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Vertex Polynomial for the Corona of Cycle Graphs with Some Standard Graphs

Sowmya. S

Assistant Professor,
Department of Mathematics,
Malankara Catholic College, Mariagiri, Tamilnadu, INDIA.
email: sowras@gmail.com.

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ABSTRACT

Let G be a graph. The vertex polynomial of the graph G is defined as $V(G,x)=\sum_{k=0}^{\Delta(G)} v_k x^k$ where $\Delta(G)=\max\{\deg(v)/v\in V(G)\}$ and v_k is the number of vertices of degree k. In this paper, I find the vertex polynomial for the corona of cycle graphs with some standard graphs such as path graph, cycle graph, wheel graph, fan graph, comb graph, complete graph and ladder graph.

Keywords: path, cycle, wheel, fan, comb, complete, ladder.

1. INTRODUCTION

Unless mentioned or otherwise, a graph in this paper shall mean a simple finite graph and without isolated vertices. We denote the vertex set by V(G) and the edge set by E(G). For $v\in V(G)$, $\deg(v)$ is the number of edges is incident with v, the maximum degree of G is defined as $\Delta(G)=\max\{\deg(v)/v\in V(G)\}$. For standard terminology and notations, we follow [1,2,3] . The corona of two graphs G_1 and G_2 is defined as $G_1 \Theta G_2$, formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2 . The corona , in particular, is the graph constructed from a copy of G, where for each vertex $v\in V(G)$, a new vertex v' and a pendant edge vv' are added.

Definition 1.1: The graph obtained by joining a single pendent edge to each vertex of a path is called Comb.

Definition 1.2: The Fan graph $F_{1,n}$ is a graph obtained by joining $\overline{K}_1$ to every vertices of a path graph P_n .

Definition 1.3: The Ladder graph $L_n = P_n \times K_2$, the Cartesian product of path of n vertices and complete graph on two vertices.

Theorem 1.4:

The vertex polynomial of $C_n \Theta C_n$ is $V(C_n \Theta C_n, x) = n^2 x^3 + nx^{n+2}$ for $n \geq 3$.

Proof: Let C_n be a cycle on n vertices. We can obtain $C_n \Theta C_n$ by attaching C_n to each vertex of C_n. Note that C_n is a graph with n(n+1) vertices and $2n^2$+n edges. In $C_n \Theta C_n$, n^2 vertices has degree 3 and n vertices has degree n+2. Hence $V(C_n \Theta C_n, x) = n^2 x^3 + nx^{n+2}$.

Example 1.5:

$C_3 \Theta C_3$:

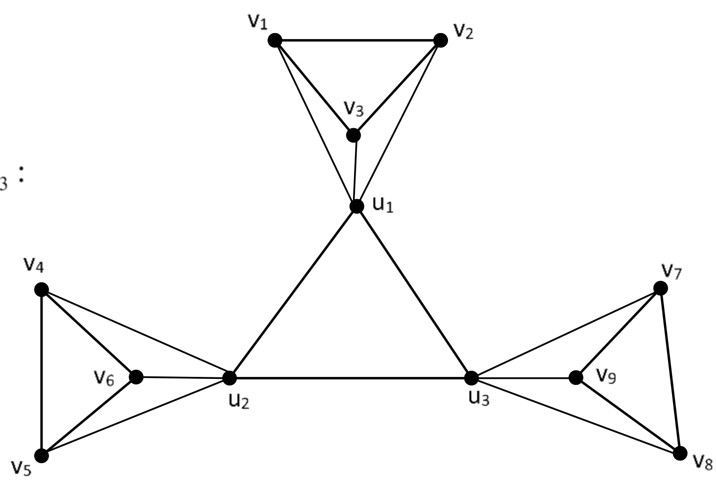

Figure 1.1

Theorem 1.6:

The vertex polynomial of $C_n \Theta P_n$ is $V(C_n \Theta P_n, x) = 2nx^2 + n(n-2)x^3 + nx^{n+2}$ for $n \geq 3$.

