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Square Difference Labeling for Some Graphs

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ABSTRACT

In this paper, we prove that two copies of star S_n with path P_k, two copies of cycle C_n with path P_k, restricted square of bistar $B_{n,n}$, restricted total graph of bistar $B_{n,n}$ and restricted middle graph of $B_{n,n}$ are square difference graphs.

Keywords: Square difference labeling, Square difference graphs, Star graph, Cycle graph, Path graph, Bistar graph $B_{n,n}$.

1. INTRODUCTION

All graphs considered in this paper are finite, simple and undirected graphs. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. We refer to Bondy and Murty[2] for the standard terminology and notations related to graph theory. A dynamic survey on graph labeling is regularly updated by Gallian[3]. The concept of square difference labeling was first introduced in[1]. The square sum labeling for some bistar related graphs[4], square difference labeling for some graphs[5] and square difference labeling of some union graphs[6] are taken for references.

Definition 1.1

A square difference labeling of a graph G is a bijection $f:V(G)\rightarrow\{0,1,2,3,4,\ldots\ldots\ldots\ldots P-1\}$, such that the induced function $f^*:E(G)\rightarrow N$ defined by $f^*(uv)=\left|[f(u)]^2-[f(v)]^2\right|$ for every $uv \in E(G)$ are all distinct. A graph which admits square difference labeling is called a square difference graph.

Definition 1.2

A walk in G is a finite non-null sequence $W = v_0 e_1 v_1 e_2 v_2 e_3 ....e_k v_k$, whose terms are alternately vertices and edges, such that, for $1 \le i \le k$, the ends of e_i are v_{i-1} and v_i . We say that W is a walk from v_0 to v_k or a (v_0, v_k)- walk.

Definition 1.3

A walk is closed if it has positive length and its origin and terminus are the same.

Definition 1.4

If the edges $e_1, e_2, ..., e_k$ of a walk W are distinct then W is called a trial. A trail is closed it its origin and terminus are the same.

Definition 1.5

If the edges $e_1, e_2, ..., e_k$ and the vertices $v_0, v_1, ..., v_k$ are distinct in a walk W then W is called a path. The path on k vertices is denoted by P_k .

Definition 1.6

A closed trail whose origin and internal vertices are distinct is a cycle.

Definition 1.7

A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y, so that each edge has one end in X and one end in Y ; such a partition (X, Y) is called a bipartition of the graph.

Definition 1.8

A complete bipartite graph is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y if $|X| = m$ and $|Y| = n,$ such a graph is denoted by $K_{m,n}$.

Definition 1.9

A star graph is the complete bipartite graph $K_{1,n}$ and it has $n+1$ vertices and n edges. The star on n vertices is denoted by S_n .

Definition 1.10

Bistar $B_{n,n}$ is the graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge. The vertex set of $B_{n,n}$ is $V(B_{n,n}) = \{u, v, u_i, v_i / 1 \le i \le n\}$, where u, v are apex vertices and u_i, v_i are pendent vertices. The edge set of $B_{n,n}$ is $E(B_{n,n}) = \{uv, uu_i, vv_i / 1 \le i \le n\}$. So, $|V(B_{n,n})| = 2n+2$ and $|E(B_{n,n})| = 2n+1$.

Definition 1.11

The restricted square of $B_{n,n}$ is a graph G with vertex set $V(G) = V(B_{n,n})$ and edge set $E(G) = E(B_{n,n}) \cup \{uu_i, vu_i / 1 \le i \le n\}$.

Definition 1.12

The restricted total graph of $B_{n,n}$ is a graph G with vertex set $V(G) = \{u, v, w, u_i, v_i, u_i', v_i' / 1 \le i \le n\}$, where u and v are apex vertices, u_i and v_i are pendent vertices w, u_i' *and* v_i' are vertices corresponding to the edge of $B_{n,n}$ and edge set *E(G)*= $E(B_{n,n}) \cup \{uw, vw, wu_i', wv_i', uu_i', vv_i', u_iu_i', v_iv_i' / 1 \le i \le n\}$.

Definition 1.13

The restricted middle graph of $B_{n,n}$ is a graph G with vertex set $V(G) = \{u, v, w, u_i, v_i, u_i', v_i' / 1 \le i \le n\}$, where u and v apex vertices, u_i and v_i are pendent vertices *w,u*$_i$*' and v*$_i$*'* are vertices corresponding to the edges of $B_{n,n}$ and edge set $E(G) = \{uw, vw, wu_i', wv_i', uu_i', vv_i', u_iu_i', v_iv_i' / 1 \le i \le n\}$.

