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Ternary Semigroups T Satisfying the Identity $aba = b$ For All $a, b \in T$

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ABSTRACT

N. Kuroki introduced and studied the notion of fuzzy semigroups. He also studied the concept of fuzzy quasi-ideals and fuzzy bi-ideals of semigroups. In this paper, we pertaining the identity $aba = b$ on ternary semigroups and we proved some properties on ternary semigroups. Mainly we prove that if μ is a left ideal of T and semi-group T is regular then μ is a fuzzy bi ideal in ternary semi-group T .

Keywords: ternary semigroup, ideal, bi ideal, sub semigroup.

1. INTRODUCTION

The formal study of semigroups began in the early 20th century. Semiring theory stands with a foot in each of two mathematical domains. On one hand, semirings are abstract mathematical structures and their study is part of abstract algebra - arising from the work of Dedekind, Macaulay, krull, and others on the theory of ideals of a commutative ring and then through the more general work of Vandiver and the tools used to study them are primarily the tools of abstract algebra. On the other, the modern interest in semirings arises primarily from fields of applied Mathematics such as optimization theory, the theory of discrete-event dynamical systems, automata theory, and formal language theory, as well as from the allied areas of theoretical computer science and theoretical physics, and the questions being asked are, for the most part, motivated by applications. In 1932, Lehmer introduced the concept of a ternary semigroup. He investigated certain ternary algebraic structures called triplexes. Santiago developed the theory of ternary semigroups and semiheaps. He studied regular and completely regular ternary semigroups. The notion of quasi ideals and bi-ideals in ternary semigroups presented by Dixit and Dewan, Kar. Maity investigated congruences of ternary semigroups and Iampan studied minimal and maximal lateral ideals of ternary semigroups. In1965 L.A Zadeh defined fuzzy sets in order to study mathematical vague situations. S. Kar and P. Sarkar studied Fuzzy quasi-ideals and fuzzy bi-ideals of ternary semigroups. This paper consists properties of fuzzy sets in ternary semigroups. We prove that the conditions for a ternary semigroup T with fuzzy structures. Through out the paper ternary semigroup T satisfing the identity $aba = b$ for any $a, b \in T$.

Definition 1.1: A non-empty set T is said to be Γ - ternary semigroup if there exists a ternary operation $\cdot : T \times T \times T \to T$ written as $(a,b,c) \to a.b.c$ satisfies the following identity $ab(cde) = a(bcd)e = (abc)de$ for any $a,b,c,d,e \in T$.

Definition 1.2 : Fuzzy subset of a non empty set is a collection of objects with each object being assigned a value between 0 and 1 by a membership function.

Definition 1.3: Let X is a non empty set. A fuzzy set μ of the set X is a function μ : $X \to [0,1]$.

Definition 1.4 : Let X is a semi-group. A map A from T to [0,1] is called a fuzzy set in T .

Definition 1.5: Let $F(T)$ denote the set of all fuzzy sets in T . For A,B,C$\in$ $F(T)$, $A \subseteq B$ and $B \subseteq C$ if and only if $A(x) \le B(x)$ *and* $B(x) \le C(x)$ in the ordering of [0,1], $\forall x \in T$.

Definition1.6: A fuzzy set $A \in F(T)$ is said to be a fuzzy sub semi-group of T if $A(xyz) \ge \min\{A(x), A(y), A(z)\} \forall x, y, z \in T$.

Definition1.7: A fuzzy set $A \in F(T)$ is said to be a fuzzy left (resp., lateral and right) ideal of T if $A(xyz) \ge A(z)$, (resp., $A(xyz) \ge A(x)$, and $A(xyz) \ge A(y)$) $\forall x, y, z \in T$.

Definition1.8: A fuzzy sub semi group $A \in F(T)$ is said to be A fuzzy bi ideal of T if $A(xuyvz) \ge \min\{A(x), A(y), A(z)\}$, $\forall x, y, z, u, v \in T$.

Definition 1.9: A non-empty subset A of an ordered ternary semi-group T is called a Left (lateral, right) ideal, of T .
If it satisfies following:
1) $TTA \subseteq A (ATT \subseteq A, TAT \subseteq A\ respectively)$

2) If $a \in A \,\&\, b \in T$ such that $b \leq a$ then $b \in A$.

