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Main Scalar, Hypersurfaces of a Finsler Space with Projective Generalized β -conformal Change

Suraj Kumar Shukla and Asmita Mishra*

Department of Mathematics & Statistics,
D.D.U Gorakhpur University Gorakhpur-273009, INDIA.
*Marwad Business School, Gorakhpur-273001, INDIA.
email: surajkumarshukla499@gmail.com, mishra.asmita55@gmail.com.

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ABSTRACT

In the present paper, we have studied the transformation of Finsler metric $L^* = f(e^{\sigma(x)}L(x,y),\beta)$ where f is positively homogeneous function of $\bar{L}[= e^{\sigma(x)}L(x,y)]$ and β of degree one, $\beta = b_i(x)y^i$ is differential one form. We call this transformation as generalized β - conformal change. Under the transformation, a necessary and sufficient condition for these Finsler spaces to be projectively related has been obtained. Further relations among main scalars of two dimensional Finsler space and hypersurfaces of two Finsler spaces under different conditions have also been discussed.

Mathematical subject classification: 53B40, 53C60.

Keywords: Finsler spaces, hypersurface, generalized β conformal change, main scalars, projective change.

1. INTRODUCTION

The conformal theory of Finsler spaces has been initiated by M. S. Kneblman[6] in 1929 and investigated in detail by many authors[3,4,7], and[8]. The conformal change is defined as L(x, y) $\rightarrow e^{\sigma(x)}L(x,y)$, where $\sigma(x)$ is function of x alone called conformal factor. In 1980, M. Matsumoto[11] discussed the projective changes of Finsler metrics. In 1976, Hashiguchi[8] studies the conformal change of a Finsler metric, namely $L^* = e^{\sigma(x)}L$, In particular, he also dealt with special conformal transformation named C-conformal. This change has been studies by Izumi[3]. In 2008 Abed[12,13], introduced the change $L^* = e^{\sigma(x)}L(x,y) + \beta(x,y)$, which is called β conformal change. In 1984, Shibata[2] considered the general case of any β change i.e. $L^* = f(L, \beta)$, thus generalizing several changes in results of Finsler geometry[1,10].

In the present paper, we construct a theory which generalizes all the above mentioned transformations. Here we construct a change of the form

$$L(x, y) \rightarrow L^*(x, y) = f(\bar{L}, \beta) \qquad (1.1)$$

where, f is positively homogeneous function of $\bar{L}[= e^{\sigma(x)} L(x, y)]$ and β of degree one. We called the change (1.1) generalized β conformal change. In this paper, we obtained

(a) Relation between main scalars of F^2 and F^{*2}.
(b) The necessary and sufficient condition for these Finsler spaces to be projectively related under generalized β conformal change.
(c) Relation between the hypersurface of a Finsler space and a hypersurface of a Finsler space whose metric is obtained by projective generalized β conformal change.

2. PRELIMINARIES

Let (M^n, L) be a Finsler space F^n, where M^n is n-dimensional differentiable manifold equipped with the fundamental function L^*(x, y) is called generalized β conformally changed space F^{*n}.

We define $f_1 = \frac{\partial f}{\partial \bar{L}}$, $f_2 = \frac{\partial f}{\partial \beta}$, $f_{12} = \frac{\partial^2 f}{\partial \bar{L} \partial \beta}$, $f_{22} = \frac{\partial^2 f}{\partial \beta^2}$

Differentiating partially (1.1) with respect to y^i, the normalized supporting element $l_i^* = \frac{\partial L^*}{\partial y^i}$ is given by

$$l_i^* = f_1 e^{\sigma(x)} l_i + f_2 b_i. \qquad (2.2)$$

Differentiating (2.2) partially with respect to y^j ,the angular metric tensor $h_{ij}^* = L^* \frac{\partial^2 L^*}{\partial y^i y^j}$ is given by

$$h_{ij}^* = e^{\sigma(x)} P h_{ij} + q_0 m_i m_j \, , \qquad (2.3)$$

where $m_i = b_i - \frac{\beta}{L^2} y^i$, $P = \frac{f f_1}{L}$, $q_0 = f f_{22}$.

