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Symmetric Periodic Fourier Series using Pentagonal Fuzzy Number

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ABSTRACT

Pentagonal fuzzy number is applied to check the symmetric property of a periodic Fourier series. We observe that pentagonal fuzzy number satisfies periodic fuzzy valued functions of the trigonometric sine and cosine fuzzy Fourier coefficients for real numbers. We verify that different types of pentagonal fuzzy number also satisfy symmetric periodic fuzzy Fourier series.

Keywords: Fuzzy Numbers; Pentagonal Fuzzy numbers; Fourier series; Fourier Coefficients.

1. INTRODUCTION

Lotfi. A. ZADEH introduced the notion of fuzziness in 1965. An approximation of analytic functions by Dubois, Prade and Yager (1978), encompasses dividing the membership functions of the algebraic operations into a left side and a right sides representation by simple analytic form. Due to the rapid growth in the study of Fuzzy sets, we have different type of fuzzy numbers namely Triangular fuzzy number[4], Trapezoidal fuzzy number[2] and Pentagonal fuzzy number[5], hexagonal, Octagonal, Diamond[12] fuzzy numbers. Operations on fuzzy numbers with real valued functions were studied by S.H. Chen in 1985 and also by Klement,[13]. Triangular fuzzy number was introduced by Michael Hanss in 2004.The Pentagonal fuzzy number and the reverse order pentagonal fuzzy number were introduced by T. Pathinathan and Ponnivalavan[6]. Fuzzy numbers are used in many applications such as control theory, signal processing and approximation theory.

Fourier series was introduced by Joseph Fourier (1807). It plays a vital role in the field of mathematics, physics, engineering and signal processing. Fourier series is applied in the study of sound, wave and light motion.

In this paper we apply pentagonal fuzzy number to define fuzzy Fourier coefficients. We have verified the cases were different types of pentagonal fuzzy numbers satisfies symmetric periodic fuzzy Fourier series. We have illustrated with some examples.

2. PRELIMINARY NOTIONS

2.1 Definition: Fuzzy set

A fuzzy set $\underset{\sim}{A}$ of X is defined as $\underset{\sim}{A} = \{(x, «_{\underset{\sim}{A}}(x))/ x \in X\}$, where *x* is a element in the universe of discourse X and $«_{\underset{\sim}{A}} : X \rightarrow [0,1]$ is a mapping called the degree of membership function and $«_{\underset{\sim}{A}}(x)$ is the membership value of $x \in X$.

2.2 Definition: Fuzzy Number

A *fuzzy number* is a fuzzy set $\underset{\sim}{A}$ on the real axis, whose membership function $«_{\underset{\sim}{A}}$ satisfies the following conditions.

(i) $«_{\underset{\sim}{A}}$ is normal; there exists $x \in \Re$ such that $«_{\underset{\sim}{A}}(x)$ =1.

(ii) $«_{\underset{\sim}{A}}$ is convex.

$$«_A[ªx_1 + (1-ª)x_2] \geq \max(«_A(x_1), «_A(x_2)) \quad \forall x_1, x_2 \in \Re \quad \text{and } ª \in [0,1]$$

(iii) $«_{\underset{\sim}{A}}$ is piecewise continuous function,

$\forall \in > 0$, there exists $¢ > 0$ such that $«_{\underset{\sim}{A}}(x_1) - «_{\underset{\sim}{A}}(x_1) < \in$ whenever $|x\text{-}x_0| < ¢$.

2.3 Periodic fuzzy Membership function

Most of the membership functions of fuzzy sets are defined on a fixed interval which is usually denoted by [0, 1]. The periodic function can be differed for some of the fuzzy approximate reasoning, time, season and direction. The membership functions of those fuzzy sets are periodic functions and the fuzzy grades gives the same value at the regular intervals,[3]. A membership function is said to be periodic with period $µ > 0$, if $«(v) = «(v + µ)$ for all variables *v* of the membership function.