2. MAIN RESULTS

In this section, we prove that two copies of star S_n with path P_k, two copies of cycle C_n with path P_k, restricted square of bistar $B_{n,n}$, restricted total graph of bistar $B_{n,n}$ and restricted middle graph of bistar $B_{n,n}$ are square difference graphs and also prove that the splitting graph S'(G), the shadow graph of bistar $B_{n,n}$, the degree splitting graph DS(G), the arbitrary super subdivision of $B_{n,n}$ and duplication of any vertex of bistar $B_{n,n}$ are not square difference graphs.

Theorem 2.1

The graph obtained by joining two copies of star S_n with the path P_k is a square difference graph.

Proof

Let S_n be the star graph with n vertices and $n-1$ edges. Let P_k be the graph with k vertices and $k-1$ edges. Let G be the graph obtained by connecting the two copies of star S_n with the path P_k.

Let $v_1, v_2, v_3, \ldots. v_n, v_{n+1}, v_{n+2}, v_{n+3}, \ldots, v_{[n+(k-2)]+n-1}, v_{2n+(k-2)}$ be the vertices of G. In G, the vertex v_n is the vertex common to the first copy of S_n and the path P_k as well as the vertex $v_{[n+(k-2)]+1}$ is the vertex common to the second copy of S_n and the path P_k.

$Let\ V(G) = \{v_i\ ;1 \le i \le 2n + (k-2)\}$

$$and \quad E(G) = \begin{cases} v_1 v_i & ;2 \le i \le n \\ v_i v_{i+1} & ;n \le i \le n+k \\ v_{n+k} v_i & ;n+(k+2) \le i \le 2n+(k-2) \end{cases}$$

Then we have $|V(G)| = 2n + (k-2)$ and $|E(G)| = 2n + (k-3)$

Define a bijection $f : V \to \{0,1,2,3,\ldots,2n+(k-3)\}$ by $f(v_i) = i-1,\ 1 \le i \le 2n+(k-2)$

For the vertex labeling f, the induced edge labeling f^* is defined as follows.

$f^*(v_i v_1) = (i-1)^2 \ ;\ 2 \le i \le n$

$f^*(v_{i+1} v_i) = 2i - 1 \ ;\ n \le i \le n+k$

$f^*(v_i v_{n+k}) = (i-1)^2 - (n+k-1)^2 \ ;n+(k+2) \le i \le 2n+(k-2)$

Clearly the edge labels are distinct. Hence the graph G is a square difference graph.

Example 1: The graph of two stars S_8 with path P_4 is a square difference graph which is shown in the figure 2.1.

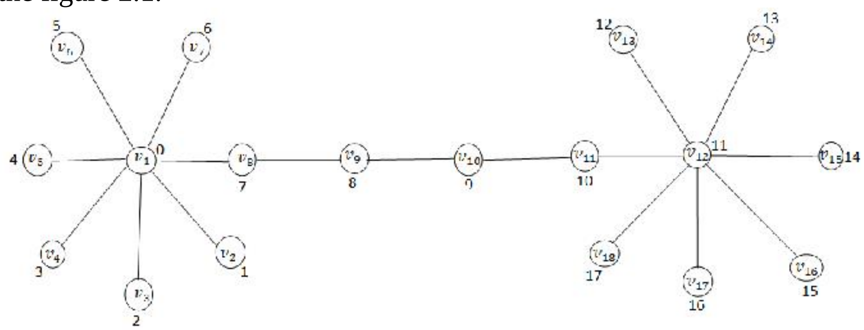

Figure 2.1: Two stars with one path

Theorem 2.2

The graph obtained by joining two copies of cycle C_n with the path P_k is a square difference graph.

Proof

Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the cycle C_n and $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path P_k. Let G be the graph obtained by connecting two copies of cycle

C_n with path P_k. Let $v_1, v_2, v_3, ... v_n, v_{n+1}, v_{n+2}, v_{n+3}, ... v_{[n+(k-2)]+n-1}, v_{2n+(k-2)}$ be the vertices of G. In G, the vertex v_n is the vertex common to the first copy of C_n and the path P_k as well as the vertex $v_{[n+(k-2)]+1}$ is the vertex common to the second copy of C_n and the path P_k.