Again, the fundamental tensor $g_{ij}^* = h_{ij}^* + l_i^* l_j^*$ is given by

$$g_{ij}^* = e^{\sigma(x)} P g_{ij} + P_o b_i b_j + e^{\sigma(x)} P_{-1} \left(b_i y_j + b_j y_i \right) + e^{\sigma(x)} P_{-2} y_i y_j \, , \qquad (2.4)$$

where

$$\left. \begin{aligned} P_0 = f_2^2 + q_0, \quad & q_o = f f_{22} \\ q_{-1} = \frac{f f_{12}}{L} \quad , \quad & P_{-1} = q_{-1} + \frac{P f_2}{f} \\ q_{-2} = \frac{f(e^{\sigma(x)} f_{11} - \frac{f_1}{L})}{L^2}, \quad & P_{-2} = q_{-2} + \frac{e^{\sigma(x)} P^2}{f^2} \end{aligned} \right\} . \qquad (2.5)$$

It is easy to see that the det (g_{ij}^*) does not vanish therefore the reciprocal tensor with component g^{*ij} comes out to be

$$g^{*ij} = \frac{1}{P} e^{-\sigma(x)} g^{ij} - s_0 b^i b^j - s_{-1} \left(y^i b^j + y^j b^i \right) - s_{-2} y^i y^j \, , \qquad (2.6)$$

where
$$\left.\begin{aligned} s_0 &= \frac{e^{-\sigma(x)} f^2 q_0}{\lambda P L^2}, \quad s_{-1} = \frac{P_{-1} f^2}{P \lambda L^2} \\ s_{-2} &= \frac{P_{-1}\left(e^{\sigma(x)} m^2 P L^2 - b^2 f^2\right)}{\lambda P \beta L^2}, \\ \lambda &= \frac{f^2\left(e^{\sigma(x)} P + m^2 q_0\right)}{L^2} \neq 0, \quad g^{ij} m_i m_j = m^i m_i = m^2. \\ and \quad & b^i = g^{ij} b_j \end{aligned}\right\} \tag{2.7}$$

Again differentiating (2.4) partially with respect to y^k, we have the cartan covariant tensor $C^*$with covariant components $C^*_{ijk} = \frac{1}{2}\frac{\partial g^*_{ij}}{\partial y^k}$ is given by

$$C^*_{ijk} = e^{\sigma(x)} P C_{ijk} + \frac{e^{\sigma(x)} P_{-1}}{2}\left(h_{ij} m_k + h_{jk} m_i + h_{ki} m_j\right) + \frac{P_{02}}{2} m_i m_j m_k, \tag{2.8}$$

where, $P_{O2} = \frac{\partial P_0}{\partial \beta}$.

Transvecting (2.8) with g^{*lk}, we have

$$\begin{aligned} C^{*l}_{ij} = \; & C^l_{ij} + \frac{1}{2}\left\{e^{-\sigma(x)} P^{-1} m^l - m^2\left(s_0 b^l + s_{-1} y^l\right)\right\}\left(P_{02} m_i m_j + e^{\sigma(x)} P_{-1} h_{ij}\right) \\ & - e^{\sigma(x)}\left(s_0 b^l + s_{-1} y^l\right)\left(P C_{ij0} + P_{-1} m_i m_j\right) + \frac{P_{-1}}{2P}\left(h^l_i m_j + h^l_j m_i\right), \end{aligned} \tag{2.9}$$

where $h^i_j = g^{il} h_{lj}$, $C_{i00} = C_{ijk} b^j b^k$.

Proposition:(2.1) Let $F^{*n} = (M^n, L^*)$ be an n-dimensional Finsler space obtained from the generalized β conformal change of Finsler space $F^n = (M^n, L)$ then the normalized supporting element l^*_i, angular metric tensor h^*_{ij},fundamental metric tensor g^*_{ij}, the inverse metric tensor g^{*ij} and (h) hv-torsion tensor $C^*_{ijk}$of F^{*n} are given by (1.2),(1.3),(1.4),(1.6) and (1.8) respectively.

