Journal of Computer and Mathematical Sciences, *Vol.8(12), 750-757 December 2017* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Multiplicative Revan and Multiplicative Hyper-Revan Indices of certain Networks

V. R. Kulli

Department of Mathematics,
Gulbarga University, Gulbarga 585106, INDIA.
email: vrkulli@gmail.com.

(Received on: December 3, 2017)

ABSTRACT

Recently, the Revan vertex degree concept was defined in Chemical Graph Theory. Furthermore, the Revan indices, connectivity Revan indices were introduced and studied well. In this paper, we propose the multiplicative Revan indices and determine exact formulae for oxide networks and honeycomb networks.

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*12.

Keywords: Revan vertex degree, multiplicative Revan indices, multiplicative hyper-Revan indices, oxide network, honeycomb network.

1. INTRODUCTION

A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have been considered in Theoretical Chemistry and many topological indices were defined by using vertex degree concept. The Zegreb, the Banhatti and the Gourava indices are the most degree based topological indices in Chemical Graph Theory. Very recently, Kulli[1] defined a novel degree concept in graph theory: the Revan vertex degree and determined exact formulae for oxide and honeycomb networks.

We consider only finite, simple and connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Let $\Delta(G)(\delta(G))$ denote the maximum (minimum) degree among the vertices of G. We refer to[2] for undefined term and notation.

The Revan vertex degree of a vertex v in G is defined as $r_G(v)=\Delta(G)+\delta(G) - d_G(v)$. The Revan edge connecting the Revan vertices u and v will be denoted by uv.

The first and second Revan indices were introduced by Kulli in[1]. They are defined as

$$R_1(G)=\sum_{uv\in E(G)}\left[r_G(u)+r_G(v)\right] \quad R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v).$$

We now define the first and second multiplicative Revan indices of a graph *G* as

$$RII_1(G)=\prod_{uv\in E(G)}\left[r_G(u)+r_G(v)\right] \qquad RII_2(G)=\prod_{uv\in E(G)} r_G(u)r_G(v).$$

We propose the multiplicative Revan vertex index of a graph *G* as

$$RII_{01}(G)=\prod_{u\in V(G)} r_G(u)^2.$$

Also we define the multiplicative Revan zero index of a graph *G* as

$$RII_0(G)=\prod_{u\in V(G)} r_G(u).$$

The first and second multiplicative hyper-Revan indices of a graph *G* are defined respectively as

$$HRII_1(G)=\prod_{uv\in E(G)}\left[r_G(u)+r_G(v)\right]^2, \qquad HRII_2(G)=\prod_{uv\in E(G)}\left[r_G(u)r_G(v)\right]^2.$$

The Revan indices were studied, for example, in[4,5,6]. Recently many topological indices were studied, for example, in[7,8,9,10,11,12,13]; the multiplicative indices were studied, for example, in[14,15,16,17,18,19,20,21,22,23,24]. In this paper, we initiate a study of some multiplicative Revan indices. For networks see[11] and references cited therein.

2. RESULTS FOR OXIDE NETWORKS

Oxide networks are very important in the study of silicate networks. A 5-dimensional oxide network is depicted in Figure 1. An oxide network of dimension n is denoted by OX_n.

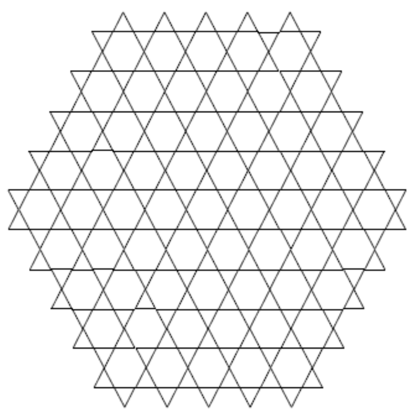

Figure 1. Oxide network of dimension 5

In the following theorem, we compute the formulae of $RII_0(OX_n)$ and $RII_{01}(OX_n)$ for oxide networks.

Theorem 1. Let OX_n be the oxide networks. Then

(1) $RII_0(OX_n) = 2^{9n(n+1)}$. (2) $RII_{01}(OX_n) = 2^{18n(n+1)}$.

