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Regular and Edge Regular Interval Valued Fuzzy Graphs

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ABSTRACT

In this paper, we study about regular, totally regular, edge regular and totally edge regular interval valued fuzzy graphs (IVFGs). We prove some theorems related to degree and total degree of vertices and edges of IVFGs. We expose some properties of regular interval valued fuzzy graphs (RIVFGs) and totally regular interval valued fuzzy graphs (TRIVFGs) through various examples. Also, we compare edge regular interval valued fuzzy graphs (ERIVFGs) and totally edge regular interval valued fuzzy graphs (TERIVFGs) and we derive a necessary and sufficient condition under which they are equivalent. We also prove some other theorems which reveal the relationship between these concepts.

Keywords: Interval valued fuzzy graph, Degree, Total degree, Edge degree, Total edge degree, Regular IVFG, Totally regular IVFG, Partially regular IVFG, Fully regular IVFG, Edge regular IVFG, Totally edge regular IVFG, Partially edge regular IVFG, Fully edge regular IVFG.

1. INTRODUCTION

Interval valued fuzzy graphs (IVFGs) defined by Hongmei and Lianhua[3] is a generalization of fuzzy graphs developed by Kaufmann[4] and Azriel Rosenfeld[11]. Both fuzzy graph theory and interval valued fuzzy graph theory are now growing and expanding its applications. Muhammad Akram and Wieslaw A. Dudek[1] defined the operations of Cartesian product, composition, union and join of IVFGS and investigated some properties. They also introduced the notion of interval valued fuzzy complete graphs and presented some properties of self complementary and self weak complementary interval valued fuzzy complete graphs. Since then, a lot of research work is being done in this area. Regularity and total regularity of edges and vertices are important topics of research in fuzzy graph theory. A. N. Gani and

K. Radha[6] introduced regular fuzzy graphs and totally regular fuzzy graphs. Also. K.Radha and N. Kumaravel[9] introduced edge regular fuzzy graphs and totally regular fuzzy graphs and studied its properties. M. Pal and H. Rashmanlou[7] defined regular and totally regular IVFGs. Total regularity of the join of two IVFGs was discussed by the authors in[12]. M. Pal, S. Samanta and H. Rashmanlou[8] defined the degree and total degree of an edge in an IVFG. S. R. Narayanan and N. R. Santhi Maheswari[10] introduced edge regular and totally edge regular IVFGs.

In this paper, we extend the notions of regularity, total regularity, edge regularity and total edge regularity to IVFGs. Several examples are given to bring out their properties. We also prove some theorems which reveal the relationship between these concepts.

2. BASIC CONCEPTS

Graph theoretic terms used in this work are either standard or are explained as and when they first appear. We consider only simple graphs. That is, graphs with multiple edges and loops are not considered.

2.1. Definition (Fuzzy Graph) [5]. Let V be a non empty set. A ***fuzzy graph*** is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. That is, $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V where $\sigma(u) \wedge \sigma(v)$ denotes minimum of $\sigma(u)\ and\ \sigma(v)$.

2.2. Definition (Interval number) [1]. An ***interval number*** D is an interval $[a^-, a^+]$ with $0 \leq a^- \leq a^+ \leq 1$.

2.3. Remark. The interval number $[a, a]$ is identified with the number $a \in [0,1]$. $D[0,1]$ denotes the set of all interval numbers.

2.4. Definition (Some operations on $\boldsymbol{D}[\mathbf{0},\mathbf{1}]$)[1]. For interval numbers $D_1 = [a_1^-, b_1^+]$ and $D_2 = [a_2^-, b_2^+]$

- $rmin(D_1, D_2) = [min\{a_1^-, a_2^-\}, min\{b_1^+, b_2^+\}]$
- $rmax(D_1, D_2) = [max\{a_1^-, a_2^-\}, max\{b_1^+, b_2^+\}]$
- $D_1 + D_2 = [a_1^- + a_2^- - a_1^- . a_2^- ,\ b_1^+ + b_2^+ - b_1^+ . b_2^+]$
- $D_1 \leq D_2 \Leftrightarrow a_1^- \leq a_2^-\ and\ b_1^+ \leq b_2^+$
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^-\ and\ b_1^+ = b_2^+$
- $D_1 < D_2 \Leftrightarrow D_1 \leq D_2\ and\ D_1 \neq D_2$
- $kD = k[a_1^-, b_1^+] = [ka_1^-,\ kb_1^+]\ where\ 0 \leq k \leq 1.$

Then $(D[0,1], \leq, \vee, \wedge)$ is a complete lattice with $[0,0]$ as the least element and $[1,1]$ as the greatest. Here $\vee$ denotes *rmax* and $\wedge$ denotes *rmin*.

2.5. Definition (Interval valued fuzzy set (IVFS)) [1]. The ***interval valued fuzzy set*** A in V is defined by $A = \{(x, [\mu_A^-(x), \mu_A^+(x)]): x \in V\}$ where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of V such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$. We shall sometimes denote the IVFS A by $[\mu_A^-(x), \mu_A^+(x)]$.