If A is a Left (Right) and lateral by $[A]$ and subset of T is defined by,

$$[A] = \{t \in T \,/\, t \leq a \text{ for some } a \in A\}$$

Definition 1.10: A fuzzy subset f of an ordered ternary semi-group T is called fuzzy left (right, lateral) ideal of T .if

1. $f(abc) \geq f(c)$ ($f(abc) \geq f(a)$ and $f(abc) \geq f(b)$); $\forall a,b,c \in T$.
2. If $a \leq b$, then $f(a) \geq f(b)$, $\forall a,b \in T.$,

Definition1.11: Let f and g be two fuzzy subset of T , define the relation $\subseteq$ between f and g respectively as,

$f \subseteq g$ if $f(x) \leq g(x)$

$(f \cup g)(x) = \max\{f(x), f(y)\}$

$(f \cap g)(x) = \min\{f(x), g(x)\}, \forall x \in T$

Definition1.12: Let a be the element of an ordered ternary semi-group T then we define new set $A_a = \{(x,y,z) \in T \times T \times T \,/\, a \leq xyz\}$

Definition1.13: The product of three fuzzy subsets f, g, h of a ternary semi group T is defined in the following way,

$$(f \circ g \circ h)(a) = \begin{cases} \underset{(u,v,w) \in A_a}{Sup} \{\min\{f(u), g(v), h(w)\}\} & if \;\; A_a \neq \phi. \\ 0 & if \;\; A_a = \phi. \end{cases}$$

Definition1.14: A ternary semi group T is called fuzzy simple, if every fuzzy ideal of T is a constant function.

i.e; For every fuzzy ideal f of T , we have $f(a) = f(b), \forall a,b \in T.$

Definition1.15: A fuzzy subset f of a semi group T is called quasi-prime if for any fuzzy right ideals $A, B,$ and C of T , $A \circ B \circ C \subset f$ implies $A \subset f$ or $B \subset f$ or $C \subset f$

Definition1.16: A fuzzy subset f of a ternary semi-group T is called Quasi-semi prime, if any fuzzy right ideal A of T where f is not constant function then $A \circ A \circ A \subset f$ implies $A \subset f$

2. MAIN RESULTS

Theorem 2.1: Let T be regular ternary semigroup and satisfy an identity $aba = b, \forall a, b \in T.$

i. $\mu(abc) = \mu(a)$

ii. $\mu(a^3) = \mu(a)$

iii. $\mu(a^{2n+1}) = \mu(a)$

for any non empty fuzzy sub set μ of T .

Proof: Let S be a regular semigroup then we know that $axaya = a, \forall a \in T$ and for some $x \in T$.

Given T satisfy $aba = b, \forall a, b \in T.$

i. Consider , $\mu(abc) = \mu(ab(aca))$ $(\because aca = c)$

$= \mu(abaca)$ for some $b, c \in T$

$= \mu(a)$

ii. Consider , $\mu(a^3) = \mu(aaa)$ $(aba = b, \forall a, b \in T)(\because b = a)$

$= \mu(a)$

iii. Consider , $\mu(a^{2n+1})$, where n=1, 2, 3……

Let us prove by mathematical induction

For n=1, $\mu(a^{2(1)+1}) = \mu(a^3) = \mu(aaa) = \mu(a)$ (proved above)

For n=2, $\mu(a^5) = \mu(a^3a^2) = \mu(a^2a^3) = \mu(aaaaa) = \mu(a)$ $(\because a = x, a = y)$ since T is regular.

In general $\mu(a^{2n+1}) = \mu(a^n a a^n) = \mu(a).$

Thus $\mu(a^{2n+1}) = \mu(a).$

Theorem 2.2: Let μ be a left ideal of T and semi group T satisfy an identity $aba = b, \forall a, b \in T$ then μ is a fuzzy sub semi group T .

