Journal of Computer and Mathematical Sciences, *Vol.10(4),738-747 April 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Some New Subclasses of Bi-Univalent Functions Defined by Al-Oboudi Differential Operator

T. Stalin[*1], M. Thirucheran[2] and M. Vinoth Kumar[3]

[1]Assistant Professor, Department of Mathematics,
Gojan School of Business and Technology, Redhills-600052.
Anna University, Chennai, INDIA.
[2]Associate Professor and Head, Post Graduate and Research Department of Mathematics,
LN Government College, Ponneri-601204, University of Madras, Chennai, INDIA.
[3]Research Scholar, Post Graduate and Research Department of Mathematics,
LN Government College, Ponneri-601204, University of Madras, Chennai, INDIA.

(Received on: April 3, 2019)

ABSTRACT

The main object of this paper is investigating a new subclass of bi-univalent function in the open unit disk U which is defined by Al-Oboudi Differential Operator. And obtained the initial two Taylor -McLaurin co-efficient $|a_2|$ and $|a_3|$ for the subclass $S_{\Sigma}^{m,n,b,¢}(\sim)$ of Bi-Univalent function.

2010 Mathematics Subject classification: 30C45, 30C55 and 30C80.

Keywords: Analytic functions, univalent functions, bivalent functions, and Al-Oboudi differential operator.

1. INTRODUCTION AND DEFINITIONS

Definition 1.1

Let A denote the class of functions f normalized by

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j \tag{1.1}$$

Which are analytic in the open unit disc $\mathrm{U} = \{z \in C : |z| < 1\}$.

For $f \in A$, Al-Oboudi[1] introduces the following operator.

$$D^0 f(z) = f(z),$$

$$D' f(z) = (1 - ¢) f(z) + ¢\, z f'(z) = D_{¢} f(z), \quad ¢ \geq 0$$

$$D^n f(z) = D_{¢}(D^{n-1} f(z)), \qquad n \in N = 1,2,3,....$$

$$\therefore D^n f(z) = z + \sum_{j=2}^{\infty} [1 + (j-1)¢]^n a_j z^j \ , \qquad n \in N_0 = N \cup \{0\}. \tag{1.2}$$

If $¢ = 1$, then we get Salagean[7] differential operator.

Koebe One -Quarter Theorem[5]

The range of every function of class A contains the disk of radius $\left\{ w : |w| < \frac{1}{4} \right\}$.

It is well known that every function $f \in A$ has an inverse f^{-1} defined by $f^{-1}(f(z)) = z$ and $f(f^{-1}(w)) = w$, $\left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4} \right)$.

For this inverse function f^{-1}, we have:

$$g(w) := f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2^3 - 5a_2 a_3 + a_4) w^4 + ... \tag{1.3}$$

Definition 1.2

If both the function f and its inverse function f^{-1} are univalent in U, then the function f is called bi-univalent**.**

For example, $\frac{z}{1-z}$,$-\log(1-z)$,$\frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$, and so on.

However, the familiar Koebe function is not a bi-univalent function.

For example, $z - \frac{z^2}{2}, \frac{z}{1-z^2}$, and so on.

Let the class $\sum$ of bi-univalent function first investigated by Levin[8] and found that $|a_2| < 1.51$.

Afterward, Brannan and Clunie[2] conjectured that $|a_2| \leq \sqrt{2}$.

Later, Brannan and Taha[3] introduced the new subclass of bi-univalent function of the class $\sum$ like the familiar subclasses $S^*(\sim)$ and $C(\sim)$ of starlike and convex functions of $\sim$. $(0 \leq \sim < 1)$,respectively. If a function $f \in A$ is in the class $S^*_{\Sigma}(\sim)$ of strongly bi-starlike function of order $\sim$. $(0 \leq \sim < 1)$ if each of the following conditions satisfied

$$f(z) \in \sum \text{ and } \left|\arg\left(\frac{zf'(z)}{f(z)}\right)\right| < \frac{\sim ®}{2} \quad , \quad (z \in U)$$

And

$$\left|\arg\left(\frac{wg(z)}{g(w)}\right)\right| < \frac{\sim ®}{2}, \quad (w \in U)$$

Where the function g is the extension of f^{-1} to U.