Let f^{t} be any fuzzy valued function defined on $[-®, ®]$ in trigonometric series. If the Fourier series of fuzzy valued functions turn to converge to f^{t}, then it is a periodic function; the level sum of this gives the required periodic extension of f^{t},[4].

2.4 Definition: (Pentagonal fuzzy number)

A pentagonal fuzzy number (PFN), $\mu_u(x)$ has a piecewise continuous graph consisting of five points in its domain, forming a pentagonal shape. As chosen, the points in the domain have the ordering $a \le b \le c \le d \le e\,; a,b,c,d,e \in R.$ of a fuzzy set $\underset{\sim}{u}$ is defined as $\underset{\sim P}{u} = \{a,b,c,d,e\}$ and its membership function is given by,

$$\mu_{\underset{\sim}{u}}(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{(x-a)}{(b-a)} & for \quad a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & for \quad b \le x \le c \\ 1 & x = c \\ \dfrac{(d-x)}{(d-c)} & for \quad c \le x \le d \\ \dfrac{(e-x)}{(e-d)} & for \quad d \le x \le e \\ 0 & for \ x > e \end{cases} \tag{1}$$

Then, the result $[u]_\alpha = [u^-(\alpha), u^+(\alpha)] = [(c-a)\alpha + a + b, (c-e)\alpha + d + e]$

3. FOURIER SERIES FOR FUZZY VALUED FUNCTIONS OF PERIOD 2π

3.1.1 Definition:

A fuzzy valued function $\tilde{f}$ is called periodic with respect to a constant P >0 for which $\tilde{f}(x+P) = \tilde{f}(x)$

for any $x,t \in [a,b]$.The condition $f_\alpha^-(t+P) = f_\alpha^-(t)$ and $f_\alpha^+(t+P) = f_\alpha^+(t)$ hold for all $\alpha \in [0,1]$ and $t \in [a,b]$ such a constant is called a period of the function $\tilde{f}$.

3.1.2 Definition:

Let $\tilde{f}$ be a 2π -periodic fuzzy valued function on a set *A*. The Fourier series of fuzzy valued function $\tilde{f}$ of period 2π is defined as follows:

$$\tilde{f}(x) = \frac{a_0}{2} \oplus \sum_{n=1}^{\infty} (a_n \cos nx \oplus b_n \sin nx) \tag{2}$$

with respect to the fuzzy coefficients a_0, a_n and b_n which converges uniformly in $\text{a} \in [0,1]$ for all $n \in N$, $x, t \in A$ .To calculate the Fourier coefficients a_0, a_n and b_n with respect to the level sets are defined as $a_0 = \frac{1}{®}\int_{-®}^{®} [\, f_{\text{a}}^{-}(t),\, f_{\text{a}}^{+}(t)]\,dt$,

$a_n = \frac{1}{®}\int_{-®}^{®} [\, f_{\text{a}}^{-}(t)\cos nt,\, f_{\text{a}}^{+}(t)\cos nt]\,dt$ and $b_n = \frac{1}{®}\int_{-®}^{®} [\, f_{\text{a}}^{-}(t)\sin nt,\, f_{\text{a}}^{+}(t)\sin nt]\,dt$.

3.2 Case: 1

Consider the periodic fuzzy pentagonal function shown below,

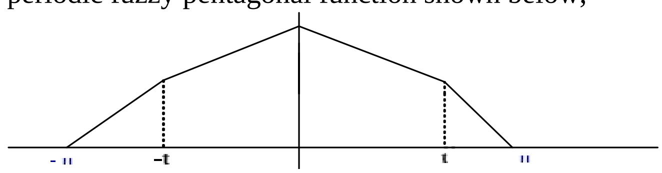

Figure 1

Let f^{t} be $2®$ -periodic fuzzy valued function on the Fuzzy integral on $[-®, ®]$ with pentagonal form defined by