Proof: Let G be the graph of oxide network OX_n. From Figure 1, we can see that the vertices of OX_n are either of degree 2 or 4. Therefore $\Delta(G)=4$ and $\delta(G)=2$. Thus $r_G(u) = 6 - d_G(u)$. By algebraic method, we obtain that $|V(OX_n)| = 9n^2+3n$ and $|E(OX_n)| = 18n^2$. We partition $V(G)$ into two sets, vertices of degree 2 and 4 respectively.

$V_2 = \{u \in V(G) \mid d_G(u) = 2\}$, $|V_2| = 6n$.

$V_4 = \{u \in V(G) \mid d_G(u) = 4\}$, $|V_4| = 9n^2 - 3n$.

Thus there are two types of Revan vertices as follows:

$RV_4 = \{u \in V(G) \mid r_G(u) = 4\}$, $|RV_4| = 6n$.

$RV_2 = \{u \in V(G) \mid r_G(u) = 2\}$, $|RV_2| = 9n^2 - 3n$.

1) By definition, we have $RII_0(G) = \prod_{u \in V(G)} r_G(u)$.

Thus, $$RII_0(OX_n) = \prod_{u \in RV_4} r_G(u) \times \prod_{u \in RV_2} r_G(u) = 4^{6n} \times 2^{9n^2-3n} = 2^{9n(n+1)}.$$

2) By definition, we have $RII_{01}(G) = \prod_{u \in V(G)} r_G(u)^2$.

Thus, $$RII_{01}(OX_n) = \prod_{u \in RV_4} r_G(u)^2 \times \prod_{u \in RV_2} r_G(u)^2 = \left(4^2\right)^{6n} \times \left(2^2\right)^{9n^2-3n} = 2^{18n(n+1)}.$$

In the following theorem, we compute the first and second multiplicative Revan indices, the first and second multiplicative hyper-Revan indices of oxide networks.

Theorem 2. Let OX_n be the oxide networks. Then

(1) $RII_1(OX_n) = 6^{12n} \times 4^{18n^2-12n}$. (2) $RII_2(OX_n) = 2^{36n^2+12n}$.

(3) $HRII_1(OX_n) = 6^{24n} \times 4^{36n^2-24n}$. (4) $HRII_2(OX_n) = 2^{72n^2+24n}$.

Proof: Let G be the graph of oxide network. By calculation, we obtain that G has $18n^2$ edges. In OX_n, by algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{24} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_{24}| = 12n$.

$E_{44} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_{44}| = 18n^2 - 12n$.

Clearly, we have $r_G(u) = \Delta(u) + \delta(u) - d_G(u) = 6 - d_G(u)$.

Thus there are two types of Revan edges based on the degree of the end Revan vertices of each Revan edge as follows:

$RE_{42} = \{uv \in E(G) \mid r_G(u) = 4, r_G(v) = 2\}$, $|RE_{42}| = 12n$.

$RE_{22} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 2\}$, $|RE_{22}| = 18n^2 - 12n$.

1) To compute $RII_1(OX_n)$, we see that

$$RII_1(OX_n) = \prod_{uv\in E(G)} \left[r_G(u)+r_G(v)\right] = \prod_{RE_{42}} \left[r_G(u)+r_G(v)\right] \times \prod_{RE_{22}} \left[r_G(u)+r_G(v)\right]$$

$$= (4+2)^{12n} \times (2+2)^{18n^2-12n} = 6^{12n} \times 4^{18n^2-12n}$$

2) To compute $RII_2(OX_n)$, we see that

$$RII_2(OX_n) = \prod_{uv\in E(G)} r_G(u)r_G(v) = \prod_{RE_{42}} \left[r_G(u)r_G(v)\right] \times \prod_{RE_{22}} \left[r_G(u)r_G(v)\right]$$

$$= 8^{12n} \times 4^{18n^2-12n} = 2^{36n^2+12n}$$

3) To compute $HRII_1(OX_n)$, we see that

$$HRII_1(OX_n) = \prod_{uv\in E(G)} \left[r_G(u)+r_G(v)\right]^2 = \prod_{RE_{42}} \left[r_G(u)+r_G(v)\right]^2 \times \prod_{RE_{22}} \left[r_G(u)+r_G(v)\right]^2$$

$$= \left(6^2\right)^{12n} \times \left(4^2\right)^{18n^2-12n} = 6^{24n} \times 4^{36n^2-24n}$$

