Journal of Computer and Mathematical Sciences, *Vol.8(11), 621-629 November 2017* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

On Fuzzy Generalized b Closed Sets

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(Received on: October 14, Accepted: November 10, 2017)

ABSTRACT

In this paper, we study the class of fuzzy generalized b closed sets and use the notion to consider new weak and stronger forms of continuities associated with these sets. We apply these notions to give new characterization of fT_{gs} spaces.

2000 AMS Subject Classification: 94D05.

Keywords: fb closed, fgb closed, contra fb continuous, ap-fb-continuous, ap-fb-closed, $fT_{1/2}$ space, fT_{gs} space.

1. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh in his classical paper[13]. Subsequently several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by Chang[7]. Pu and Liu[9] introduced the concept of quasi coincidence and q neighbourhoods by which the expansions of functions in fuzzy setting can very interestingly and effectively be carried out.

Ahmad al-Omari *et al*.[1] defined generalized b closed sets and studied its properties. The aim of this paper is to introduce the notion of fuzzy generalized b closed sets and its various characterizations in Section 3. In Section 4, we study various forms of fuzzy continuity associated to fuzzy generalized b closed sets. Finally in Section 5 we characterize the fuzzy T_{gs} spaces in terms of these notions of continuity.

2. PRELIMINARIES

A family τ of fuzzy sets of X is called a fuzzy topology (briefly ft)[7] on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open (briefly fo) sets and their complements are called fuzzy closed (briefly fc) sets.

Throughout this paper (X, τ), (Y,σ) and (Z,η) (or simply X,Y,Z) always mean fuzzy topological space on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy set A of (X, τ), cl(A) and int(A) denote the closure and interior of A respectively. By 0_X and 1_X we mean fuzzy sets with constant function 0(zero function) and 1 (unit function) respectively.

The following definitions are useful in the sequel.

Definition 2.1: A fuzzy set A of (X, τ) is called:
1) Fuzzy semi open (briefly fso) if $A \leq cl(int(A))$ and fuzzy semi closed (briefly fsc) if $int(cl(A)) \leq A$ [2]
2) Fuzzy pre open (briefly fpo) if $A \leq int(cl(A))$ and fuzzy pre closed(briefly fpc) if $cl(int(A)) \leq A$ [6]
3) Fuzzy α open (briefly f αo) if $A \leq int(cl(int(A)))$ and fuzzy α closed (briefly f αc) if $cl(int(cl(A))) \leq A$ [6]
4) Fuzzy b open (briefly fbo) if $A \leq cl(int(A)) \vee int(cl(A))$ and fuzzy b closed (briefly fbc) if $Cl(int(A)) \wedge int(cl(A)) \leq A$ [5]

The intersection of all fbc sets of X containing A is called fuzzy b closure of A and is denoted by fbcl(A). The union of all fbo sets of X contained in A is called fuzzy b interior of A and is denoted by fbint(A). The family of all fbo(respy fso, fpo, f αo) subsets of a space X is denoted by fbO(X) (respy fsO(X),fpO(X),f αo(X)) and the collection of all fbo subsets of X containing a fixed point x is denoted by fbO(X,x). The sets fsO(X,x), fpO(X,x), f αo(X,x) are defined analogously.

Definition 2.2: A fuzzy subset A of a space (X,τ) is called
1. a fuzzy generalized closed set (briefly fgc) if $cl(A) \leq U$ whenever $A \leq U$ and U is fo in X [3]
2. a fuzzy α generalized closed set(briefly fαgc) if $f\alpha cl(A) \leq U$ whenever $A \leq U$ and U is fo in X [10].
3. a fuzzy generalized pre closed set(briefly fgpc) if $fpcl(A) \leq U$ whenever $A \leq U$ and U is fo in X [6].

Complement of fgc(respy fgαc, fgpc) sets are called fg open(respy fgα open,fgp open)(briefly fgo, fgαo,fgpo) sets

Definition 2.3: A function $f:X \to Y$ is said to be fuzzy b continuous(briefly fb continuous) if for each $x \in X$ and each fo set V of Y containing f(x), there exists $U \in fbO(X,x)$ such that $f(U) \leq V$ [8].

Definition 2.4: A function f:X→Y is said to be contra fuzzy continuous(briefly contra f continuous) (respy contra fuzzy b continuous or briefly contra fb continuous) if $f^{-1}(V)$ is fc (respy fbc) in X for each fo set V of Y [8].