$$f_{\text{a}}^{\text{t}}(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{(x+®)}{(-t+®)} & for \quad -® \le x \le -t \\ \dfrac{(x+t)}{(0+t)} & for \quad -t \le x \le 0 \\ 1 & x = 0 \\ \dfrac{(t-x)}{(t-0)} & for \quad 0 \le x \le t \\ \dfrac{(®-x)}{(®-t)} & for \quad t \le x \le ® \\ 0 & for\ x > e \end{cases} \tag{3}$$

which is fuzzy integral on $[-®, ®]$ for each $x, t \in [a, b]$ and $\text{a} \in [0,1]$. By using Definition 4, the level set $[\, f^{\text{t}} \,]_{\text{a}}$ of the membership function of f^{t} can be defined as,

$$\left[f^{\text{t}}\right]_{ª} = \left[f_{ª}^{-}(\text{t}), f_{ª}^{+}(\text{t})\right] = \left[®\,(ª-1)-\text{t}, \text{t}-®\,(ª-1)\right] \tag{4}$$

The fuzzy Fourier coefficients $a_0, a_n, and\ b_n$ given by

$$a_0 = \frac{1}{®}\left[\int_{-®}^{®}\left[®\,(ª-1)-\text{t}\right]dt, \int_{-®}^{®}\left[\text{t}-®\,(ª-1)\right]dt\right]$$

$$= \left[2®\left(ª-\frac{3}{2}\right), 2®\left(\frac{3}{2}-ª\right)\right]$$

$$a_n = \frac{1}{®}\left[\int_{-®}^{®}\left[®\,(ª-1)-\text{t}\right]\cos nt\ dt, \int_{-®}^{®}\left[\text{t}-®\,(ª-1)\right]\cos nt\ dt\right]$$

$$= \frac{2}{®}\left[-\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right], \left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]\right]$$

$$b_n = \frac{1}{®}\left[\int_{-®}^{®}\left[®\,(ª-1)-\text{t}\right]\sin nt\ dt, \int_{-®}^{®}\left[\text{t}-®\,(ª-1)\right]\sin nt\ dt\right]$$

$$= \frac{2}{®}\left\{\left[\frac{(-1)^n}{n}\left[-®\,(ª-1)+®\right]+\frac{®\,(ª-1)}{n}\right], \left[\frac{(-1)^n}{n}\left[®\,(ª-1)-®\right]-\frac{®\,(ª-1)}{n}\right]\right\}$$

$$= \frac{2}{®}\left\{\left[\frac{(-1)^n}{n}\left[®\,(-ª+2)\right]+\frac{®\,(ª-1)}{n}\right], \left[\frac{(-1)^n}{n}\left[®\,(ª-2)\right]-\frac{®\,(ª-1)}{n}\right]\right\}$$

By using the above coefficients $a_0, a_n, and\ b_n$ in (2), we have

$$f_{ª}^{\text{t}}(x) = \frac{\left[2®\left(ª-\frac{3}{2}\right), 2®\left(\frac{3}{2}-ª\right)\right]}{2} \oplus \left\{\begin{array}{l} \sum_{n=1}^{\infty}\frac{2}{®}\left[-\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right], \left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]\right]\cos nt \oplus \\ \sum_{n=1}^{\infty}\frac{2}{®}\left\{\left[\frac{(-1)^n}{n}\left[®\,(-ª+2)\right]+\frac{®\,(ª-1)}{n}\right], \left[\frac{(-1)^n}{n}\left[®\,(ª-2)\right]-\frac{®\,(ª-1)}{n}\right]\right\}\sin nt \end{array}\right\}$$

$$f_{ª}^{\text{t}}(x) = \left[\begin{array}{l} ®\left(ª-\frac{3}{2}\right)+\frac{4}{®}\left(\frac{\cos t}{1^2}+\frac{\cos 3t}{3^2}+\frac{\cos 5t}{5^2}....\right)+4\left(ª-\frac{3}{2}\right)\left(\sin \text{t}+\frac{\sin 3\text{t}}{3}+\frac{\sin 5\text{t}}{5}........\right)+2\left(\frac{\sin 2\text{t}}{2}+\frac{\sin 4\text{t}}{4}+........\right), \\ ®\left(\frac{3}{2}-ª\right)-\frac{4}{®}\left(\frac{\cos t}{1^2}+\frac{\cos 3t}{3^2}+\frac{\cos 5t}{5^2}....\right)+4\left(\frac{3}{2}-ª\right)\left(\sin \text{t}+\frac{\sin 3\text{t}}{3}+\frac{\sin 5\text{t}}{5}........\right)-2\left(\frac{\sin 2\text{t}}{2}+\frac{\sin 4\text{t}}{4}+......\right) \end{array}\right] \tag{5}$$

