1 . While f is also onto and fb*-closed then $f[f^{-1}(\alpha)] = \alpha$ is fbCS in X_2 . It shows that each rgfbCS set is fbCS. Therefore X_2 is rgfb $T_{1/2}$ -space.

Theorem 3.14: Let $f : X_1 \rightarrow X_2$ be onto, rgfb-irresolute and fb-closed. X_2 is rgfb $T_{1/2}^*$ -space, when X_1 is rgfb $T_{1/2}^*$ -space.

Proof: Follows the above theorem

Theorem 3.15: If a function $f : X_1 \rightarrow X_2$ is rgfb-continuous then $f[\text{rgfbCl}(\alpha)] \leq \text{cl}[f(\alpha)]$ for each fuzzy set α in X_1 .

Proof: Assume α be any fuzzy set in X_1 . $\text{cl}[f(\alpha)]$ is a closed fuzzy set in X_2 . While f is rgfb-continuous, $f^{-1}[\text{cl}(f(\alpha))]$ is rgfbCS in X_1 . $\alpha \leq f^{-1}[\text{cl}(f(\alpha))]$ that implies $\text{rgfbCl}(\alpha) \leq f^{-1}[\text{cl}(f(\alpha))]$. Hence $f[\text{rgfbCl}(\alpha)] \leq \text{cl}[f(\alpha)]$.

Theorem 3.16: If a function $f : X_1 \rightarrow X_2$ is rgfb-continuous then $\text{rgfbCl}[f^{-1}(\alpha)] \leq f^{-1}[\text{cl}(\alpha)]$ for each fuzzy set α in X_2 .

Proof: Assume α is any fuzzy set in X_2 . $\text{cl}(\alpha)$ is a closed fuzzy in X_2 . While f is rgfb-continuous $f^{-1}[\text{cl}(\alpha)]$ is rgfbCS in X_1 . From the definition of closure $f^{-1}(\alpha) \leq f^{-1}[\text{cl}(\alpha)]$. For rgfbCS set $f^{-1}(\alpha) = \text{rgfbCl}[f^{-1}(\alpha)]$. Hence $\text{rgfbCl}[f^{-1}(\alpha)] \leq f^{-1}[\text{cl}(\alpha)]$.

Theorem 3.17: A function $f : X_1 \rightarrow X_2$ is rgfb-continuous if $f^{-1}[\text{Int}(\alpha)] \leq \text{rgfbInt}[f^{-1}[\text{cl}(\alpha)]]$ for each fuzzy set α in X_2 .

Proof: Assume α be any fuzzy set in X_2 . $\text{Int}(\alpha)$ is open fuzzy in X_2 . While f is rgfb-continuous, $f^{-1}[\text{Int}(\alpha)]$ is rgfbOS set in X_1 . We have

$f^{-1}[\text{Int}(\alpha)] = \text{rgfbInt}[f^{-1}(\text{Int}(\alpha))]$. But $\text{rgfbInt}[f^{-1}(\text{Int}(\alpha))] \leq \text{rgfbInt}[f^{-1}(\alpha)]$. Hence $f^{-1}[\text{Int}(\alpha)] \leq \text{rgfbInt}[f^{-1}(\alpha)]$.

On the other hand, Let α be any open in X_2 . We have $\text{rgfbInt}[f^{-1}(\alpha)] \leq f^{-1}[\text{Int}(\alpha)] = f^{-1}(\alpha)$. so $\text{rgfbInt}[f^{-1}(\alpha)] \leq f^{-1}(\alpha)$. But $f^{-1}(\alpha) \leq \text{rgfbInt}[f^{-1}(\alpha)]$ from the definition. Therefore $f^{-1}(\alpha) = \text{rgfbInt}[f^{-1}(\alpha)]$. Hence $f^{-1}(\alpha)$ is rgfbOS in X_1 . This follows that f is rgfb-continuous.

Theorem 3.18: A function $f : X_1 \rightarrow X_2$ is rgfb-continuous iff $\text{Int}[f(\alpha)] \leq f[\text{rgfbInt}(\alpha)]$ for each fuzzy set α in X_1 .

Proof: Assume α be any fuzzy set in X_1 . $\text{Int}[f(\alpha)]$ is open fuzzy in X_2 . While f is rgfb-continuous $f^{-1}[\text{Int}(f(\alpha))]$ is rgfbOS in X_1 . From the definition $f^{-1}[\text{Int}(f(\alpha))] \leq \text{rgfbInt}[f^{-1}(f(\alpha))] \leq \text{rgfbInt}(\alpha)$. Therefore $f^{-1}[\text{Int}(f(\alpha))] \leq \text{rgfbInt}(\alpha)$. Hence $\text{Int}(f(\alpha)) \leq f[\text{rgfbInt}(\alpha)]$.

On the other hand, Let α be any fuzzy set in X_2 . We have $f[\text{rgfbInt}(f^{-1}(\alpha))] \geq \text{Int}[f(f^{-1}(\alpha))] = \text{Int}(\alpha) = \alpha$. Therefore $\text{rgfbInt}(f^{-1}(\alpha)) \geq f^{-1}(\alpha)$. Also we have $f^{-1}(\alpha) \geq \text{rgfbInt}(f^{-1}(\alpha))$. This implies that $f^{-1}(\alpha) = \text{rgfbInt}(f^{-1}(\alpha))$. Hence $f^{-1}(\alpha)$ is rgfbOS in X_1 and f is rgfb-continuous.