4) To compute $HRII_2(OX_n)$, we see that

$$HRII_2(OX_n) = \prod_{uv\in E(G)} \left[r_G(u)r_G(v)\right]^2 = \prod_{RE_{42}} \left[r_G(u)r_G(v)\right]^2 \times \prod_{RE_{22}} \left[r_G(u)r_G(v)\right]^2$$

$$= \left(8^2\right)^{12n} \times \left(4^2\right)^{18n^2-12n} = 2^{72n^2+24n}$$

3. RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n where n is the number of hexagons between central and boundary hexagon. A honeycomb network of dimension 4 is depicted in Figure 2.

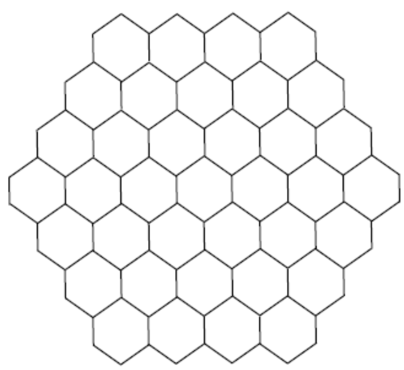

Figure 2. A 4-dimensional honeycomb network

In the following theorem, we compute the formulae of $RII_0(HC_n)$, $RII_{01}(HC_n)$ for honeycomb networks.

Theorem 3. Let HC_n be the honeycomb networks. Then

(1) $RII_0(HC_n) = 3^{6n} \times 2^{6n^2-6n}$ (2) $RII_{01}(HC_n) = 3^{12n} \times 2^{12n^2-12n}$.

Proof: Let H be the graph of honeycomb network HC_n. From Figure 2, we can see that the vertices of HC_n are either of degree 2 or 3. Therefore $\Delta(H) = 3$ and $\delta(H) = 2$. Thus $r_H(u) = 5 - d_H(u)$. By algebraic method, we obtain that $|V(HC_n)| = 6n^2$ and $|E(HC_n)| = 9n^2 - 3n$. We partition $V(H)$ into two sets, vertices of degree 2 and 3 respectively.

$V_2 = \{u \in V(H) \mid d_H(u) = 2\}$, $|V_2| = 6n$.

$V_3 = \{u \in V(H) \mid d_H(u) = 3\}$, $|V_3| = 6n^2 - 6n$.

Thus there are two types of Revan vertices as follows:

$RV_3 = \{u \in V(H) \mid r_H(u) = 3\}$, $|RV_3| = 6n$.

$RV_2 = \{u \in V(H) \mid r_H(u) = 2\}$, $|RV_2| = 6n^2 - 6n$.

1) By definition, we have $RII_0(H) = \prod_{u \in V(H)} r_H(u)$.

Thus, $RII_0(HC_n) = \prod_{u \in RV_3} r_H(u) \times \prod_{u \in RV_2} r_H(u) = 3^{6n} \times 2^{6n^2-6n}$.

2) By definition, we have $RII_{01}(H) = \prod_{u \in V(H)} r_H(u)^2$.

Thus, $RII_{01}(HC_n) = \prod_{u \in RV_3} r_H(u)^2 \times \prod_{u \in RV_2} r_H(u)^2 = 3^{12n} \times 2^{12n^2-12n}$.

In the following theorem, we compute the first and second multiplicative Revan indices, and the first and second multiplicative hyper-Revan indices of honeycomb networks.