Example: 2

Let the membership value a be 0.6 and $t = 30°$ in (5). For the pentagonal fuzzy number of five parameters we can obtain the Fourier fuzzy valued function is a symmetric periodic function.

$$\left[f^{30°} \right]_{0.6} = \left[-3.817, 3.817 \right]$$

3.3 Case: 2

Consider the periodic fuzzy pentagonal function shown below,

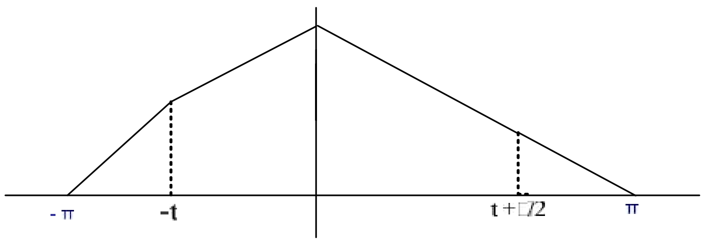

Figure 2

Let f^{t} be 2® -periodic fuzzy valued function on the Fuzzy integral on $\left[-\text{®}, \text{®}\right]$ with pentagonal form defined by,

$$f^{t}_{a}(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{(x+\text{®})}{(-t+\text{®})} & for \quad -\text{®} \le x \le -t \\ \dfrac{(x+t)}{(0+t)} & for \quad -t \le x \le 0 \\ 1 & x = 0 \\ \dfrac{\left(t+\dfrac{\text{®}}{2}-x\right)}{\left(t+\dfrac{\text{®}}{2}-0\right)} & for \quad 0 \le x \le t+\dfrac{\text{®}}{2} \\ \dfrac{(\text{®}-x)}{\left(\text{®}-t-\dfrac{\text{®}}{2}\right)} & for \quad t+\dfrac{\text{®}}{2} \le x \le \text{®} \\ 0 & for \ x > e \end{cases} \tag{6}$$

which is fuzzy integral on $[-®,®]$ for each $x,t \in [a,b]$ and $ª \in [0,1]$. By using Definition 4, the level set $[f^t]_ª$ of the membership function of f^t can be defined as,

$$[f^t]_ª = [f_ª^-(t), f_ª^+(t)] = \left[®(ª-1)-t, t+®\left(\frac{3}{2}-ª\right)\right] \quad (7)$$

The fuzzy Fourier coefficients $a_0, a_n, and\ b_n$ are

$$a_0 = \frac{1}{®}\left[\int_{-®}^{®}\left[®(ª-1)-t, t+®\left(\frac{3}{2}-ª\right)\right]dt\right]$$

$$= \left[2®\left(ª-\frac{3}{2}\right), 2®(2-ª)\right]$$

$$a_n = \frac{1}{®}\left[\int_{-®}^{®}\left[®(ª-1)-t\right]\cos nt\ dt, \int_{-®}^{®}\left[t+®\left(\frac{3}{2}-ª\right)\right]\cos nt\ dt\right]$$

$$= \frac{2}{®}\left[-\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right], \left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]\right]$$

$$b_n = \frac{1}{®}\left[\int_{-®}^{®}\left[®(ª-1)-t\right]\sin nt\ dt, \int_{-®}^{®}\left[t+®\left(\frac{3}{2}-ª\right)\sin nt\right]dt\right]$$