3. MAIN SCALAR OF GENERALIZED β CONFORMAL CHANGE OF TWO DIMENSIONAL FINSLER SPACE

The (h) hv torsion tensor for a two dimensional Finsler space F^2 is given by[9]:

$$C_{ijk} = I m_i m_j m_k \quad , \tag{3.1}$$

where $I = C_{222}$ is main scalar of F^2.

Similarly, the (h) hv- torsion tensor for a two dimensional Finsler space F^{*2} is given by

$$C^*_{ijk} = I^* m^*_i m^*_j m^*_k \, , \tag{3.2}$$

Where I^* is the main scalar of $F^{*2}$and m^*_i is the unit vector orthogonal to l^*_i in two dimensional Finsler space .Putting $l = j$ in equation (2.9) we have

$$C^*_i = C_i - e^{\sigma(x)} P s_0 C_{i00} + \eta m_i, \tag{3.3}$$

where
$$\eta = \frac{(n+1)P_{-1}}{2P} - \frac{3 e^{\sigma(x)} P_{-1} s_0}{2} + \frac{P_{02}}{2\left(e^{\sigma(x)} P + q_0\right)} \quad . \tag{3.4}$$

The normalized torsion vectors are $m^i = \frac{C^i}{C}\ in\ F^2$ and $m^{*i} = \frac{C^{*i}}{C}$ inF^{*2}, where C and $C^*$are the length of C^i & C^{*i}in F^2 & F^{*2} respectively. The equation (3.3) can also be written as

$$m_i^* = (\mu + \psi)m_i - \gamma C_{i00} \quad , \tag{3.5}$$

where $\mu = \frac{C}{C^*}, \quad \gamma = \frac{e^{\sigma(x)} P s_0}{C^*}, \ \psi = \frac{\eta}{C^*}$.

Transvecting equation (3.5) by g^{*ij} we have

$$m^{*j} = \frac{1}{P} e^{-\sigma(x)}(\mu + \psi)m^j + B^j , \tag{3.6}$$

where $B^j = \frac{-\gamma}{P} e^{-\sigma(x)} g^{ij} C_{i00} - s_0(\mu + \psi)b^i b^j m_i + s_0 \gamma C_{i00} b^i b^j - s_{-1}(\mu + \psi)m_i\left(y^i b^j + y^j b^i\right) + s_{-2}(\mu + \psi)m_i y^i y^j + s_{-2}\gamma C_{i00} y^i y^j$. (3.7)

From (3.2), we have

$$I^* m_i^* m_j^* m_k^* = (e^{\sigma(x)} PI + \frac{3}{2} e^{\sigma(x)} P_{-1} + \frac{P_{02}}{2}) m_i m_j m_k$$

Transvecting the above equation by $m^{*i} m^{*j} m^{*k}$ and using (3.6) we have,

$$I^* = (e^{\sigma(x)} PI + \frac{3}{2} e^{\sigma(x)} P_{-1} + \frac{P_{02}}{2})(\Theta^3 + \Omega) , \tag{3.8}$$

where,
$$\Theta = \frac{1}{P} e^{-\sigma(x)}(\mu + \psi) , \tag{3.9}$$

and $\Omega = m_i m_j m_k [R^2(m^i m^j B^k + m^i m^k B^j + B^i m^j m^k) + \Theta(m^i B^j B^k + m^i m^j B^k + m^j B^i B^k) + B^i B^j B^k]$. (3.10)

Theorem:(3.1) Let $F^{*n} = (M^n, L^*)$ be an n-dimensional Finsler space from the generalized β conformal change of Finsler space $F^n = (M^n, L)$, then the relationship between main scalars I^* and I of these Finsler spaces is given by (3.8).