Proof: Let P_n be a path on n vertices. We can obtain $C_n \Theta P_n$ by attaching P_n to each vertex of C_n. Note that $C_n \Theta P_n$ is a graph with n^2+n vertices and $2n^2$ edges. In $C_n \Theta P_n$, 2n vertices has degree 2 and n(n-2) vertices has degree 3 and n vertices has degree n+2. Hence $V(C_n \Theta P_n, x) = 2nx^2 + n(n-2)x^3 + nx^{n+2}$.

Example 1.7:

$C_3\Theta P_3$:

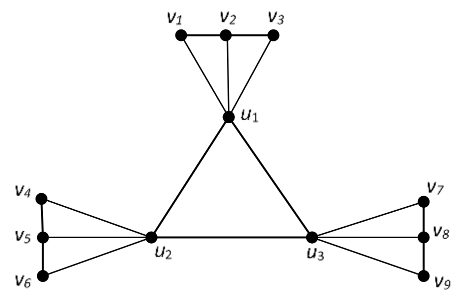

Figure 1.2

Theorem 1.8:

The vertex polynomial of $C_n \Theta K_n$ is $V(C_n \Theta K_n, x) = n^2 x^n + nx^{n+2}$ for $n \geq 3$.

Proof: Let K_n be a complete graph on n vertices. We can obtain $C_n \Theta K_n$ by attaching K_n to each vertex of C_n. Note that $C_n \Theta K_n$ is a graph with n vertices and nC_2 edges. In $C_n \Theta K_n$, n^2 vertices has degree n and n vertices has degree n+2. Hence $V(C_n \Theta K_n, x) = n^2 x^n + nx^{n+2}$.

Example 1.9:

$C_3\Theta K_4$:

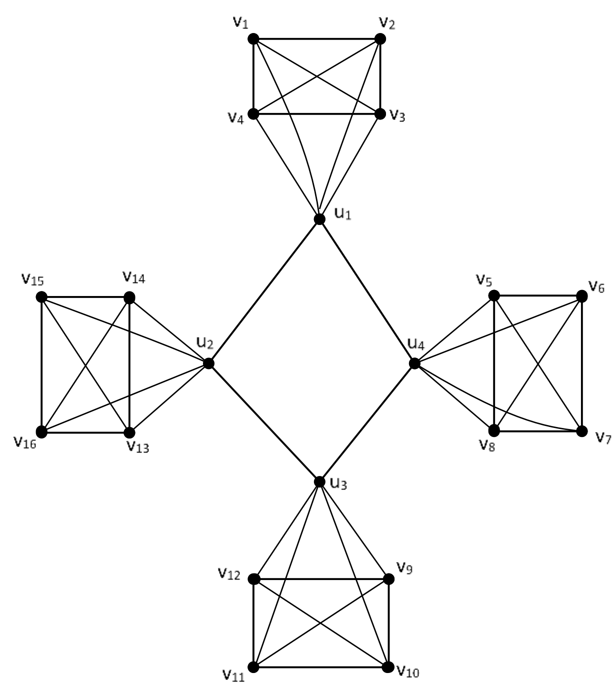

Figure 1.3

Example 1.11:

$C_3 \Theta G$ where $G = P_3 \Theta K_1$:

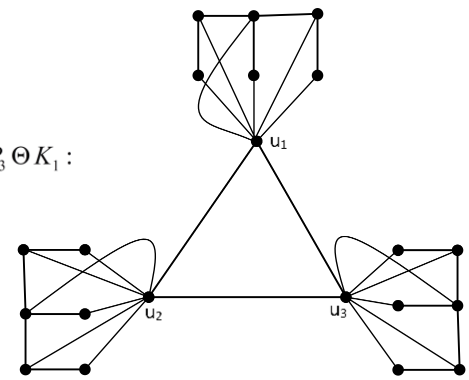

Figure 1.4

Theorem 1.12:

Let G be a Fan graph with order n+1. Then the vertex polynomial of $C_n \Theta G$ is $V(C_n \Theta G, x) = 2nx^3 + n(n-2)x^4 + nx^{n+1} + nx^{n+3}$ for all $n \geq 3$.