Similarly, a function $f \in A$ is in the class $C_{\Sigma}(\sim)$ of strongly bi-convex function of order $\sim$. $(0 \le \sim < 1)$ if each of the following conditions satisfied

$$f(z) \in \sum \text{ and } \left|\arg\left(1+\frac{zf''(z)}{f'(z)}\right)\right| < \frac{\sim ®}{2}, \qquad (z \in U)$$

And

$$\left|\arg\left(1+\frac{wg''(z)}{g'(w)}\right)\right| < \frac{\sim ®}{2}, \quad (w \in U)$$

Where the function g is the extension of f^{-1} to U. For each function $S_{\Sigma}^{*}(\sim)$ and $C_{\Sigma}(\sim)$. They found non-sharp estimates on the first two Taylor -McLaurin co-efficient $|a_2|$ and $|a_3|$.

Recently, Qing-Hua Xu, Ying-Chun Gui and H.M. Srivastava[9], B.A. Frasin and M.K. Aouf[6], Seker.B[10] investigated some subclasses of bi-univalent function and obtained non-sharp estimates on the first two co-efficient.

By motivated this study we introduce and obtained the initial two co-efficient $|a_2|$ and $|a_3|$ for the subclass $S_{\Sigma}^{m,n,b,¢}(\sim)$.

Definition: 1.3

A function $f(z)$ given by (1.1) is said to be in the class

$f \in S_{\Sigma}^{m,n,b,¢}(\sim)$, $(m \in N, n \in N_0, m > n,\ 0 < \sim \le 1, b \in C - \{0\}, ¢ \ge 0$

if the following conditions are satisfied:

$$f(z) \in A \text{ and } \left|\arg\left(1+\frac{1}{b}\left(\frac{D^m f(z)}{D^n f(z)}-1\right)\right)\right| < \frac{\sim ®}{2} \quad , \qquad (z \in U) \tag{1.4}$$

And

$$\left|\arg\left(1+\frac{1}{b}\left(\frac{D^m g(w)}{D^n g(w)}-1\right)\right)\right| < \frac{\sim ®}{2} \qquad , \quad (w \in U) \tag{1.5}$$

Where the function g is given by (1.3).

Definition: 1.4

A function $f(z)$ given by (1.1) is said to be in the class

$f \in S_{\Sigma}^{m,n,b,\text{¢}}(\sim)$, $(m \in N, n \in N_0, m > n,\ 0 < \sim \leq 1, b \in C - \{0\}, \text{¢} \geq 0$

if the following conditions are satisfied:

$$f(z) \in A \text{ and } \Re\left(1 + \frac{1}{b}\left(\frac{D^m f(z)}{D^n f(z)} - 1\right)\right) > \text{¥} \quad , \ (z \in U) \qquad (1.6)$$

And

$$\Re\left(1 + \frac{1}{b}\left(\frac{D^m g(w)}{D^n g(w)} - 1\right)\right) > \text{¥} \qquad , \qquad (w \in U) \qquad (1.7)$$

Where the function g is given by (1.3).

Definition: 1.5

A function $h(z), p(z) : U \to C$ satisfy the conditions

$\min\{\Re(h(z)), \Re(p(z))\} > 0$, $z \in U$ and $h(0) = p(0) = 1$

For a function $f \in A$ defined by (1.1) ,

we say that $f \in S_{\Sigma}^{m,n,b,\text{¢}}(\sim)$, $(m \in N, n \in N_0, m > n,\ 0 < \sim \leq 1, b \in C - \{0\}, \text{¢} \geq 0$

if the following conditions are satisfied:

$$f(z) \in A \text{ and } \left(1 + \frac{1}{b}\left(\frac{D^m f(z)}{D^n f(z)} - 1\right)\right) \in h(z) \quad , \ (z \in U) \qquad (1.8)$$

And

$$\left(1 + \frac{1}{b}\left(\frac{D^m g(w)}{D^n g(w)} - 1\right)\right) \in p(z) \qquad , \qquad (w \in U) \qquad (1.9)$$

Where the function g is given by (1.3).

Remarks

(i). $S_{\Sigma}^{m,n,1,1}(\sim) = H_{\Sigma}^{m,n}(\sim)$,(Seker.B[10])

(ii). $S_{\Sigma}^{1,0,1,1}(\sim) = S_{\Sigma}^{*}(\sim)$, (Brannan and Taha[3])

(iii). $S_{\Sigma}^{2,1,1,1}(\sim) = C_{\Sigma}(\sim)$, (Brannan and Taha[3])

2. MAIN RESULT

To derive our main results, we should recall the following lemma[4].

Lemma 2.1

Let $h \in P$ the family of all functions h analytic in U for which $\mathrm{Re}\{h(z) > 0\}$ and have the form $h(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + ...$ for $z \in U$ .Then $|p_n \le 2|$, for each n.