Let V(G) = $\{v_i ; 1 \leq i \leq 2n+(k-2)$

$$and \quad E(G) = \begin{cases} v_i v_{i+1} \; ; 1 \leq i \leq 2n+(k-3) \\ v_1 v_n \\ v_{[n+(k-2)]+1} v_{2n+(k-2)} \end{cases}$$

Then we have $|V(G)| = 2n+(k-2)$ and $|E(G)| = 2n+(k-1)$

Define a bijection $f : V \rightarrow \{0,1,2,....,2n+(k-3)\}$ by $f\,(v_i) = i-1, 1 \leq i \leq 2n+(k-2)$

For the vertex labeling f, the induced edge labeling f^* is defined as follows.

$$f^*(v_{i+1} v_i) = 2i-1; \;\; 1 \leq i \leq 2n+(k-3)$$
$$f^*(v_{2n+(k-2)} v_{[n+(k-2)]+1}) = [2n+(k-3)]^2 - [n+(k-2)]^2$$
$$f^*(v_n v_1) = (n-1)^2$$

Clearly the edge labels are distinct. Hence the graph G is a square difference graph.

Example 2: The graph of two cycles C_9 with path P_6 is a square difference graph which is shown in the figure 2.2.

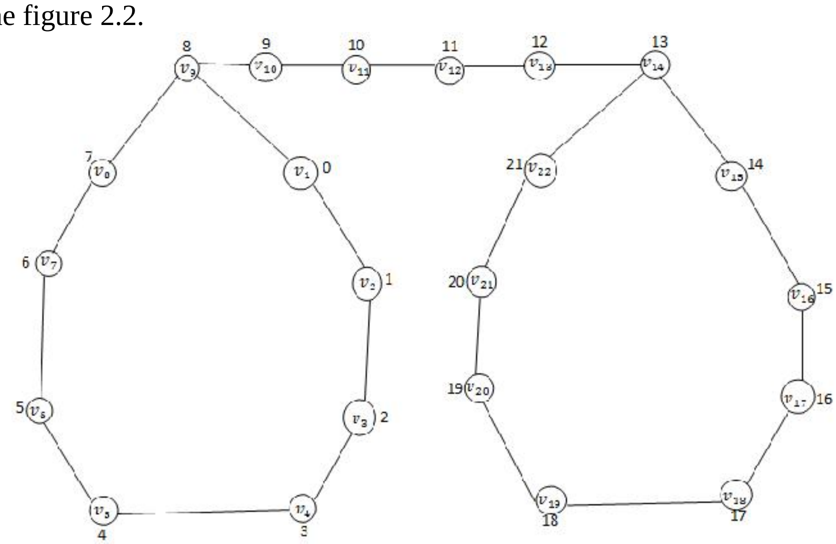

Figure 2.2: Two cycles with one path

Theorem 2.3

The restricted square of bistar $B_{n,n}$ is a square difference graph.

Proof

Let G be the restricted square of bistar $B_{n,n}$ with vertex set $V(G) = V(B_{n,n})$ and egde set $E(G) = E(B_{n,n}) \cup \{uv_i, vu_i / 1 \leq i \leq n\}$.

Then we have $|V(G)| = 2n+2$ and $|E(G)| = 4n+1$.

Define a bijection $f : V \to \{0,1,2,3,............2n+1\}$ by

$$f(u) = 1,$$

$$f(u_i) = 2i ; 1 \leq i \leq n,$$

$$f(v) = 0,$$

$$f(v_i) = 2i+1 ; 1 \leq i \leq n.$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows.

$$f^*(uv) = 1,$$

$$f^*(u_i u) = (2i)^2 - 1 \;;\; 1 \leq i \leq n,$$

$$f^*(v_i v) = (2i+1)^2 \;;\; 1 \leq i \leq n,$$

$$f^*(u_i v) = (2i)^2 \;;\; 1 \leq i \leq n,$$

$$f^*(v_i u) = (2i+1)^2 - 1 \;;\; 1 \leq i \leq n.$$

Clearly the edge labels are distinct. Hence the graph G is a square difference graph.

Example 3: The graph of restricted square of bistar $B_{6,6}$ is a square difference graph which is shown in the figure 2.3.

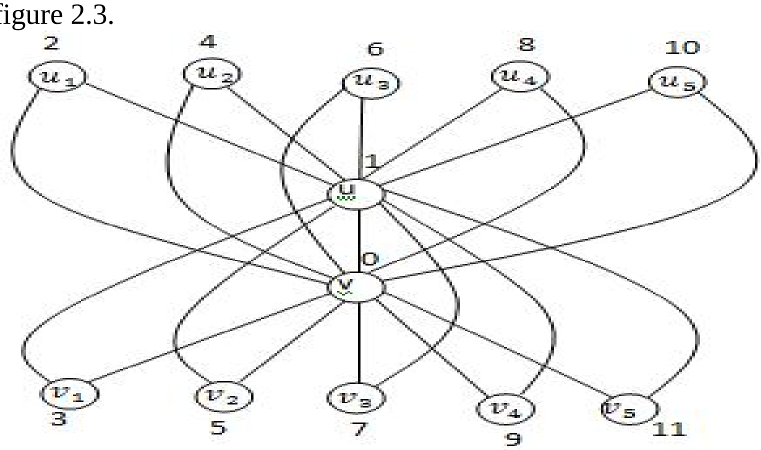

Figure 2.3: Restricted square of bistar $B_{6,6}$.