Definition 2.5: A function f:X→Y is said to be fuzzy b irresolute(briefly fb irresolute) if for each fbo set of Y, $f^{-1}(V)$ is fbo in X.

Definition 2.6: A function f:X→Y is said to be fuzzy b closed function(briefly fb closed function)(respy fuzzy b open function or briefly fb open function) if for every fbc subset(respy fbo subset) A of X,f(A) is fbc(respy fbo) in Y.

Definition 2.7: A function f:X→Y is said to be contra fuzzy b closed function(briefly contra fb closed function) (respy contra fuzzy b open function or briefly contra fb open function) if f(U) is fbo(respy fbc) for each fc subset(respy fbo subset) U of X.

3. FUZZY GENERALIZED b CLOSED SETS

In this section, we investigate the class of fuzzy generalized b closed sets and study some of its fundamental properties. Several characterizations of fuzzy generalized b closed sets are given.

Definition 3.1: Let X be a space. A fuzzy subset A of X is called fuzzy generalized b closed set(briefly fgbc set) if fbcl(A) ≤ U whenever A≤ U and U is fo.
The complement of fuzzy generalized b closed set is fuzzy generalized b open(briefly fgbo)
Every fbc is fgbc but the converse is not true. The collection of all fgbc (respy fgbo) subsets of X is denoted by fgbc(X) (respy fgbo(X))

Example 3.1: Let X={a,b}. Let A and B be fuzzy sets defined by A(a) = 0.7,A(b) = 0.6. B(a)=0.7,B(b) =0.8. τ ={0,A,1}. Then B is fgbc but not fbc.
Now we give necessary and sufficient condition for a fgbc to be fbc

Theorem 3.1: Let A be a fgbc subset of X. Then fbcl(A)-A does not contain any nonempty fc sets.

Proof: Let F∊C(X) such that F≤fbcl(A)-A. Then X-F is fo.A≤1-F and A is fgbc. It follows that fbcl(A) ≤1-F. Hence F≤1-fbcl(A). This implies F≤(1-fbcl(A))Λ(fbcl(A)-A). So F=0

Corollary 3.1: Let A be fgbc. Then A is fbc iff fbcl(A)-A is fc.

Proof: Let A be fgbc. If A is fbc ,then fbcl(A)-A=0, which is fc.
Conversely, let fbcl(A)-A be fc. Then by theorem 3.1,fbcl(A)-A does not contain any nonempty fc subset and since fbcl(A)-A is fc subset of itself, then fbcl(A)-A=0. This implies A=fbcl(A) and so A is fbc.

Definition 3.2: Let A be a fuzzy subset of X. A point x∊X is said to be fuzzy b limit point(briefly fb limit point) of A if for each fbo set U containing x, we have UΛ(A-{x})≠0. The set of all fb limit points of A is called the fb derived set of A and is denoted by $D_{fb}(A)$.

Since every fo set is fbo, we have $D_{fb}(A) \leq D(A)$, for any fuzzy subset A≤X, where D(A) is the fuzzy derived set of A. Moreover , since every fc set is fbc, wehave A≤fbcl(A) ≤cl(A).

Lemma 3.1: If $D(A)=D_{fb}(A)$, then we have cl(A)=fbcl(A).

Proof: Let xϵcl(A)=AvD(A). Then either xϵA or xϵD(A). If xϵA , then xϵfbclA).If xϵD(A), then xϵ $D_{fb}(A)$. Hence xϵfbcl(A) as fbcl(A)=Av$D_{fb}(A)$.

Corollary 3.2: If D(A) ≤ $D_{fb}(A)$ for every fuzzy subset A of X , then for any fuzzy subsets F and B of X, we have fbcl(FvB)=fbcl(F)vfbcl(B).

Corollary 3.3: If A and B are fgbc sets such that D(A) ≤$D_{fb}(A)$ and D(B) ≤$D_{fb}(B)$, then AvB is fgbc.

Proof: Let U be fo set such that AvB≤U. Then since A and B are fgbc sets, we have fbcl(A) ≤U and fbcl(B) ≤U. Since D(A) ≤$D_{fb}(A)$,thus D(A)=$D_{fb}(A)$ and by lemma 3.1,cl(A)=fbcl(A). Similarly , Cl(B)=fbcl(B). Thus fbcl(AvB) ≤cl(AvB)=cl(A)vcl(B)=fbcl(A)vfbcl(B) ≤U, which implies AvB is fgbc. Let B≤A≤X. Then we say B is fgbc relative to A if $fbcl_A(B)$ ≤U, where B≤U and U is fo in A.