Proof: Let μ be a left ideal of T .

$$\mu(xyz) \geq \mu(z),\ forsome\ x, y, z \in T \tag{1}$$

And S satisfies an identity $aba = b, \forall a, b \in T.$

To prove μ is a fuzzy sub semi group.

i.e, $\mu(xyz) = \mu(xyxzx)$ $(\because z = xzx)$

$= \mu((xyx)zx)$ from (1)

$$= \mu(x) \tag{2}$$

$\mu(xyz) = \mu(xyyzy)$ $((\because z = yzy))$

$= \mu((xyy)zy)$ from (1)

$= \mu(y)$ (3)

From Esq. (2) and (3)

$\mu(xyz) \wedge \mu(xyz) \geq \mu(x) \wedge \mu(y)$

$\mu(xyz) \geq \mu(x) \wedge \mu(y)$

$\mu(xyz) \wedge \mu(xyz) \geq \mu(x)\mu(y)\mu(xyz)$ From (1)

$\mu(xyz) \geq \mu(x)\mu(y)\mu(z), \forall x, y, z \in T$

$\therefore$ Thus μ is a fuzzy sub semi group of T .

Theorem 2.3 : Let μ be a fuzzy subset of a semi group T and T satisfy an identity $aba = b, \forall a, b \in T$ then $\mu \circ \mu \circ \mu \leq \mu$.

Proof: Let μ be a fuzzy subset in a semi group T .

Given T satisfy the identity $aba = b, \forall a, b \in T$. .

We know that, $\mu(abc) \geq \mu(a) \wedge \mu(b) \wedge \mu(c)$, $\forall a, b, c \in T$

Now, $\mu \circ \mu \circ \mu(x) = \vee\{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$ [Let $x = abc$]

$= \vee\{\mu(a) \wedge \mu(b) \wedge \mu(c)\}$

$= \mu(a) \wedge \mu(b) \wedge \mu(c)$

$= \mu(abc)$

$= \mu(x)$

$\therefore \mu \circ \mu \circ \mu \leq \mu(x)$.

Theorem 2.4 : Let μ be a fuzzy left ideal of T and semi group T satisfy an identity $aba = b, \forall a, b \in T$ then

$\mu(abc) = \mu(bca) = \mu(cab) = \mu(acb) = \mu(cba) = \mu(bac).$

Proof: Let μ be a fuzzy sub semi-group of T .

And T satisfies identity $aba = b, \forall a, b \in T$.

Consider, $\mu(abc) = \mu(abaca)$ $(\because c = aca)$

$= \mu((aba)ca)$

$= \mu(bca)$

And $\mu(abc) = \mu(cacbc)$ $(\because a = cac)$

$= \mu(c(acb)c)$

$= \mu(acb)$

$\mu(abc) = \mu(cacbc)$ $(\because a = cac)$

$= \mu(ca(cbc))$

$= \mu(cab)$

Similarly $\mu(abc) = \mu(cba) = \mu(bac)$

$\therefore \mu(abc) = \mu(bca) = \mu(acb) = \mu(cab) = \mu(cba) = \mu(bac).$

Theorem 2.5: Let μ be a left ideal of T and semi-group T is regular. Then μ is a fuzzy bi ideal in ternary semi-group T .

Proof: Let μ be a left ideal of semi-group T .

$$\mu(xyz) \geq \mu(z),\ for some\ x, y, z \in T \tag{1}$$

T is regular, then $axaya = a, \forall a \in T$

Since from above theorem

$$\mu(xyz) = \mu(xzy) \geq \mu(y) \tag{2}$$

And $\mu(xyz) = \mu(yzx) \geq \mu(z)$ (3)

And now $\mu(xuyvz) = \mu(x(uyv)z) \geq \mu(z)$ (4)

$$\mu(xuyvz) = \mu(x(uzv)y) \geq \mu(y) \tag{5}$$

$$\mu(xuyvz) = \mu(x)$$

From eqs (1), (2) & (3)

$$\mu(xyz) \wedge \mu(xyz) \wedge \mu(xyz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z)$$

$$\mu(xyz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z)$$

(Or)

$$\mu(xuyvz) = \min\{\mu(x), \mu(y), \mu(z)\}$$

$\therefore \mu$ is a fuzzy bi ideal in a ternary semigroup T .

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