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# Morphism, Continuity and Linearity in Binary Č ech Closure Spaces

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**ABSTRACT**

In this paper we define and study the concept binary morphism, binary continuity in Binary Č ech Closure Spaces. Also we distinguish the concepts of morphism and continuity in Binary Č ech Closure Spaces. We further define Binary Linear Č ech Closure Spaces.

**AMS Subject Classification:** 54A05, 54A99.

**Keywords:** Binary Č ech closure spaces, binary morphism, binary continuity, binary linear Č ech closure space.

## 1. INTRODUCTION

Closure spaces were introduced by E.Čech[1] and then studied by many authors. He introduced the concept of continuity in closure spaces. T. A. Sunitha[3] renamed the continuity as morphism. In[10] we, Tresa and D. Susha introduced and studied the notion of Linear Čech closure spaces. P. Thangavelu and Nithanantha Jothi introduced the concept of binary topology[5]. In[11] we extended the concept of Čech closure spaces to Binary Čech Closure Spaces and in[9] we studied the concept of binary linear topological spaces(BLTS) and their properties. In this paper we define binary morphism and binary continuity in Binary Čech closure spaces and establish the difference between the two concepts. Also we combine the concepts of binary Čech closure and Linear Čech closure and introduce Binary Linear Čech Closure Space.

This paper is divided as follows: Section 2 contains the pre-requisites for the work. In section 3 we introduce binary morphism and mention its relevant properties. In section 4 we define binary continuity and establish the relation that every binary morphism is binary continuous. Also we prove that binary continuity neednot imply binary morphism. We define and introduce the concept and properties of Binary Linear Čech Closure Spaces in section 5.

## 2. PRELIMINARIES

**Definition 1** *[1] Let $X$ be a set and $\wp(X)$ be its powerset. A function $c: \wp(X) \rightarrow \wp(X)$ is called a Čech closure operator for $X$ if*

1. $c(\phi) = \phi$
2. $A \subseteq c(A)$
3. $c(A \cup B) = c(A) \cup c(B), \forall A, B \subseteq X$

Then $(X, c)$ is called Čech closure space or simply closure space.

If in addition

4. $c(c(A)) = c(A), \forall A \subseteq X,$

the space $(X, c)$ is called a Kuratowski (topological) space.

If further

5. for any family of subsets of $X, \{A_i\}_{(i\in I)}, c(\cup_{i\in I} A_i) = \cup_{i\in I} c(A_i)$, the space is called a total closure space.

A subset $A$ of a closure space $(X, c)$ will be closed if $c(A) = A$ and open if its complement is closed, i.e. if $c(X - A) = X - A$.

If $(X, c)$ is a closure space, we denote the associated topology on $X$ by $t$. i.e. $t = \{A^c : c(A) = A\}$.

**Definition 2** *[3] A map $f: (X, c) \rightarrow (Y, c')$ is said to be a $c - c'$ morphism or just a morphism if $f(c(A)) \subseteq c' f(A)$.*

**Definition 3** *[3] A homeomorphism is a bijective mapping $f$ such that both $f$ and $f^{-1}$ are morphisms.*

**Definition 4** *Let $V$ be a vector space and $c$ be a closure operator on $V$ such that*

1. $c(A) + c(B) \subseteq c(A + B), \forall A, B \subset V$
2. $\lambda c(A) \subseteq c(\lambda A), \forall A \subset V$ and $for\ all\ scalars\ \lambda$

Then $c$ is called a linear Čech closure operator and $(V, c)$ is called a linear Čech closure space (LČCS).

**Definition 5** *Let $X$ and $Y$ be two sets. A function $\breve{b}: \wp(X) \times \wp(Y) \rightarrow \wp(X) \times \wp(Y)$ is called a binary closure (monotone) operator if*

$\breve{b}(\phi, \phi) = (\phi, \phi)(A, B) \subseteq \breve{b}(A, B)(A, B) \subseteq (C, D) \Rightarrow \breve{b}(A, B) \subseteq \breve{b}(C, D)$.