Theorem 2.2

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,¢}(h,p)$ .Then

$$|a_2| \le \min\left\{\frac{2|b|}{[(1+¢)^m - (1+¢)^n]}, \sqrt{\frac{2|b|}{[((1+2¢)^m - (1+2¢)^n) - ((1+¢)^{m+n} - (1+¢)^{2n})]}}\right\} \quad (2.1)$$

and

$$|a_3| \le \min\left\{\begin{array}{l}\frac{4|b|^2}{[(1+¢)^m - (1+¢)^n]^2} + \frac{2|b|}{[(1+2¢)^m - (1+2¢)^n]}, \\ \frac{2|b|}{[((1+2¢)^m - (1+2¢)^n) - ((1+¢)^{m+n} - (1+¢)^{2n})]} + \frac{2|b|}{((1+2¢)^m - (1+2¢)^n)}\end{array}\right\} \quad (2.2)$$

Proof

Let consider the function h and p Satisfying the conditions of the definition (1.5) with the form

$$h(z) = 1 + h_1 z + h_2 z^2 + h_3 z^3 + ... \ , z \in U$$

and

$$p(w) = 1 + p_1 w + p_2 w^2 + p_3 w^3 + ... \ , \ w \in U \ \text{respectively.}$$

Since $f \in S_{\Sigma}^{m,n,b,¢}(h,p)$,then

$$\left(1 + \frac{1}{b}\left(\frac{D^m f(z)}{D^n f(z)} - 1\right)\right) = h(z) \ , \ (z \in U) \quad (2.3)$$

and

$$\left(1 + \frac{1}{b}\left(\frac{D^m g(w)}{D^n g(w)} - 1\right)\right) = p(z), \quad (w \in U) \quad (2.4)$$

respectively.

By equating the coefficient of (2.3 and (2.4), we get

$$((1+¢)^m - (1+¢)^n)a_2 = bh_1 \quad (2.5)$$

$$\left((1+2¢)^m-(1+2¢)^n\right)a_3-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)a_2^2=bh_2 \tag{2.6}$$

$$-\left((1+¢)^m-(1+¢)^n\right)a_2=bp_1 \tag{2.7}$$

and

$$-\left((1+2¢)^m-(1+2¢)^n\right)a_3+\left[2\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]a_2^2=bp_2 \tag{2.8}$$

From (2.5)&(2.7),

$$h_1=-p_1 \tag{2.9}$$

and

$$2\left[(1+¢)^m-(1+¢)^n\right]^2a_2^2=b^2(h_1^2+p_1^2) \tag{2.10}$$

$$\therefore a_2^2=\frac{b^2(h_1^2+p_1^2)}{2\left((1+¢)^m-(1+¢)^n\right)^2} \tag{2.11}$$

From (2.6)&(2.8),

$$2\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]a_2^2=b(h_2+p_2) \tag{2.12}$$

$$\therefore\ a_2^2=\frac{b(h_2+p_2)}{2\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]} \tag{2.13}$$

$$2\left((1+2¢)^m-(1+2¢)^n\right)a_3=2\left((1+2¢)^m-(1+2¢)^n\right)a_2^2+b(h_2-p_2) \tag{2.14}$$

$$\therefore a_3=\frac{b^2(h_1^2+p_1^2)}{2\left((1+¢)^m-(1+¢)^n\right)^2}+\frac{b(h_2-p_2)}{2\left((1+2¢)^m-(1+2¢)^n\right)} \tag{2.15}$$

and

$$\therefore a_3=\frac{b(h_2+p_2)}{2\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\frac{b(h_2-p_2)}{2\left((1+2¢)^m-(1+2¢)^n\right)} \tag{2.16}$$

From Lemma (2.1),(2.11),(2.13),(2.15) and (2.16), we get

$$|a_2|\le\frac{2|b|}{\left[(1+¢)^m-(1+¢)^n\right]},$$

$$|a_2|\le\sqrt{\frac{2|b|}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}}\ ,$$

$$|a_3|\le\frac{4|b|^2}{\left[(1+¢)^m-(1+¢)^n\right]^2}+\frac{2|b|}{\left[(1+2¢)^m-(1+2¢)^n\right]}$$

and

$$|a_3|\le\frac{2|b|}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\frac{2|b|}{2\left((1+2¢)^m-(1+2¢)^n\right)}$$

This completes the theorem (2.2).