Corollary:(3.1) For $\sigma(x) = 0$, i.e for generalized β-change the relationship between the main scalars I^* & I of $F^{*2}\ and\ F^2$ is given by

$$I^* = (PI + \frac{3}{2} P_{-1} + \frac{P_{02}}{2})(\Theta^3 + \Omega)$$

Corollary :(3.2) For β=0, i.e. for generalized conformal change, the relation between the main scalars I^* & I of $F^{*2}\ and\ F^2$ is given by

$$I^* = (e^{\sigma(x)} PI + \frac{3}{2} e^{\sigma(x)} P_{-1} + \frac{P_{02}}{2})(\Theta^3 + \Omega) \mid_{at\ \beta=0}$$

4. RELATION BETWEEN PROJECTIVE CHANGE AND GENERAIZED B-CONFORMAL CHANGE

For two Finsler spaces $F^n = (M^n, L)$ and $F^{*n} = (M^{*n}, L)$, if any geodesic of $F^n$is also a geodesic of F^{*n} and vice-versa, then the change L→$L^*$of the metric is called projective.
i.e. The change L→$L^*$is projective if

$$G^{*i} = G^i + P(x, y)y^i \quad . \tag{4.1}$$

Where P(x, y) is homogeneous scalar function of degree one in y^i, called a projective factor.

Partially differentiation of (4.1) with respect to y^j gives,

$$G_j^{*i} - G_j^i = P_j y^j + P\delta_j^i \quad . \tag{4.2}$$

A geodesic of F^n is given by the system of differentiation equations.

$$\frac{d^2x^i}{dt^2} + 2G^i(x,y) = \emptyset(t) y^i \quad , \tag{4.3}$$

Where $\emptyset(t) = \frac{\frac{d^2s}{dt^2}}{\frac{ds}{dt}}, \quad y^i = \frac{dx^i}{dt}$ *and t is a parameter.*

The Euler Lagrange's equation for the space F^{*n} is given by

$$\frac{\partial L^*}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L^*}{\partial y^i}\right) = 0 \quad . \tag{4.4}$$

Using, $L^* = f(e^{\sigma(x)}L, \beta)$ in (4.4) we have,

$$\frac{\partial}{\partial x^i}\left[f\left(e^{\sigma(x)}L, \beta\right) - \frac{d}{dt}\left[\frac{\partial}{\partial y^i}\left\{f\left(e^{\sigma(x)}L, \beta\right)\right\}\right] = 0 \quad . \tag{4.5}$$

$$\Leftrightarrow \quad f_1\left(\sigma_i e^{\sigma(x)}L + e^{\sigma(x)}\frac{\partial L}{\partial x^i}\right) + f_2\left(N_i^r b_r + b_{j|i}y^j\right) - \frac{d}{dt}\left(e^{\sigma(x)}f_1 l_i + f_2 b_i\right) = 0 \, . \tag{4.6}$$

$$\Leftrightarrow \quad f_1 e^{\sigma(x)}\left[\frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial y^i}\right)\right] + f_1 L\sigma_i e^{\sigma(x)} - f_1\sigma_0 e^{\sigma} l_i + f_2\left(b_{j|i} - b_{i|j}\right)y^j$$

$$-l_i e^{\sigma}\frac{df_1}{dt} - b_i\frac{df_2}{dt} = 0,$$

where, $\sigma_r y^r = \sigma_0$.

$$\Leftrightarrow \quad f_1 e^{\sigma(x)}\left[\frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial y^i}\right)\right] + A_i = 0,$$

where $A_i = f_1 L\sigma_i e^{\sigma(x)} - f_1\sigma_0 e^{\sigma} l_i + f_2\left(b_{j|i} - b_{i|j}\right)y^j - l_i e^{\sigma}\frac{df_1}{dt} - b_i\frac{df_2}{dt}$.

If $A_i = 0$, then $\left[\frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial y^i}\right)\right] = 0$. Since $e^{\sigma(x)}f_1 \neq 0$.

Thus, we conclude that

Theorem: (4.1) A Finsler space $F^n = (M^n, L)$ and a Finsler space $F^{*n} = (M^n, L^*)$ where $L^* = f(e^{\sigma(x)}L, \beta)$ are protectively related iff the covariant vector A_i vanishes identically.