Then $(X, Y, \breve{b})$ is called a binary closure (monotone) space. The binary closure operator is a Binary Čech Closure Operator(BČCO) if it satisfies the property

$\breve{b}[(A, B) \cup (C, D)] = \breve{b}(A, B) \cup \breve{b}(C, D)$. Then $(X, Y, \breve{b})$ is called a Binary Čech Closure Space(BČCS).

**Definition 6** *A set $(A, B) \in \wp(X) \times \wp(Y)$ is $\breve{b}$-closed if $\breve{b}(A, B) = (A, B)$ and a set $(C, D)$ is $\breve{b}$-open if $\breve{b}(X \backslash C, Y \backslash D) = (X \backslash C, Y \backslash D)$.*

Throughout this paper we consider vector spaces over the same field K.

## 3. BINARY MORPHISM

**Definition 7** *A binary mapping is a map* $f:(X_1,Y_1,\breve{b}_1)\to(X_2,Y_2,\breve{b}_2)$*, given by* $f(A,B)=(f_X(A),f_Y(B))$ *where* $f_X$ *is a mapping from* $X_1$ *to* $X_2$ *and* $f_Y$ *is a mapping from* $Y_1$ *to* $Y_2$*. It is denoted by* $f=(f_X,f_Y)$*.*
For $(C,D)\subseteq(X_2,Y_2), f^{-1}(C,D)=(f_X^{-1}C,f_Y^{-1}D)$. In other words $f^{-1}(C,D)=\{(x,y)\in(X_1,Y_1): f(x,y)\in(C,D)\}$

**Definition 8** *Let* $f=(f_X,f_Y)$ *be a binary mapping of a BČCS* $(X_1,Y_1,\breve{b}_1)$ *into a BČCS* $(X_2,Y_2,\breve{b}_2)$*. Then* $f$ *is a binary morphism if*
$f[\breve{b}_1(A,B)]\subseteq\breve{b}_2 f(A,B), \forall\ (A,B)\subseteq(X_1,Y_1)$

**Proposition 1** *Let* $f:(X_1,Y_1,\breve{b}_1)\to(X_2,Y_2,\breve{b}_2)$*. Then* $f$ *is a binary morphism if and only if for all* $(C,D)\subseteq(X_2,Y_2), \breve{b}_1[f^{-1}(C,D)]\subseteq f^{-1}[\breve{b}_2(C,D)]$*.*
*Proof.* Assume that $f[\breve{b}_1(A,B)]\subseteq\breve{b}_2 f(A,B), \forall\ (A,B)\subseteq(X_1,Y_1)$.
Now let $(C,D)\subseteq(X_2,Y_2)$ and $(E,F)=f^{-1}(C,D)$.
Then

$$\begin{aligned}&f[\breve{b}_1(E,F)]\subseteq\breve{b}_2 f(E,F)\\&=\breve{b}_2 f[f^{-1}(C,D)]\\&=\breve{b}_2(C,D)\\&\Rightarrow\breve{b}_1(E,F)\subseteq f^{-1}[\breve{b}_2(C,D)]\\&\text{i.e.}\ \breve{b}_1 f^{-1}(C,D)\subseteq f^{-1}[\breve{b}_2(C,D)]\end{aligned}$$

Conversely assume that $\breve{b}_1[f^{-1}(C,D)]\subseteq f^{-1}[\breve{b}_2(C,D)], \forall\ (C,D)\subseteq(X_2,Y_2)$.
Let $(A,B)\subseteq(X_1,Y_1), (C,D)=f(A,B)$ and $(A',B')=f^{-1}(C,D)$.
Then $\breve{b}_1(A',B')=\breve{b}_1[f^{-1}(C,D)]\subseteq f^{-1}[\breve{b}_2(C,D)]$

$$\Rightarrow f[\breve{b}_1(A',B')]\subseteq\breve{b}_2(C,D)=\breve{b}_2 f(A',B')$$