$$= \frac{2}{®}\left\{\left[\frac{(-1)^n}{n}\left[-®(ª-1)+®\right]+\frac{®(ª-1)}{n}\right], \left[-\frac{(-1)^n}{n}\left[®\left(\frac{3}{2}-ª\right)+®\right]+\frac{®\left(\frac{3}{2}-ª\right)}{n}\right]\right\}$$

$$= \frac{2}{®}\left[\frac{(-1)^n}{n}\left[®(-ª+2)\right]+\frac{®(ª-1)}{n}\right], \left\{\left[-\frac{(-1)^n}{n}\left[®\left(\frac{5}{2}-ª\right)\right]+\frac{®\left(\frac{3}{2}-ª\right)}{n}\right]\right\}$$

By using the above coefficients $a_0, a_n, and\ b_n$ in (2), we have

$$f_ª^t(x) = \frac{\left[2®\left(ª-\frac{3}{2}\right), 2®(2-ª)\right]}{2} \oplus$$

$$\left\{\begin{array}{l}\sum_{n=1}^{\infty}\frac{2}{®}\left[-\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right], \left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]\right]\cos nt \oplus \\ \sum_{n=1}^{\infty}\frac{2}{®}\left\{\left[\frac{(-1)^n}{n}\left[®(-ª+2)\right]+\frac{®(ª-1)}{n}\right], \left[\frac{(-1)^n}{n}\left[®(ª-2)\right]-\frac{®(ª-1)}{n}\right]\right\}\sin nt\end{array}\right\}$$

$$f^{t}_{\alpha}(x)=\frac{\left[2\pi\left(\alpha-\frac{3}{2}\right),2\pi\left(2-\alpha\right)\right]}{2}\oplus$$

$$\left\{\begin{array}{l}\sum_{n=1}^{\infty}\frac{2}{\pi}\left[-\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right],\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]\right]\cos nt\oplus\\ \sum_{n=1}^{\infty}\frac{2}{\pi}\left\{\left[\frac{(-1)^n}{n}\left[\pi\left(-\alpha+2\right)\right]+\frac{\pi\left(\alpha-1\right)}{n}\right],\left[\frac{(-1)^n}{n}\left[\pi\left(\alpha-\frac{5}{2}\right)\right]+\frac{\pi\left(\frac{3}{2}-\alpha\right)}{n}\right]\right\}\sin nt\end{array}\right\}$$

$$f_{\alpha}^{t}(x)=\left[\begin{array}{l}\pi\left(\alpha-\frac{3}{2}\right)+\frac{4}{\pi}\left(\frac{\cos t}{1^2}+\frac{\cos 3t}{3^2}+\frac{\cos 5t}{5^2}+....\right)+4\left(\alpha-\frac{3}{2}\right)\left(\sin t+\frac{\sin 3t}{3}+\frac{\sin 5t}{5}........\right)+2\left(\frac{\sin 2t}{2}+\frac{\sin 4t}{4}+........\right),\\ \pi\left(2-\alpha\right)-\frac{4}{\pi}\left(\frac{\cos t}{1^2}+\frac{\cos 3t}{3^2}+\frac{\cos 5t}{5^2}+....\right)+4\left(2-\alpha\right)\left(\sin t+\frac{\sin 3t}{3}+\frac{\sin 5t}{5}........\right)-2\left(\frac{\sin 2t}{2}+\frac{\sin 4t}{4}+......\right)\end{array}\right] \quad (8)$$

Example: 3

Let the membership value α be 0.6 and $t = 30°$ in (8). For the pentagonal fuzzy number of five parameters we can obtain the Fourier fuzzy valued function is not a symmetric periodic function.

$$\left[f^{30°}\right]_{0.6}=\left[-3.817,7.259\right]$$

CONCLUSION

In this paper we have verified the similarity of Fuzzy Fourier coefficients using pentagonal fuzzy number. We conclude that case 1 satisfies the symmetric property of fuzzy Fourier series using pentagonal fuzzy number and case 2 can be a periodic Fourier fuzzy valued function but it does not satisfy the symmetric condition of pentagonal fuzzy number.

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