5. HYPERSURFACE GIVEN BY A PROJECTIVE GENERALIZED β - CONFORMAL CHANGE

A hypersurface of the $M^{n-1}$of the underlying smooth manifold M^n may be represented parametrically by the equations $x^i = x^i(\alpha)$, where u^α are Gaussian coordinate on $M^{n-1}$and i=1,2,3…,n and α=1,2,3,…n-1.The rank of the matrix of projection factor $B_\alpha^i = \frac{\partial x^i}{\partial u^\alpha}$ is supposed to be component of (n-1) linearly independents vectors tangent to F^{n-1}. If the supporting element y^i at a point $u = (u^\alpha)$ of F^{n-1}then y^i maybe written as $y^i = B_\alpha^i(u)v^\alpha$ so that $v = (v^\alpha)$ is thought of the supporting element at the point u^α of M^{n-1}. The function $\underline{L} = L\left(x(u), y(u,v)\right)$ give rise to a Finsler metric on M^{n-1}. Thus we get an n-dimensional Finsler space $F^{n-1} = (M^{n-1}, \underline{L}(u,v))$ the unit normal vector $N^i(u,v)$ at each point u^α of F^{n-1} is defined by

(a) $g_{ij}B^i_\alpha N^j = 0$ (b) $g_{ij}N^iN^j = 1.$ (5.1)

Let us define $B^\alpha_i = B^\alpha_i(u,v)$ by

$$B^\alpha_i = g^{\alpha\beta}g_{ij}B^j_\beta \quad . \tag{5.2}$$

This, in view of $g_{ij}B^j_\beta B^i_\gamma = g_{\beta\gamma}$.

$$B^i_\alpha B^\beta_i = \delta^\beta_\alpha \quad . \tag{5.3}$$

From (5.1), (5.2) and (5.3), we have

$$\left.\begin{array}{l} B^i_\alpha N_i = 0,\ N^iB^\alpha_i = 0\,, N^iN_j = 1 \\ B^i_\alpha B^\alpha_j + N^iN_j = \delta^i_j,\ N_i = g_{ij}N^j \end{array}\right\}. \tag{5.4}$$

The second fundamental h-tensor $H_{\alpha\beta}$ and the normal curvature vector H_α for the induced cartan's connection ICΓ = $(F^\alpha_{\beta\gamma}, G^\alpha_\beta, C^\alpha_{\beta\gamma})$ on F^{n-1} are given by

$$H_{\alpha\beta} = N_i\left(B^i_{\alpha\beta} + F^i_{jk}B^j_\alpha B^k_\beta\right) + M_\alpha M_\beta\ , \tag{5.5}$$

and

$$H_\alpha = N_i(B^i_{0\alpha} + G^i_jB^j_\alpha) \quad , \tag{5.6}$$

where, $M_\alpha = C_{ijk}B^i_\alpha N^jN^k,$ $B^i_{\alpha\beta} = \frac{\partial^2x^i}{\partial u^\alpha\partial u^\beta}$ and $B^i_{0\alpha} = B^i_{\beta\alpha}v^\beta$.

From (5.5) and (5.6), we have

(a) $H_{0\alpha} = H_{\beta\alpha}v^\beta = H_\alpha$ (b)$H_{\alpha0} = H_{\alpha\beta}v^\beta = H_\alpha + M_\alpha H_0$. (5.7)

Consider Finslerian hypersurface $F^{n-1} = \left(M^{n-1}, \underline{L^*}(u,v)\right)$ of F^{*n}. The functions $B^i_\alpha(u)$ may be considered as the component of (n-1) linearly independent vectors tangent to F^{n-1} . Since N^i is the unit normal vector at a point u^α of F^{n-1}, the unit normal vector $N^{*i}(u,v)$ of $F^{*(n-1)}$ and the inverse projection factor $B^{*\alpha}_i$ along $F^{*(n-1)}$ are uniquely determined by

$$g^*_{ij}B^i_\alpha N^{*j} = 0, \qquad g^*_{ij}N^{*i}N^{*j} = 1, \tag{5.8}$$

and

$$B^{*\alpha}_i = g^{*\alpha\beta}g^*_{ij}B^j_\beta\ , \tag{5.9}$$

where $g^{*\alpha\beta}$ is the inverse of the metric tensor $g^*_{\alpha\beta}$ of $F^{*(n-1)}$.