Theorem 2.3

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,¢}(\sim)$. Then (using definition (1.3))

$$|a_2| \le \min\left\{\frac{2|b|\sim}{\left[(1+¢)^m-(1+¢)^n\right]}, \sqrt{\frac{2|b|\sim}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}}\right\} \quad (2.17)$$

and

$$|a_3| \le \min\left\{\begin{array}{l}\frac{4|b|^2\sim^2}{\left[(1+¢)^m-(1+¢)^n\right]^2}+\frac{2|b|\sim}{\left[(1+2¢)^m-(1+2¢)^n\right]}, \\ \frac{2|b|\sim}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\frac{2|b|\sim}{\left((1+2¢)^m-(1+2¢)^n\right)}\end{array}\right\} \quad (2.18)$$

Proof

$$|a_2| \le \frac{2|b|\sim}{\left[(1+¢)^m-(1+¢)^n\right]},$$

$$|a_2| \le \sqrt{\frac{2|b|\sim}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}},$$

$$|a_3| \le \frac{4|b|^2\sim^2}{\left[(1+¢)^m-(1+¢)^n\right]^2}+\frac{2|b|\sim}{\left[(1+2¢)^m-(1+2¢)^n\right]}$$

and

$$|a_3| \le \frac{2|b|\sim}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\frac{2|b|\sim}{2\left((1+2¢)^m-(1+2¢)^n\right)}$$

This completes the theorem (2.3).

Theorem 2.4

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,¢}(¥)$. Then (using definition (1.4))

$$|a_2| \le \min\left\{\frac{2|b|(1-¥)}{\left[(1+¢)^m-(1+¢)^n\right]}, \sqrt{\frac{2|b|(1-¥)}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}}\right\} \quad (2.19)$$

and

$$|a_3| \le \min\left\{\begin{array}{l} \dfrac{4|b|^2(1-¥)^2}{\left[(1+¢)^m-(1+¢)^n\right]^2}+\dfrac{2|b|(1-¥)}{\left[(1+2¢)^m-(1+2¢)^n\right]}, \\ \dfrac{2|b|(1-¥)}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\dfrac{2|b|(1-¥)}{\left((1+2¢)^m-(1+2¢)^n\right)} \end{array}\right\} \qquad (2.20)$$

Proof

$$|a_2| \le \frac{2|b|(1-¥)}{\left[(1+¢)^m-(1+¢)^n\right]},$$

$$|a_2| \le \sqrt{\frac{2|b|(1-¥)}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}},$$

$$|a_3| \le \frac{4|b|^2(1-¥)^2}{\left[(1+¢)^m-(1+¢)^n\right]^2}+\frac{2|b|(1-¥)}{\left[(1+2¢)^m-(1+2¢)^n\right]}$$

and

$$|a_3| \le \frac{2|b|(1-¥)}{\left[\left((1+2¢)^m-(1+2¢)^n\right)-\left((1+¢)^{m+n}-(1+¢)^{2n}\right)\right]}+\frac{2|b|(1-¥)}{2\left((1+2¢)^m-(1+2¢)^n\right)}$$

This completes the theorem (2.4).

Letting $¢ = 1$ in theorem (2.2), (2.3) and (2.4), we obtain the following corollaries.

Corollary 2.5

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,1}(h,p)$.

Then
$$|a_2| \le \min\left\{\frac{2|b|}{\left[(2)^m-(2)^n\right]}, \sqrt{\frac{2|b|}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}}\right\} \qquad (2.21)$$

and

$$|a_3| \le \min\left\{\begin{array}{l} \dfrac{4|b|^2}{\left[(2)^m-(2)^n\right]^2}+\dfrac{2|b|}{\left[(3)^m-(3)^n\right]}, \\ \dfrac{2|b|}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}+\dfrac{2|b|}{\left((3)^m-(3)^n\right)} \end{array}\right\} \qquad (2.22)$$

Corollary 2.6

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,1}(\sim)$.

Then (using definition (1.3))

$$|a_2| \le \min\left\{\frac{2|b|\sim}{\left[(2)^m-(2)^n\right]}, \sqrt{\frac{2|b|\sim}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}}\right\} \qquad (2.23)$$

and

$$|a_3| \le \min \left\{ \begin{array}{l} \dfrac{4|b|^2 \sim^2}{\left[(2)^m - (2)^n\right]^2} + \dfrac{2|b|\sim}{\left[(3)^m - (3)^n\right]}, \\ \dfrac{2|b|\sim}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]} + \dfrac{2|b|\sim}{\left((3)^m - (3)^n\right)} \end{array} \right\} \tag{2.24}$$

Corollary 2.7

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,b,1}(\text{¥})$. Then (using definition (1.4))

$$|a_2| \le \min \left\{ \frac{2|b|(1-\text{¥})}{\left[(2)^m - (2)^n\right]}, \sqrt{\frac{2|b|(1-\text{¥})}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]}} \right\} \tag{2.25}$$

and

$$|a_3| \le \min \left\{ \begin{array}{l} \dfrac{4|b|^2 (1-\text{¥})^2}{\left[(2)^m - (2)^n\right]^2} + \dfrac{2|b|(1-\text{¥})}{\left[(3)^m - (3)^n\right]}, \\ \dfrac{2|b|(1-\text{¥})}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]} + \dfrac{2|b|(1-\text{¥})}{\left((3)^m - (3)^n\right)} \end{array} \right\} \tag{2.26}$$

Letting $\text{¢} = 1 \,\&\, b = 1$ in theorem (2.2), (2.3) and (2.4), we obtain the following corollaries Seker[6].