Now since $(A,B)\subseteq(A',B'), \breve{b}_1(A,B)\subseteq\breve{b}_1(A',B')$ and
$f[\breve{b}_1(A,B)]\subseteq f[\breve{b}_1(A',B')]$.
Hence

$$\begin{aligned}&f[\breve{b}_1(A,B)]\subseteq f[\breve{b}_1(A',B')]\\&\subseteq\breve{b}_2 f(A',B')\\&=\breve{b}_2 f(A,B)\end{aligned}$$

**Corollary 1** *Every binary mapping from a discrete BČCS into any BČCS is a binary morphism and every binary mapping of any BČCS into an indiscrete BČCS is a binary morphism.*
*Proof.* Let $f:(X_1,Y_1,\breve{b}_1)\to(X_2,Y_2,\breve{b}_2)$ be a binary mapping where $(X_1,Y_1,\breve{b}_1)$ is the discrete space.
Then $\breve{b}_1(A,B)=(A,B), \forall(A,B)\subseteq(X_1,Y_1)$.

Now $f\breve{b}_1(A,B) = f(A,B) \subseteq \breve{b}_2[f(A,B)]$, by the property of $\breve{b}_2$.
Thus $f\breve{b}_1(A,B) \subseteq \breve{b}_2[f(A,B)]$, showing that $f$ is a a binary morphism.
Assume that $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is a binary mapping where $(X_2,Y_2,\breve{b}_2)$ is the indiscrete space.
$f\breve{b}_1(\phi,\phi) = f(\phi,\phi) = (\phi,\phi) = \breve{b}_2(\phi,\phi) = \breve{b}_2 f(\phi,\phi)$.
Let $(A,B) \neq (\phi,\phi)$ and $f\breve{b}_1(A,B) = (C,D)$. Then $(C,D) \neq (\phi,\phi)$ and
$\breve{b}_2(C,D) = (X_2,Y_2)$,since the space is indiscrete.
Thus $f\breve{b}_1(A,B) \subseteq (X_2,Y_2) = \breve{b}_2[f(A,B)]$ for all $(A,B)(\neq(\phi,\phi)) \subseteq (X_1,Y_1)$, showing that $f$ is a a binary morphism.

**Corollary 2** *A BČCO $\breve{b}$ from a set $X$ to a set $Y$ is coarser than a BČCO $\check{c}$ from $X$ to $Y$, if and only if the identity mapping, $i:(X,Y,\check{c}) \to (X,Y,\breve{b})$ is a morphism.*
*Proof.* By definition of identity mapping we have,
$i[\check{c}(A,B)] \subseteq \breve{b}[i(A,B)], \forall\,(A,B) \in (X,Y) \Leftrightarrow \check{c}(A,B) \subseteq \breve{b}(A,B), \forall\,(A,B) \in (X,Y)$

**Proposition 2** *Composition of two binary morphisms is again a binary morphism.*
*Proof.* Let $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ and $g:(X_2,Y_2,\breve{b}_2) \to (X_3,Y_3,\breve{b}_3)$ be two binary morphisms. Then $g \circ f:(X_1,Y_1,\breve{b}_1) \to (X_3,Y_3,\breve{b}_3)$ is also a binary morphism.

$$(g \circ f)\breve{b}_1(A,B) = g[f\{\breve{b}_1(A,B)\}]$$
$$\subseteq g[\breve{b}_2\{f(A,B)\}]$$
$$\subseteq \breve{b}_3[g\{f(A,B)\}]$$
$$= \breve{b}_3[(g \circ f)(A,B)]$$

Thus $g \circ f$ is a binary morphism.