From (5.8) and (5.9), we have

$$g^*_{ij}B^{*i}_\alpha = \delta^\beta_\alpha, B^i_\alpha N^*_i = 0, N^{*i}B^{*\alpha}_i = 0, N^{*i}N^*_i = 1. \tag{5.10}$$

From (5.10), we have $B^i_\alpha B^{*\alpha}_j + N^{*i}N^*_j = \delta^i_j$. (5.11)

Transvection of (5.1) (a) with v^α we have

$$y_jN^j = 0\ . \tag{5.12}$$

Transvecting equation (2.4) by N^iN^j and paying attention to

$$g_{ij}B^i_\alpha N^j = 0, g_{ij}N^iN^j = 1, l_iN^i = 0.$$

We have, $g^*_{ij}N^iN^j = [e^{\sigma(x)}P + P_0\left(b_iN^i\right)^2]$. (5.13)

This implies that $\frac{N^i}{\sqrt{e^{\sigma(x)}P+P_0(b_iN^i)^2}}$ is a unit normal vector.

Again transvecting (2.4) by $B^i_\alpha N^j$ and using $g_{ij}B^i_\alpha N^j = 0$ *and* $y_jN^j = 0$ we have,

$$g^*_{ij}B^i_\alpha N^j = (b_jN^j)\left[P_0(b_iB^i_\alpha) + e^{\sigma(x)}P_{-1}y_iB^i_\alpha\right]. \quad (5.14)$$

Which shows that N^j is normal to $F^{*(n-1)}$ iff

$$(b_jN^j)\left[P_0(b_iB^i_\alpha) + e^{\sigma(x)}P_{-1}y_iB^i_\alpha\right] = 0 \ . \quad (5.15)$$

Which implies that $(b_jN^j) = 0$ or $\left[P_0(b_iB^i_\alpha) + e^{\sigma(x)}P_{-1}y_iB^i_\alpha\right] = 0.$

Contracting $\left[P_0(b_iB^i_\alpha) + e^{\sigma(x)}P_{-1}y_iB^i_\alpha\right] = 0$, by v^α and using $y^i = B^i_\alpha v^\alpha$, we have

$$(f_2^2 + ff_{22}) + e^{\sigma(x)}(ff_{12} + f_1f_2)L = 0$$, This is a contradiction.

Therefore, $(b_jN^j) = 0$. Thus the vector N^j normal to $F^{*(n-1)}$ iff b_j is tangent to $F^{(n-1)}$.

Hence, we can say that $\frac{N^i}{\sqrt{e^{\sigma(x)}P}}$ is a unit normal of $F^{*(n-1)}$, now in the view of $N^{*i} = \frac{N^i}{\sqrt{e^{\sigma(x)}P}}$,

$$since \qquad N^*_i = g^*_{ij}N^{*j} \ .$$

Now in the view of $N^{*i} = \frac{N^i}{\sqrt{e^{\sigma(x)}P}}$, $y_jN^j = 0$ *and* $b_jN^j = 0$

$$N^*_i = \sqrt{e^{\sigma(x)}P}\ N_i \ . \quad (5.16)$$

Hence, we conclude that

Theorem: (5.1) Let F^{*n} be Finsler space obtained from F^n by β- conformal change. If $F^{*(n-1)}$ and $F^{(n-1)}$are hypersurfaces of these respectively then the vector b_i is tangential to the hypersurface $F^{(n-1)}$ iff every vector normal to $F^{(n-1)}$ is also normal to $F^{*(n-1)}$ spaces.