Proof: Let $G = F_{1,n}$ be a Fan graph on n+1 vertices . We can obtain by attaching G to each vertex of C_n. Note that $C_n \Theta G$ is a graph with n(3n+1) vertices and n(n+2) edges. In $C_n \Theta G$, 2n vertices has degree 3, n(n-2) vertices has degree 4 and n vertices has degree n+1 and n vertices has degree n+3. Hence $V(C_n \Theta G, x) = 2nx^3 + n(n-2)x^4 + nx^{n+1} + nx^{n+3}$.

Example 1.13:

$C_3 \Theta F_{1,4}$:

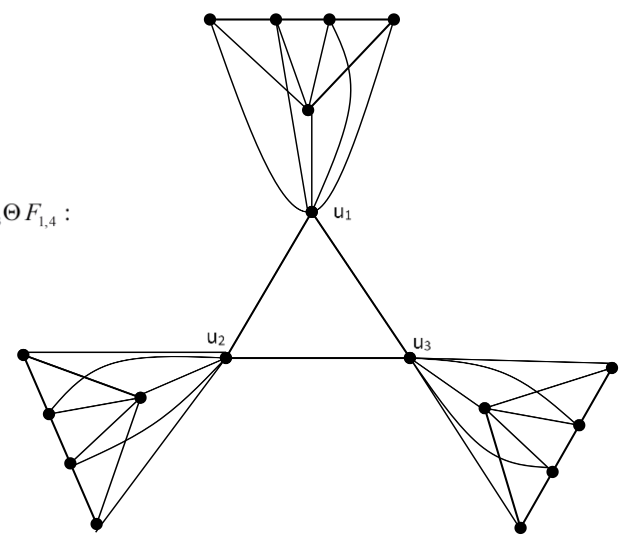

Figure 1.5

Theorem 1.14:

Let G be a Ladder graph with order 2n. Then the vertex polynomial of $C_n \Theta G$ is $V(C_n \Theta G, x) = 4nx^3 + n(2n-4)x^4 + nx^{2n+2}$ for all n $\geq$ 3.

Proof: Let G $= L_n$ be a Ladder graph on 2n vertices . We can obtain $C_n \Theta G$ by attaching G to each vertex of C_n. Note that $C_n \Theta G$ is a graph with n(2n+1) vertices and n(5n-1) edges. In $C_n \Theta G$, 4n vertices has degree 3 , n(2n-4) vertices has degree 4 and n vertices has degree 2n+2 vertices. Hence $V(C_n \Theta G, x) = 4nx^3 + n(2n-4)x^4 + nx^{2n+2}$.

Example 1.15:

$C_3 \Theta L_4$:

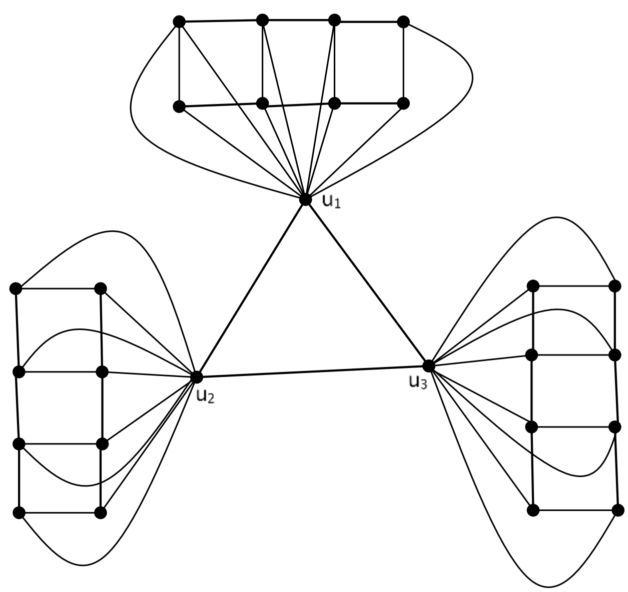

Figure 1.6

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