Theorem 3.2: Let B≤A≤X, where A is fgbc and fo. Then B is fgbc relative to A iff B is fgbc in X.

Proof: We first note that, since B≤A and A is both a fgbc and fo set ,then fbcl(A) ≤A and thus fbcl(B) ≤fbcl(A) ≤A. Now from the fact that AΛfbcl(B)=$fbcl_A(B)$, we have fbcl(B)=$fbcl_A(B)$ ≤A

If B is fgbc relative to A and U is fo subset of X such that B≤U, then B=BΛA≤UΛA, where UΛA is fo in A. Hence as B is fgbc relative to A , fbcl(B)=$fbcl_A(B)$ ≤UΛA≤U. So, B is fgbc in X.

Conversely, if B is fgbc in X and U is a fo subset of A such that B≤U, then U=VΛA for some fo subset V of X. As B≤V and B is fgbc in X,fbcl(B) ≤V. Thus $fbcl_A(B)$=fbcl(B)ΛA≤VΛA=U. So B is fgbc relative to A.

Corollary 3.4: Let A be fo,fgbc set. Then AΛF is fgbc whenever FϵfbC(X).

Proof: Since A is fgbc and fo, fbcl(A) ≤A and thus A is fbc. Hence AΛF is fbc which implies AΛF is fgbc in X.

Theorem 3.3: If A is fgbc set and B is any fuzzy set such that A≤B≤fbcl(A),then B is a fgbc set.

Proof: Let B≤U where U is fo set. Since A is fgbc and A≤U, then fbcl(A) ≤U and also fbcl(A)=fbcl(B). Hencefbcl(B) ≤U and so B is a fgbc set.

Theorem 3.4: A fuzzy subset A ≤X is fgbo iff F≤fbint(A), whenever F is fc and F≤A.

Proof: Let A be fgbo set and suppose F≤A, where F is fc. T hen 1-A is fgbc set contained in the fo 1-F. Hence fbcl(X-A) ≤1-F and 1-fbint(A) ≤1-F. Thus F≤fbint(A).

Conversely, if F is a fc set with F≤ fbint(A) and F≤A. Then 1-fbint(A) ≤1-F.Thus fbcl(1-A) ≤1-F.
Hence 1-A is fgbc set and A is a fgbo set.

4. Ap FUZZY b CONTINUOUS, ap FUZZY b CLOSED AND CONTRA FUZZY b CONTINUOUS MAPS.

We introduce the following notion.

Definition 4.1: A map f:X→Y is said to be approximately fuzzy b continuous(briefly ap-fb continuous) if fbcl(F) ≤ f^{-1}(U) whenever U is fo subset of Y and F is fgbc subset of X such that F≤ f^{-1}(U).

Definition 4.2: A map f:X→Y is said to be approximately fuzzy b closed (briefly ap-fb closed) if f(F) ≤fbint(V) whenever V is a fgbo subset of Y, F is a fc subset of X and f(F) ≤V.

Definition 4.3: A map f:X→Y is said to be approximately fuzzy b open (briefly ap-fb open) if fbcl(F) ≤f(U) whenever U ia a fo subset of X, F is a fgbc subset of Y and F≤ f(U).

Theorem 4.1: Let f:X→Y be a map. Then:
1. If f is contra fb continuous, then f is ap-fb continuous
2. If f is contra fb closed, then f is ap-fb closed
3. If f is contra fb open, then f is ap-fb open

Proof: 1) Let F≤f^{-1}(U), where U is fo in Y and F is a fgbc subset of X. So
fbcl(F) ≤ fbcl(f^{-1}(U)).Since f is contra fb continuous, we have fbcl(F) ≤ fbcl(f^{-1}(U))=f^{-1}(U). Hence f is ap-fb continuous.
2) Let f(F) ≤V, where F is a fc subset of X and V is a fgbo subset of Y. Therefore
f(F) =fbint(f(F)) ≤fbint(V). Thus f is ap-fb closed.
3)Let F≤f(U), where F is a fgbc subset of Y and U is a fo subset of X. Since f is contra fb open fbcl(F) ≤fbcl(f(U))=f(U). So f is ap-fb open.
The converse of theorem 4.1 is not true. The following examples illustrate them.