Theorem 4. Let HC_n be the honeycomb networks. Then

(1) $RII_1(HC_n) = 6^6 \times 5^{12n-12} \times 4^{9n^2-15n+6}$ (2) $RII_2(HC_n) = 9^6 \times 6^{12n-12n} \times 4^{9n^2-15n+6}$

(3) $HRII_1(HC_n) = 6^{12} \times 5^{24n-24} \times 4^{18n^2-30n+12}$ (4) $HRII_2(HC_n) = 9^{12} \times 6^{24n-24} \times 4^{18n^2-30n+12}$.

Proof: Let H be the graph of honeycomb network. By calculation, we obtain that H has $9n^2 - 3n$ edges. In HC_n, by algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in E(H) \mid d_H(u) = d_H(v) = 2\}$, $|E_{22}| = 6$.

$E_{23} = \{uv \in E(H) \mid d_H(u) = 2, d_H(v) = 3\}$, $|E_{23}| = 12n - 12$.

$E_{33} = \{uv \in E(H) \mid d_H(u) = d_H(v) = 3\}$, $|E_{33}| = 9n^2 - 15n + 6$.

Clearly, we have $\Delta(H) = 3$ and $\delta(H) = 2$. Therefore $r_H(u) = 5 - d_H(u)$. Thus there are three types of Revan edges based on the degree of the end Revan vertices of each Revan edge as follows:

$RE_{33} = \{uv \in E(H) \mid r_H(u) = r_H(v) = 3\}$, $|RE_{33}| = 6$.
$RE_{32} = \{uv \in E(H) \mid r_H(u) = 3, r_H(v) = 2\}$, $|RE_{32}| = 12n - 12$.
$RE_{22} = \{uv \in E(H) \mid r_H(u) = r_H(v) = 2\}$, $|RE_{22}| = 9n^2 - 15n + 6$.

1) To compute $RII_1(HC_n)$, we see that

$$RII_1(HC_n) = \prod_{uv \in E(H)} \left[r_H(u) + r_H(v)\right]$$

$$= \prod_{RE_{33}} \left[r_H(u) + r_H(v)\right] \times \prod_{RE_{32}} \left[r_H(u) + r_H(v)\right] \times \prod_{RE_{22}} \left[r_H(u) + r_H(v)\right]$$

$$= (3+3)^6 \times (3+2)^{12n-12} \times (2+2)^{9n^2-15n+6}$$

$$= 6^6 \times 5^{12n-12} \times 4^{9n^2-15n+6}.$$

2) To compute $RII_2(HC_n)$, we see that

$$RII_2(HC_n) = \prod_{uv \in E(H)} r_H(u) r_H(v) = \prod_{RE_{33}} \left[r_H(u) r_H(v)\right] \times \prod_{RE_{32}} \left[r_H(u) r_H(v)\right] \times \prod_{RE_{22}} \left[r_H(u) r_H(v)\right]$$

$$= 9^6 \times 6^{12n-12} \times 4^{9n^2-15n+6}.$$

3) To compute $HRII_1(HC_n)$, we see that

$$HRII_1(HC_n) = \prod_{uv \in E(H)} \left[r_H(u) + r_H(v)\right]^2$$

$$= \prod_{RE_{33}} \left[r_H(u) + r_H(v)\right]^2 \times \prod_{RE_{32}} \left[r_H(u) + r_H(v)\right]^2 \times \prod_{RE_{22}} \left[r_H(u) + r_H(v)\right]^2$$

$$= (3+3)^{2\times 6} \times (3+2)^{2(12n-12)} \times (2+2)^{2(9n^2-15n+6)}$$

$$= 6^{12} \times 5^{24n-24} \times 4^{18n^2-30n+12}.$$

4) To compute $HRII_2(HC_n)$, we see that

$$HRII_2(HC_n) = \prod_{uv \in E(H)} \left[r_H(u) r_H(v)\right]^2$$

$$= \prod_{RE_{33}} \left[r_H(u) r_H(v)\right]^2 \times \prod_{RE_{32}} \left[r_H(u) r_H(v)\right]^2 \times \prod_{RE_{22}} \left[r_H(u) r_H(v)\right]^2$$

$$= 9^{12} \times 6^{24n-24} \times 4^{18n^2-30n+12}.$$

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