Theorem 2.4

The restricted total graph of $B_{n,n}$ is a square difference graph.

Proof

Let G be the restricted total graph of $B_{n,n}$ with vertex set $V(G) = V(B_{n,n}) \cup \{w, u_i', v_i' / 1 \le i \le n\}$ where u and v are apex vertices, u_i and v_i are pendent vertices, w, u_i' and v_i' are vertices related to edges and edge set $E(G) = E\ (B_{n,n}) \cup \{uw, vw, wu_i', wv_i', uu_i', vv_i', u_iu_i', v_iv_i' / 1 \le i \le n.\}$

Then we have $|V(G)| = 4n + 3$ and $|E(G)| = 8n + 3$.

Define a bijection $f : V \to \{0, 1, 2, 3, ...4n + 2\}$ by

$f(u) = 1,$

$f(u_i) = 2(i-1)\ ;\ 1 \le i \le n,$

$f(u_i') = 2n + 2(i-1)\ ;\ 1 \le i \le n,$

$f(v) = 4n + 2,$

$f(v_i) = 2i + 1\ ;\ 1 \le i \le n,$

$f(v_i') = 2n + 2i + 1\ ;\ 1 \le i \le n,$

$f(w) = 4n.$

For the vertex labeling f, the induced edge labeling f^* is defined as follows.

$$f^*(uw) = (4n)^2 - 1,$$
$$f^*(vw) = (4n+2)^2 - (4n)^2,$$
$$f^*(vu) = (4n+2)^2 - 1,$$
$$f^*(u_iu) = [2(i-1)]^2 - 1\ ;\ 1 \le i \le n,$$
$$f^*(u_i'u) = [2n + 2\ (i-1)]^2 - 1\ ;\ 1 \le i \le n,$$
$$f^*(vv_i') = (4n+2)^2 - (2n + 2i + 1)^2 ;\ 1 \le i \le n,$$
$$f^*(wv_i') = (4n)^2 - (2n + 2i + 1)^2 ;\ 1 \le i \le n,$$
$$f^*(wu_i') = (4n)^2 - [2n + 2\ (i-1)]^2 ;\ 1 \le i \le n,$$
$$f^*(vv_i) = (4n+2)^2 - (2i+1)^2 ;\ 1 \le i \le n,$$
$$f^*(u_i'u_i) = [2n + 2(i-1)]^2 - [2(i-1)]^2 ;\ 1 \le i \le n,$$
$$f^*(v_i'v_i) = (2n + 2i + 1)^2 - (2i+1)^2 ;\ 1 \le i \le n.$$

Clearly the edge labels are distinct. Hence the graph G is a square difference graph.

Example 4: The restricted total graph of bistar $B_{6,6}$ is a square difference graph which is shown in the figure 2.4.

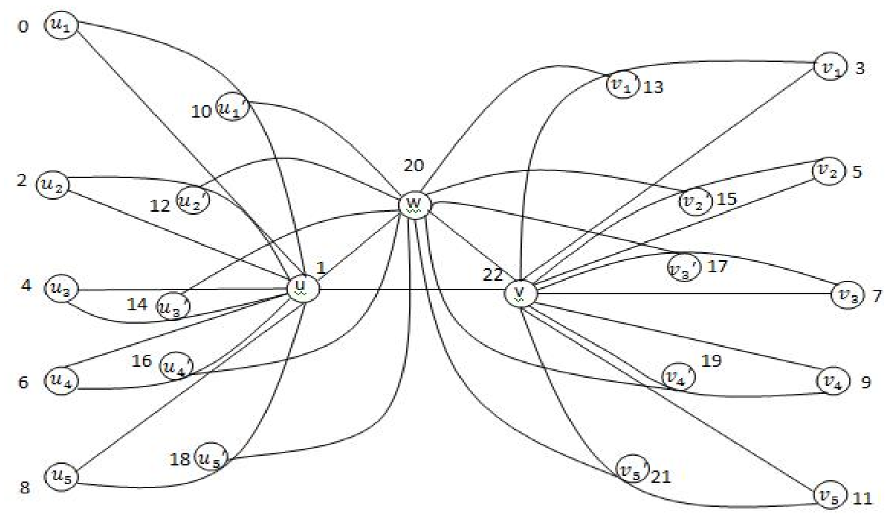

Figure 2.4: Restricted total graph of bistar $B_{6,6}$

Theorem 2.5

The restricted middle graph of bistar $B_{n,n}$ is a square difference graph.