Example 4.1: Let X={a,b} .Let A be a fuzzy set defined by
A(a)=0.7,A(b)=0.6. Let τ={0,A,1}. Let f:(X,τ)→(X,τ) be the identity map. Then f is ap-fb continuous but not contra fb continuous.

Example 4.2: Take X,A,τ,f as above. Then f is ap-fb closed but not contra fb closed.

Example 4.3: Take X,A,τ,f as in example 4.1. Then f is ap-fb open but not contra fb open.
Definition 4.4: A function f:X→Y is said to be perfectly fuzzy continuous(respy perfectly fuzzy closed) if the inverse image(respy image) of every fo set in Y(respy fc set in X) is fuzzy clopen in X(respy fuzzy clopen in Y)

Theorem 4.2: Let f:X→Y be a function. Then:
1. If fo and fbc sets of X coincide, then f is ap-fb continuous iff f is contra fb continuous.

2. If fo and fbc sets of Y coincide, then f is ap-fb closed iff f is contra fb closed.
3. If fo and fbc sets of Y coincide, then f is ap-fb open iff f is contra fb open.

Proof: 1. Assume f is ap-fb continuous. Let A be an arbitrary fuzzy subset of X such that A≤U, where U is fo in X. Then by hypothesis fbcl(A) ≤fbcl(U)=U. Therefore all fuzzy subsets of X are fgbc(and hence all are fgbo).So, for any fo set V of Y, we have $f^{-1}(V)$ is fgbc in X. Since f is ap-fb continuous fbcl($f^{-1}(V)$) ≤$f^{-1}(v)$. Hence fbcl($f^{-1}(V)$)=$f^{-1}(V)$. Thus $f^{-1}(V)$ is fbc in X and f is contra fb continuous.
The converse is obvious from theorem 4.1.
2. Assume f is ap-fb closed. As in 1 we obtain all subsets of Y are fgbo. So, for any fc subset F of X,f(F) is fgbo in Y. Since f is ap-fb closed, we have f(F) ≤fbint(f(F)). Hence f(F)=fbint(f(F)) and hence f is contra fb closed.
The converse is obvious by theorem 4.1.
3. Analogous to 1 and 2 making obvious changes.

Lemma 4.1[2]: Let A be a fuzzy subset of X. Then:
1.fbcl(A)=fscl(A)Λfpcl(A)=Av[int(cl(A))vcl(int(A))]
2.fbint(A)=fsint(A)vfpint(A)=AΛ[int(cl(A))vcl(int(A))]
3. fbcl(1-A)=1-fbint(A)
4. fbint(1-A)=1-fbcl(A)

Theorem 4.3: If a map f:X→Y is surjective fb irresolute and ap-fb closed, then the inverse image of each fgbc(respy fgbo) set in Y is fgbc(respy fgbo) in X.

Proof: Let A be a fgbc subset of Y. Suppose that $f^{-1}(A)$ ≤U, where U is fo subset of X. Taking complements we have, 1_X-U≤$f^{-1}(1_Y\text{-}A)$ or f(1_X-U) ≤1_Y-A. Since f is ap-fb closed and by lemma 4.1, we have f(1_X-U) ≤fbint(1_Y-A)=1_Y-fbcl(A). It follows that 1_X-U≤1_X-f^{-1}(fbcl(A)) and hence f^{-1}(fbcl(A)) ≤U. Since f is fb irresolute f^{-1}(fbcl(A)) is fbc. Thus we have fbcl($f^{-1}(A)$) ≤ fbcl [f^{-1}(fbcl(A))] = f^{-1}(fbcl(A)) ≤U. This implies $f^{-1}(A)$ is fgbc in X. A similar argument shows that inverse images of fgbo sets are fgbo.

Theorem 4.4: If a map f:X→Y is surjective fb irresolute and ap-fb open , then the inverse image of each fgbo set in Y is fgbo in X.

Proof: Analogous to theorem 4.3 making obvious changes.

Theorem 4.5: If a map f:X→Y is ap-fb continuous and fb closed, then the image of each fgbc set in X is fgbc in Y.

Proof: Let F be a fgbc subset of X. Let f(F) ≤ V, where V is fo set in Y. Then F ≤ $f^{-1}(V)$ holds. Since f is ap-fb continuous, we have fbcl(F) ≤$f^{-1}(V)$. Then f(fbcl(F)) ≤V. So we have fbcl(f(F)) ≤fbcl[f(fbcl(F))]=f(fbcl(F)) ≤V.
Hence f(F) is fgbc in Y.