**Definition 9** *Let $f = (f_X, f_Y)$ be a one-one binary morphism of a BČCS $(X_1,Y_1,\breve{b}_1)$ onto a BČCS $(X_2,Y_2,\breve{b}_2)$. Then $f$ is a binary homeomorphism if $f^{-1} = (f_X^{-1}, f_Y^{-1})$ is a binary morphism from $(X_2,Y_2,\breve{b}_2)$ onto $(X_1,Y_1,\breve{b}_1)$.*

## 4. BINARY CONTINUITY

**Definition 10** *A binary mapping $f = (f_X, f_Y)$ from a Binary Čech closure space, $(X_1,Y_1,\breve{b}_1)$ to another BČCS $(X_2,Y_2,\breve{b}_2)$ is said to be binary continuous if for each binary point $(x,y)$ in $(X_1,Y_1,\breve{b}_1)$ the inverse image of every $\breve{b}_2$-neighbourhood of $f(x,y)$ is a $\breve{b}_1$-neighbourhood of $(x,y)$, or equivalently every $\breve{b}_2$-neighbourhood of $f(x,y)$ contains the image of a $\breve{b}_1$-neighbourhood of $(x,y)$.*

**Proposition 3** *A binary mapping, $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is binary continuous if and only if the inverse image of every $\breve{b}_2$-open subset is a $\breve{b}_1$-open subset.*
*Proof.* If $(U_2,V_2)$ is $\breve{b}_2$-open, then it is a $\breve{b}_2$-neighbourhood of each of its binary points. By the binary continuity of $f$, $f^{-1}(U_2,V_2)$ is a $\breve{b}_1$-neighbourhood of each of its binary points. Hence it is a $\breve{b}_1$-open subset.

Conversely suppose that the inverse image of every $\breve{b}_2$-open subset is a $\breve{b}_1$-open subset. Let $(x,y)$ in $(X_1,Y_1)$ and $(U_2,V_2)$ be a $\breve{b}_2$-neighbourhood of $f(x,y)$. Then there exists a $\breve{b}_2$-open set $(U,V)$ such that $f(x,y) \in (U,V) \subseteq (U_2,V_2)$. Then $f^{-1}(U,V)$ is $\breve{b}_1$-open and $(x,y) \in f^{-1}(U,V)$. Thus $(x,y) \in f^{-1}(U,V) \subseteq f^{-1}(U_2,V_2)$, showing that $f^{-1}(U_2,V_2)$ is a $\breve{b}_1$-neighbourhood of $(x,y)$.

**Corollary 3** *A binary mapping, $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is binary continuous if and only if the inverse image of every $\breve{b}_2$-closed subset is a $\breve{b}_1$-closed subset.*

*Proof.* Let $(C_2,D_2)$ be a $\breve{b}_2$-closed set. Then $(E_2,F_2) = (X_2 \backslash C_2, Y_2 \backslash D_2)$ is $\breve{b}_2$-open. $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is binary continuous if and only if $(E_1,F) = f^{-1}(E_2,F_2)$ is $\breve{b}_1$-open.
Let $(C_1,D_1) = f^{-1}(C_2,D_2) = (f_X^{-1}C_2, f_Y^{-1}D_2)$.
Now

$$\begin{aligned}(E_1,F_1) &= f^{-1}(E_2,F_2)\\ &= f^{-1}(X_2 \backslash C_2, Y_2 \backslash D_2)\\ &= (f_X^{-1}[X_2 \backslash C_2], f_Y^{-1}[Y_2 \backslash D_2])\\ &= (X_1 \backslash f_X^{-1}(C_2), Y_1 \backslash f_Y^{-1}(D_2))\\ &= (X_1 \backslash C_1, Y_1 \backslash D_1)\end{aligned}$$

Thus $(C_1,D_1)$ is $\breve{b}_1$-closed if and only if $(E_1,F_1)$ is $\breve{b}_1$-open.

**Proposition 4** *If a binary mapping $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is a binary morphism, then it is binary continuous.*

*Proof.* Let $(C,D)$ be a $\breve{b}_2$-closed set and $(A,B) = f^{-1}(C,D)$.
If $f$ is a binary morphism,
$f(\breve{b}_1(A,B)) \subseteq \breve{b}_2 f(A,B) = \breve{b}_2(C,D) = (C,D)$, since $(C,D)$ is $\breve{b}_2$-closed.
So $\breve{b}_1(A,B) \subseteq f^{-1}(C,D) = (A,B)$.
By definition of binary closure, $(A,B) \subseteq \breve{b}_1(A,B)$.
Hence $(A,B) = \breve{b}_1(A,B)$, showing that $(A,B)$ is $\breve{b}_1$-closed.
Thus the inverse image under $f$ of a $\breve{b}_2$-closed set is $\breve{b}_1$-closed proving the binary continuity of $f$.