Proof

Let G the restricted middle graph of bistar $B_{n,n}$ with vertex set $V(G) = \{u, v, w, v_i, u_i, u_i', v_i' / 1 \le i \le n\}$ where u and v are apex vertices, u_i and v_i are pendent vertices w, u_i' and v_i' are vertices corresponding to the edges of $B_{n,n}$ and edge set $E(G) = \{uw, vw, wu_i', wv_i', uu_i', vv_i', u_iu_i', v_iv_i' / 1 \le i \le n\}$.

Then we have $|V(G)| = 4n+3$ and $|E(G)| = 6n+2$

Define a bijection $f : V \to \{0,1,2,3,.....,4n+2\}$ by

$$f(u) = 1,$$
$$f(u_i) = 2(i-1) \quad ; 1 \le i \le n,$$
$$f(u_i') = 2n + 2\,(i-1)\,; 1 \le i \le n,$$
$$f(v) = 4n+2,$$
$$f(v_i) = 2i+1\,; 1 \le i \le n,$$
$$f(v_i') = 2n + 2i + 1\,; 1 \le i \le n,$$
$$f(w) = 4n.$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows.

$$f^*(wv_i') = (4n)^2 - (2n+2i+1)^2 \ ; \ 1 \le i \le n,$$
$$f^*(vv_i') = (4n+2)^2 - (2n+2i+1)^2 \ ; \ 1 \le i \le n,$$
$$f^*(v_i'v_i) = (2n+2i+1)^2 - (2i+1)^2 \ ; \ 1 \le i \le n,$$
$$f^*(vw) = (4n+2)^2 - (4n)^2 ,$$
$$f^*(wu_i') = (4n)^2 - [2n+2(i-1)]^2 \ ; \ 1 \le i \le n,$$
$$f^*(wu) = (4n)^2 - 1,$$
$$f^*(u_i'u_i) = [2n+2(i-1)]^2 - [2(i-1)]^2 \ ; \ 1 \le i \le n,$$
$$f^*(u_i'u) = [2n+2(i-1)]^2 - 1 \ ; \ 1 \le i \le n.$$

Clearly the edge labels are distinct. Hence the graph G is a square difference graph.

Example 5: The restricted middle graph of bistar $B_{6,6}$ is a square difference graph which is shown in the figure 2.5.

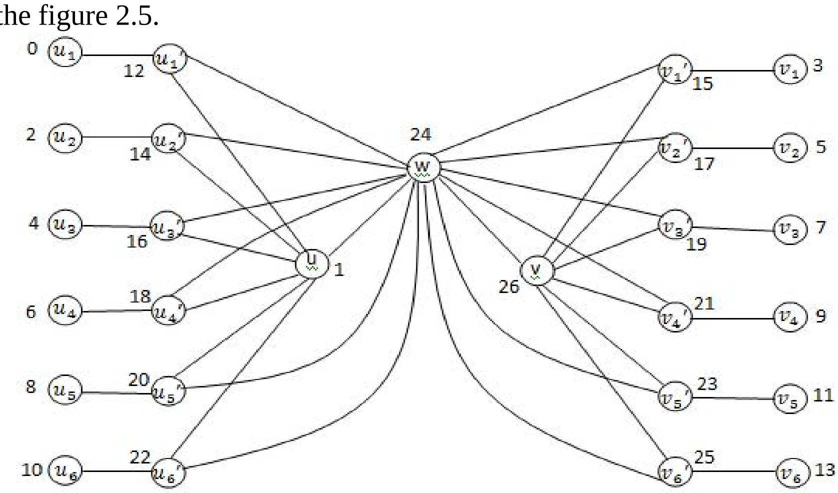

Figure 2.5: Restricted middle graph of bistar $B_{6,6}$

3. CONCLUSION

It is very interesting to study graphs which admit square difference labeling. Here we have proved that two copies of star S_n with P_k, two copies of cycle C_n with P_k, the restricted square of bistar $B_{n,n}$, the restricted total graph of $B_{n,n}$ and the restricted middle graph of bistar $B_{n,n}$ are square difference graphs. To investigate equivalent results of different graph families are an open area of research.

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