Theorem 4.6: If f:X→Y is fuzzy continuous and fb closed map, then f(A) is fgbc in Y for every fgbc subset A of X.

Proof: Let A be fgbc in X. Let f(A)≤ V, where V is fo in Y. Since f is fuzzy continuous $f^{-1}(V)$ is fo in X and $A \leq f^{-1}(V)$. Then we have $fbcl(A) \leq f^{-1}(V)$ and so f(fbcl(A)) ≤V. Since f is fb closed, f(fbcl(A)) is fbc in Y and hence fbcl(f(A)) ≤fbcl[f(fbcl(A))]=f(fbcl(A)) ≤V. This implies f(A) is fgbc in Y.

Definition 4.5: A map f:X→Y is said to be contra fb irresolute if $f^{-1}(U)$ is fbc in X for each fbo set U of Y.

Theorem 4.7: Let f:X→Y and g:Y→Z be two maps such that gof:X→Z,
1. If g is fb continuous and f is contra fb irresolute, then gof is contra fb continuous
2. If g is fb irresolute anf f is contra fb irresolute, then gof is contra fb irresolute

Proof: Straight forward
In an analogous way, we have the following

Theorem 4.8: Let f:X→Y and g:Y→Z be two maps such that gof:X→Z,
1. If f is fuzzy closed and g is ap-fb closed, then gof is ap-fb closed
2. If f is ap-fb closed g is fb open and g^{-1} preserves fgbo sets, then gof is ap-fb closed
3. If f is ap-fb continuous and g is fuzzy continuous, then gof is ap-fb continuous

Proof: 1. Let B be arbitrary fc subset of X and A be a fgbo subset of Z such that (gof)(B) ≤A. f(B) is fc in Y.
Since g is ap-fb closed,g(f(B)) ≤ fbint(A). This implies gof is ap-fb closed.
2. Let B be an arbitrary fc subset of X and A be a fgbo subset of Z such that (gof)(B) ≤A.
Hence $f(B) \leq g^{-1}(A)$.Then $f(B) \leq fbint(g^{-1}(A))$ because $g^{-1}(A)$ is fgbo and f is ap-fb closed.
Thus $(gof)(B)=g(f(B)) \leq g(fbint(g^{-1}(A)) \leq fbint(gg^{-1}(A)) \leq fbint(A)$. This implies gof is ap-fb closed.
3. Suppose F is an arbitrary fgbc subset of X and U be a fo subset of Z such that $F \leq (gof)^{-1}(U)$.
$g^{-1}(U)$ is fo in Y as g is fuzzy continuous. Since f is ap-fb continuous, we have
$fbcl(F) \leq f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$. This shows that gof is ap-fb continuous.

5. A CHARACTERIZATION OF fT_{gs} SPACE

In the following theorems, we give a characterization of a class of fuzzy topological space called fuzzy T_{gs} (briefly fT_{gs}) space by using the concepts of ap-fb continuous maps and ap-fb closed maps.

Definition 5.1[14]: A fuzzy topological space X is said to be fT_{gs} space iff every fgbc set is fbc.

Definition 5.2: A fuzzy topological space X is said to be $fT_{1/2}$ space if every fgc set is fc or equivalently every singleton is fo or fc.

Theorem 5.1: Every $fT_{1/2}$ space is fT_{gs} space.

Proof: Let X be $fT_{1/2}$ space and suppose A be a fuzzy subset of X is not a fbc set. Let xϵfbcl(A)-A. Then {x}≤fbcl(A)-A. Since X is $fT_{1/2}$,{x} is a fc set and hence by theorem 3.1, A is not a fgbc set. This proves the theorem.

Definition 5.3: A fuzzy topological space X is said to be $fbT_{1/2}$ space iff each singleton is either fbo or fbc.

Example 5.1: Let X={a,b}. Let A be a fuzzy subset of X defined by A(a)=0.3,A(b)=0.4. Let τ={0,A,1}. Every fgbc set is fbc. Hence X is T_{gs} space. Let B be a singleton fuzzy subset of X defined by B(a)=0.4,B(b)=0.0. It is not either fo or fc. Hence X is not $fT_{1/2}$ space.