Theorem 3.19: If a function $f : X_1 \rightarrow X_2$ is rgfb-irresolute then $[\text{rgfbCl}(\beta)] \leq \text{rgfbCl}[f(\beta)]$ for each fuzzy set β in X_1 .

Proof: Assume β be any fuzzy set in X_1 . Then $\text{rgfbCl}[f(\beta)]$ is rgfbCS in X_2 . While f is rgfb-irresolute $f^{-1}[\text{rgfbCl}(f(\beta))]$ is a rgfbCS in X_1 . Further $\beta \leq f^{-1}[f(\beta)] \leq f^{-1}[\text{rgfbCl}(f(\beta))]$, it follows from the definition that $\text{rgfbCl}(\beta) \leq f^{-1}[\text{rgfbCl}(f(\beta))]$. Hence $f[\text{rgfbCl}(\beta)] \leq \text{rgfbCl}(f(\beta))$.

Theorem 3.20: If a function $f : X_1 \rightarrow X_2$ is rgfb-irresolute then $\text{rgfbCl}[f^{-1}(\beta)] \leq f^{-1}[\text{rgfbCl}(\beta)]$ for each fuzzy set β in X_2 .

Proof: Assume β be any fuzzy set in X_2 . Then $\text{rgfbCl}(\beta)$ is rgfb closed set in X_2 . While f is rgfb-irresolute $f^{-1}[\text{rgfbCl}(\beta)]$ is rgfbCS in X_1 . But $f^{-1}(\beta) \leq f^{-1}[\text{rgfbCl}(\beta)]$ it follows from the definition that $\text{rgfbCl}[f^{-1}(\beta)] \leq f^{-1}[\text{rgfbCl}(\beta)]$.

Theorem 3.21: A function $f : X_1 \rightarrow X_2$ is rgfb-irresolute iff $f^{-1}[\text{rgfbInt}(\beta)] \leq \text{rgfbInt}[f^{-1}(\beta)]$ for each fuzzy set β in X_2 .

Proof: Assume β be any fuzzy set in X_2 . Then $\text{rgfbInt}(\beta)$ is rgfb open in X_2 . While f is rgfb-irresolute, $f^{-1}[\text{rgfbInt}(\beta)]$ is rgfbOS in X_1 . From the property we have $f^{-1}[\text{rgfbInt}(\beta)] = \text{rgfbInt}[f^{-1}(\text{rgfbInt}(\beta))]$. But $\text{rgfbInt}[f^{-1}(\text{rgfbInt}(\beta))] \leq \text{rgfbInt}[f^{-1}(\beta)]$. Hence $f^{-1}[\text{rgfbInt}(\beta)] \leq \text{rgfbInt}[f^{-1}(\beta)]$.

On the other hand, Let β be rgfbOS in X_2 . We have $\beta = \text{rgfbInt}(\beta)$ and $f^{-1}(\beta) = f^{-1}[\text{rgfbInt}(\beta)]$. But $f^{-1}[\text{rgfbInt}(\beta)] \leq \text{rgfbInt}(f^{-1}(\beta))$. So that $f^{-1}(\beta) \leq \text{rgfbInt}(f^{-1}(\beta))$. While $f^{-1}(\beta) \geq \text{rgfbInt}(f^{-1}(\beta))$. Hence $f^{-1}(\beta) = \text{rgfbInt}(f^{-1}(\beta))$ and f is rgfb-irresolute.

Theorem 3.22: A function $f : X_1 \rightarrow X_2$ be rgfb-irresolute iff $\text{rgfbInt}[f(\beta)] \leq f[\text{rgfbInt}(\beta)]$ for each fuzzy set β in X_1 .

Proof: Assume β be any fuzzy set in X_1 , $\text{rgfbInt}[f(\beta)]$ is rgfbOS in X_2 . While f is rgfb-irresolute $f^{-1}[\text{rgfbInt}(f(\beta))]$ is rgfbOS set in X_1 . From the above theorem, $f^{-1}[\text{rgfbInt}(f(\beta))] \leq \text{rgfbInt}[f^{-1}(f(\beta))] \leq \text{rgfbInt}(\beta)$. So that $f^{-1}[\text{rgfbInt}(f(\beta))] \leq \text{rgfbInt}(\beta)$. Hence $\text{rgfbInt}(f(\beta)) \leq f[\text{rgfbInt}(\beta)]$.

on the other hand, Let β be rgfbOS in X_2 . From the property we have $\beta = \text{rgfbInt}(\beta)$. We have $f[\text{rgfbInt}(f^{-1}(\beta))] \geq \text{rgfbInt}[f(f^{-1}(\beta))] = \text{rgfbInt}(\beta) = \beta$. Therefore

$\text{rgfbInt}(f^{-1}(\beta)) \geq f^{-1}(\beta)$. By definition we have $\text{rgfbInt}(f^{-1}(\beta)) \leq f^{-1}(\beta)$. Hence $f^{-1}(\beta) = \text{rgfbInt}[f^{-1}(\beta)]$. This shows that $f^{-1}(\beta)$ is rgfbOS in X_1 and f is rgfb-irresolute.

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