Further, suppose the generalized β-conformal change metric is projective. We call such change of metric as projective generalized β-conformal change of metric .Let us denote the difference of Cartan's connection coefficients $F^i_{jk}$of the Finsler space F^n and the Cartan's connection coefficients $F^{*i}_{jk}$of the Finsler space F^{*n} by D^i_{jk}

i.e.
$$D^i_{jk} = F^{*i}_{jk} - F^i_{jk} \ . \quad (5.17)$$

Transvecting by y^k and using $F^i_{jk}y^j = G^i_{jk}y^j = G^i_k$, we have

$$G^{*i}_j = G^i_j + D^i_{0j} \ , \quad (5.18)$$

where $D^i_{0j} = D^i_{kj}y^k$.

From (5.18) and (4.2) we have

$$D^i_{0j} = P_jy^j + P\delta^i_j \ . \quad (5.19)$$

Transvecting (5.19) with $N^iB^\alpha_i = 0$, $N_i = g_{ij}N^j$,

and $y_jN^j = 0$.

we have, $N_iD^i_{0j}B^j_\alpha = 0$. (5.20)

If each geodesic of hypersurface F^{n-1} with respect to induced metric is also a geodesic of a Finsler space F^n , then F^{n-1} is called totally geodesic hypersurface[5].

A totally geodesic hypersurface is characterized by $H_\alpha = 0$.

The normal curvature vector H^*_α of $F^{*(n-1)}$ is given by

$$H^*_\alpha = N^*_i (B^i_{0\alpha} + G^{*i}_j B^j_\alpha) . \tag{5.21}$$

Using (5.18), (5.16) in (5.21) we have

$$H^*_\alpha = H_\alpha \sqrt{e^{\sigma(x)} P} . \tag{5.22}$$

Since $\sqrt{e^{\sigma(x)} P} \neq 0$, therefore $H_\alpha = 0$ iff $H^*_\alpha = 0$.

Theorem:(5.2)- Let F^{*n} be the Finsler space obtained from the Finsler space F^n(n> 3)by a projective generalized β- conformal change metric then the hypersurface $F^{*(n-1)}$ of $F^{*n}$is totally geodesic iff the hypersurface $F^{(n-1)}$ of F^n is totally geodesic.

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REFERENCES

1. C.Shibata, On Finsler spaces with kropina metric, *Rep. Math. Phys*.13, 117-128 (1978).
2. C.Shibata, on invariants tensors of β-changes of Finsler metrics, *Math. Kyoto Univ*., 24, 163-188 (1984).
3. H. Izumi, Conformal transformation of Finsler spaces 1, *Tensor, N.S*.,31, pp-33-41 (1977).
4. H.Izumi, Conformal transformation of Finsler spaces 2, *Tensor*, 33, pp-337-359 (1980).
5. H. Rund, the differential geometry of Finsler spaces, Springer Verlag (1959).
6. M.S. Knebelman, conformal geometry generalized metric spaces, *Proc. Nat. Acad. Sci*. USA 15, pp 33-41 (1929).
7. M.Kitayama, Geometry of transformation of Finsler metrics, Hokkaido University of education, Kushiro campus, Japan (2000).
8. M.Hashiguchi, on conformal transformation of Finsler metric, *Math. Kyoto Univ*, 16, pp. 25-60 (1976).
9. M. Matsumoto, Foundation of Finsler geometry and special Finsler spaces, Springer – Verlag (1959).
10. M.Matsumoto, On Finsler spaces with Rander metric and special forms of important tensors, *Math. Kyoto Univ*., 14, 477-498 (1974).
11. M. Matsumoto, Projective changes of Finsler metrics and protectively flat Finsler spaces, *Tensor N.S*., 34, 303-315 (1980).
12. S.H.Abed, Conformal β-changes in Finsler spaces, *Proc. Math. Phys. Soc. Egypt*, 86, 79-89 (2008).
13. S.H.Abed, Cartan's connection associated with a β −conformal change in Finsler geometry, *Tensor N.S*., 70, 146-158 (2008).