Corollary 2.8

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,1,1}(h, p)$.

Then $$|a_2| \le \min \left\{ \frac{2}{\left[(2)^m - (2)^n\right]}, \sqrt{\frac{2}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]}} \right\} \tag{2.27}$$

and

$$|a_3| \le \min \left\{ \begin{array}{l} \dfrac{4}{\left[(2)^m - (2)^n\right]^2} + \dfrac{2}{\left[(3)^m - (3)^n\right]}, \\ \dfrac{2}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]} + \dfrac{2}{\left((3)^m - (3)^n\right)} \end{array} \right\} \tag{2.28}$$

Corollary 2.9

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,1,1}(\sim)$. Then (using definition (1.3))

$$|a_2| \le \min \left\{ \frac{2\sim}{\left[(2)^m - (2)^n\right]}, \sqrt{\frac{2\sim}{\left[\left((3)^m - (3)^n\right) - \left((2)^{m+n} - (2)^{2n}\right)\right]}} \right\} \tag{2.29}$$

and

$$|a_3| \leq \min \left\{ \begin{array}{l} \frac{4\sim^{2}}{\left[(2)^m-(2)^n\right]^2}+\frac{2\sim}{\left[(3)^m-(3)^n\right]}, \\ \frac{2\sim}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}+\frac{2\sim}{\left((3)^m-(3)^n\right)} \end{array} \right\} \tag{2.30}$$

Corollary 2.10

Let the function $f(z)$ given by (1.1) be in the bi-univalent function class $S_{\Sigma}^{m,n,1,1}(¥)$. Then (using definition (1.4))

$$|a_2| \leq \min \left\{ \frac{2(1-¥)}{\left[(2)^m-(2)^n\right]}, \sqrt{\frac{2(1-¥)}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}} \right\} \tag{2.31}$$

and

$$|a_3| \leq \min \left\{ \begin{array}{l} \frac{4(1-¥)^2}{\left[(2)^m-(2)^n\right]^2}+\frac{2(1-¥)}{\left[(3)^m-(3)^n\right]}, \\ \frac{2(1-¥)}{\left[\left((3)^m-(3)^n\right)-\left((2)^{m+n}-(2)^{2n}\right)\right]}+\frac{2(1-¥)}{\left((3)^m-(3)^n\right)} \end{array} \right\} \tag{2.32}$$

ACKNOWLEDGMENT

The authors thank the referees for their valuable suggestions which lead to the improvement of this paper.

REFERENCES

1. Al-Oboudi., On univalent functions defined by a generalized Salagean operator, *Int. Journal of Mathematics and Mathematical Science*, Vol.27, 1429-1436 (2004).
2. Brannan D.A, Clunie, Aspects of contemporary complex analysis, Academic press New York, London (1980).
3. Brannan D.A, Taha T.S, On some classes of bi-valent functions, *Studia Univ Babes-Bolyoni Math*, Vol.31, 70-77 (1986).
4. Ch. Pommerenke, Univalent functions, Vandenhoeck and Rupercht, Gottingen (1975).
5. Duren P.L, Univalent functions, Springer-Verlag, New York (1983).
6. Frasin B.A. and M.K. Aouf, New Subclasses of bi-univalent functions, *Applied Mathematics Letters*, Vol.24, 1569-1573 (2011).
7. G. St. Salagean, Subclasses of univalent functions, *Lecture Notes in Math*. Vol.1013, Springer-Verlag, Berlin, 362-372 (1983).
8. Lewin M. On a coefficient problem for bi-valent functions, *Proc Amer Math Soc*, Vol.18, 63-68 (1967).
9. Qing-Hua Xu, Ying-Chun Gui and H.M. Srivastava, Coefficient estimates for a certain subclass of analytic and bi-univalent functions, *Applied Mathematics Letters*, Vol.25, 990-994 (2012).
10. Seker.B, On a new Subclasses of bi-univalent functions defined by using Salagean operator, *Turk J Math*, Vol.42, 2891-2896 (2018).