**Remark 1** *But binary continuity neednot imply binary morphism.*
Consider the following example.
**Example 1** *Let* $X_1 = \{0,1,2\}, Y_1 = \{p,q\}$.
Let $\breve{b}_1: \wp(X_1) \times \wp(Y_1) \to \wp(X_1) \times \wp(Y_1)$ be defined as follows:
$\breve{b}_1(\{0\},\varphi) = (\{0,1\},\varphi)$, $\breve{b}_1(\{1\},\varphi) = (\{0,1\},\{p\})$, $\breve{b}_1(\{2\},\varphi) = (\{0,2\},\{q\})$, $\breve{b}_1(\varphi,\{p\}) = (\{0\},\{p\})$, $\breve{b}_1(\varphi,\{q\}) = (\{0\},\{q\})$
For all other $(A,B) \in \wp(X_1) \times \wp(Y_1)$,

$$\breve{b}_1(A,B) = [\cup_{x \in A} \breve{b}_1(\{x\},\varphi)] \cup [\cup_{y \in B} \breve{b}_1(\varphi,\{y\})]$$

Let $X_2 = \{3,4\}, Y_2 = \{r,s\}$.
Let $\breve{b}_2: \wp(X_2) \times \wp(Y_2) \to \wp(X_2) \times \wp(Y_2)$ be defined as follows:
$\breve{b}_2(\{3\},\varphi) = (\{3\},\{r\})$, $\breve{b}_2(\{4\},\varphi) = (\{4\},\{s\})$, $\breve{b}_2(\varphi,\{r\}) = (\{3\},\{r\})$, $\breve{b}_2(\varphi,\{s\}) = (\{3\},\{s\})$

For all other $(A,B) \in \wp(X_2) \times \wp(Y_2)$,

$$\breve{b}_2(A,B) = [\cup_{x\in A} \breve{b}_2(\{x\},\phi)] \cup [\cup_{y\in B} \breve{b}_2(\phi,\{y\})]$$

If $f:(X_1,Y_1,\breve{b}_1) \to (X_2,Y_2,\breve{b}_2)$ is a binary mapping $f=(f_X,f_Y)$ defined as $f_X(0) = 3, f_X(1) = 3, f_X(2) = 4, f_Y(p) = r, f_Y(q) = s$, then $f$ is binary continuous since the $\breve{b}_2$-closed sets are $(\phi,\phi),(X_2,Y_2),(\{3\},\{r\})$ and $(\{3\},Y_2)$ and their pre-images $(\phi,\phi) = f^{-1}(\phi,\phi),(X_1,Y_1) = f^{-1}(X_2,Y_2),(\{0,1\},\{p\}) = f^{-1}(\{3\},\{c\})$ and $(\{0,1\},Y_1) = f^{-1}(\{3\},Y_2)$ are $\breve{b}_1$-closed.

But $f$ is not a binary morphism since,

$f\breve{b}_1(\{2\},\phi) \not\subseteq \breve{b}_2 f(\{2\},\phi)$.

$$f\breve{b}_1(\{2\},\phi) = f(\{0,2\},\{q\}) = (X_2,\{s\})$$

but $\breve{b}_2 f(\{2\},\phi) = \breve{b}_2(\{4\},\phi) = (\{4\},\{s\})$.

## 5. BINARY LINEAR ČECH CLOSURE SPACES

**Definition 11** *A binary Čech closure operator, $\breve{v}$ between two vector spaces $V_1$ and $V_2$ over the same field $K$ is said to be binary linear if*

$\breve{v}(A,B) + \breve{v}(C,D) \subseteq \breve{v}(A+C,B+D)$ where $(A,B),(C,D) \subseteq (V_1,V_2)$ and

$\lambda\breve{v}(A,B) \subseteq \breve{v}(\lambda A,\lambda B), \lambda \in K$.