Example 5.2: Let X={a,b}. Let A and B be fuzzy subsets of X defined by A(a)=0.7,A(b)=0.6. B(a)=0.7,B(b)=0.8. Let τ={0,A,1}. X is $fbT_{1/2}$ space. B is fgbc but not fbc. Hence X is not fT_{gs} space.

Theorem 5.2: For a fuzzy topological space X, if every fgbc subset of X is fc, then X is $fT_{1/2}$ space.

Proof: Let x϶X. If {x} is not fc, then A=1-{x} is not fo and A is fgb closed, since the only fo set containing A is 1. Thus A is fc and {x} is fo and so X is $fT_{1/2}$ space.

Definition 5.4: A map f:X→Y is said to be fgb continuous(fgb irresolute) if $f^{-1}(V)$ is fgbc in X for every fc (respy fgbc) set V of Y.
Clearly f:X→Y fgb continuous(respy fgb irresolute) if $f^{-1}(V)$ is fgbo in X for every fo(respy fgbo) set V of Y.

Theorem 5.3: Let f:X→Y be a map. Then:
1. If f is fgb irresolute and X is fT_{gs} space, then f is fb irresolute
2. If f is fgb continuous and X is fT_{gs} space, then f is fb continuous.

Proof: 1. Let V be fbc in Y. Then V is fgbc in Y and since f is fgb irresolute, then $f^{-1}(V)$ fgbc in X.Since X is fT_{gs} space, $f^{-1}(V)$ is fbc in X. Hence f is fb irresolute.
2. Similar to 1.

Theorem 5.4: If the bijective map f: X→Y is fb irresolute and fuzzy open, then f is fgb irresolute

Proof: Let V be fgbc and $f^{-1}(V) \le U$, where U is fo in X. Clearly $V \le f(U)$, since f(U) is fo(as f is fuzzy open map) and since V is fgbc in Y, then $fbcl(V) \le f(U)$ and thus $f^{-1}(fbcl(V)) \le U$. Since f is fb irresolute and fbcl(V) is a fbc set , then $f^{-1}(fbcl(V))$ is a fbc set in X. Thus $fbcl(f^{-1}(V)) \le fbcl[f^{-1}(fbcl(V))] = f^{-1}(fbcl(V)) \le V$. So $f^{-1}(V)$ is fgbc and f is fgb irresolute.

Theorem 5.5: If f:X→Y is a fuzzy open, fb irresolute and fb closed surjective map and X is a fT_{gs} space, then Y is fT_{gs} space.

Proof: Let F be fgbc set of Y. Let G be a fo subset of X such that $f^{-1}(F) \le G$. Then $F \le f(G)$ and f(G) is fo in Y. Since F is fgbc, then $fbcl(F) \le f(G)$ and $f^{-1}(fbcl(F)) \le G$. But f is fb irresolute,then $f^{-1}(fbcl(F))$ is fbc and $fbcl[f^{-1}(fbcl(F))]=f^{-1}(fbcl(F)) \le G$. Alsofbcl$(f^{-1}(F)) \le fbcl[f^{-1}(fbcl(F))] \le G$. Thus $f^{-1}(F)$ is fgbc in X. Since X is fT_{gs} space, $f^{-1}(F)$ is fbc in X. $F=f(f^{-1}(F))$ is fbc in Y. Hence Y is fT_{gs} space.

Theorem 5.6: Let X be a fuzzy topological space. Then the following are equivalent.
1.X is fT_{gs} space
2. For every fuzzy topological space Y and every map f:X→Y, f is ap-fb continuous.

Proof: 1. ⇒2. Let F be a fgbc subset of X and suppose that $F \leq f^{-1}(U)$,where U is fo in Y. Since X is fT_{gs} space, then F is fbc. Hence $fbcl(F) = F \leq f^{-1}(U)$. So f is ap-fb continuous.
2.⇒1. Let B be a fgbc subset of X and let Y be the fuzzy set with fuzzy topology σ={0,B,1}. Let f:X→Y be the identity map. By assumption f is ap-fb continuous. Since B is a fgbc set in X and fo in Y and $B \leq f^{-1}(B)$, it follows that $fbcl(B) \leq f^{-1}(B)=B$. Hence B is fbc in X.Hence X is a fT_{gs} space.

Theorem 5.7: Let Y be a fuzzy topological space. Then the following statements are equivalent:
1. Y is fT_{gs} space
2. For every fuzzy topological space X and every map f:X→Y,f is ap-fb closed(or ap-fb open)

Proof: Analogues to theorem 5.6 making obvious changes.

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