If $\breve{v}$ is a binary linear Čech closure operator between two vector spaces $V_1$ and $V_2$, then the triplet $(V_1,V_2,\breve{v})$ is called a binary linear Čech closure space(BLČCS).

**Proposition 5** *If $(V_1,V_2,\breve{v})$ is a binary linear Čech closure space(BLČCS), then for a fixed binary point $(a,b)$ in $(V_1,V_2)$,*

$T_{(a,b)}:(V_1,V_2,\breve{v}) \to (V_1,V_2,\breve{v})$ defined by $T_{(a,b)}(A,B) = (a,b) + (A,B)$

is a homeomorphism.

*Proof.* $T_{(a,b)}$ is a morphism if $T_{(a,b)}\breve{v}(A,B) \subseteq \breve{v}[T_{(a,b)}(A,B)]$ Here

$$\begin{aligned} T_{(a,b)}\breve{v}(A,B) &= (a,b) + \breve{v}(A,B) \\ &\subseteq \breve{v}(\{a\},\{b\}) + \breve{v}(A,B), \text{since}(a,b) \in (\{a\},\{b\}) \\ &\subseteq \breve{v}[(\{a\},\{b\}) + (A,B)], \text{since} \breve{v} \text{isaBL}\breve{C}\text{O} \\ &= \breve{v}[(a,b) + (A,B)] \\ &= \breve{v}[T_{(a,b)}(A,B)] \end{aligned}$$

Thus $T_{(a,b)}$ is a morphism. Since $V_1$ and $V_2$ are vector spaces for $(a,b)$ in $(V_1,V_2)$ there exists $(-a,-b)$ in $(V_1,V_2)$ and $T_{(-a,-b)}$ is the inverse morphism for $T_{(a,b)}$. Hence $T_{(a,b)}$ is a homeomorphism.

**Corollary 4** *If $(V_1,V_2,\breve{v})$ is a binary linear Čech closure space(BLČCS), then for a fixed $\lambda \in K, \lambda \neq 0$,*

$M_\lambda:(V_1,V_2,\breve{v}) \to (V_1,V_2,\breve{v})$ defined by $M_\lambda(A,B) = \lambda(A,B)$ is a homeomorphism.

**Proposition 6** *Let $(V_1,V_2,\breve{v})$ is a binary linear Čech closure space(BLČCS). A binary set $(A,B)$ is $\breve{v}$-open if and only if $(a,b)+(A,B)$ is $\breve{v}$-open for all $(a,b) \in (V_1,V_2)$.*

*Proof.* Since $T_{(a,b)}$ is a homeomorphism, for all $(a,b) \in (V_1, V_2)$ and every morphism is continuous by Proposition 4, $(A,B)$ is v̆-open if and only if $(a,b) + (A,B)$ is v̆-open.

**Proposition 7** *Let $(V_1, V_2, \breve{v})$ is a binary linear Čech closure space(BLČCS). The sum of any binary set and a $\breve{v}$-open set is $\breve{v}$-open in $(V_1, V_2, \breve{v})$.*

*Proof.* Let $(A,B), (C,D) \subseteq (V_1, V_2)$ and $(C,D)$ is v̆-open.

Since $(C,D)$ is v̆-open, $(a,b) + (C,D)$ is v̆-open for all $(a,b) \in (A,B)$.

Then $(A,B) + (C,D) = \cup \{(a,b) + (C,D) : (a,b) \in (A,B)\}$ is v̆-open, being arbitrary union of v̆-open sets.

**Proposition 8** *The composition of two binary linear Čech closure operators is again a binary linear Čech closure operator.*

*Proof.* The composition of two binary Čech closure operators is again a binary Čech closure operator. Also the composition of two linear Čech closure operators is again a linear Čech closure operator. Hence the composition of two binary linear Čech closure operators is again a binary linear Čech closure operator.

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