For any two IVFSs $A = [\mu_A^-(x), \mu_A^+(x)]$ and $B = [\mu_B^-(x), \mu_B^+(x)]$ in V, we define

- $A \cup B = \left\{\left(x, max\left(\mu_A^-(x), \mu_B^-(x)\right), max\left(\mu_A^+(x), \mu_B^+(x)\right)\right) : x \in V\right\}$
- $A \cap B = \left\{\left(x, min\left(\mu_A^-(x), \mu_B^-(x)\right), min\left(\mu_A^+(x), \mu_B^+(x)\right)\right) : x \in V\right\}$

2.6. Definition (Interval valued fuzzy relation (IVFR)) [1]. If $G^* = (V, E)$ is a graph, then by an ***interval valued fuzzy relation*** B on the set E we mean an IVFS such that $\mu_B^-(xy) \leq min\left(\mu_A^-(x), \mu_A^-(y)\right)$ *and* $\mu_B^+(xy) \leq min\left(\mu_A^+(x), \mu_A^+(y)\right)$ for all $xy \in E$.

2.7. Definition (Interval valued fuzzy graph (IVFG)) [1]. By an ***interval valued fuzzy graph*** of a graph $G^* = (V, E)$, we mean a pair $G = (A, B)$, where $A = [\mu_A^-(x), \mu_A^+(x)]$ is an IVFS on $V$and $B = [\mu_B^-(x), \mu_B^+(x)]$ is an IVFR on E.

2.8. Definition (Complete Interval valued fuzzy graph (CIVFG)) [1]. An IVFG $G = (A, B)$ is said to be a ***complete interval valued fuzzy graph*** if $\mu_B^-(xy) = min\left(\mu_A^-(x), \mu_A^-(y)\right)$ and $\mu_B^+(xy) = min\left(\mu_A^+(x), \mu_A^+(y)\right)$ for all $x, y \in V$.

In what follows $G = (A, B)$ denotes such an IVFG of a graph $G^* = (V, E)$ where V is a non-empty finite set and $E \subseteq V \times V$.

2.9. Definition (Order of an IVFG)[7]. The ***order*** of G denoted by $O(G)$ is defined by $O(G) = [O^-(G), O^+(G)]$ where $O^-(G) = \sum_{u \in V} \mu_A^-(u)$ and $O^+(G) = \sum_{u \in V} \mu_A^+(u)$.

2.10. Definition (Size of an IVFG) [7]. The ***size*** of G denoted by $S(G)$ is defined by $\mathcal{S}(G) = [S^-(G), S^+(G)]$ where $S^-(G) = \sum_{\substack{uv \in E \\ u \neq v}} \mu_B^-(uv)$ and $S^+(G) = \sum_{\substack{uv \in E \\ u \neq v}} \mu_B^+(uv)$.

2.11. Definition (Degree of a vertex) [7]. The ***negative degree*** of a vertex $u \in V$ is defined by $d^-(u) = \sum_{uv \in E} \mu_B^-(uv)$. Similarly, ***positive degree*** of a vertex $u \in V$ is defined by $d^+(u) = \sum_{uv \in E} \mu_B^+(uv)$. Then the ***degree*** of the vertex $u \in V$ is defined as $d(u) = [d^-(u), d^+(u)]$.

2.12. Definition (Total degree (TD) of a vertex) [7]. The total ***degree*** of the vertex $u \in V$ is defined as

$td(u) = [td^-(u), td^+(u)]$ where,

$$td^-(u) = \sum_{uv \in E} \mu_B^-(uv) + \mu_A^-(u) = d^-(u) + \mu_A^-(u) \text{ and}$$

$$td^+(u) = \sum_{uv \in E} \mu_B^+(uv) + \mu_A^+(u) = d^+(u) + \mu_A^+(u).$$

$\therefore td(u) = [d^-(u) + \mu_A^-(u), d^+(u) + \mu_A^+(u)] = [d^-(u), d^+(u)] + [\mu_A^-(u), \mu_A^+(u)]$

$= d(u) + [\mu_A^-, \mu_A^+](u)$

1.13. Definition (Degree of an edge) [8] The ***degree of an edge*** uv is defined as $d_G(uv) = [d_G^-(uv), d_G^+(uv)]$ where $d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2\,\mu_B^-(uv)$ and $d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2\,\mu_B^+(uv)$. Equivalently,

$$d_G^-(uv) = \sum_{\substack{uw \in E \\ w \neq v}} \mu_B^-(uw) + \sum_{\substack{wv \in E \\ w \neq u}} \mu_B^-(wv),$$

$$d_G^+(uv) = \sum_{\substack{uw\in E\\ w\neq v}} \mu_B^+(uw) + \sum_{\substack{wv\in E\\ w\neq u}} \mu_B^+(wv)$$

2.14. Definition (Total degree of an edge) [8]. The total degree of an edge uv is defined as $td_G(uv) = [td_G^-(uv), td_G^+(uv)]$where $td_G^-(uv) = d_G^-(u) + d_G^-(v) - \mu_B^-(uv)$ and $td_G^+(uv) = d_G^+(u) + d_G^+(v) - \mu_B^+(uv)$.
This is equivalent to:

$$td_G^-(uv) = \sum_{\substack{uw\in E\\ w\neq v}} \mu_B^-(uw) + \sum_{\substack{wv\in E\\ w\neq u}} \mu_B^-(wv) + \mu_B^-(uv) = d_G^-(uv) + \mu_B^-(uv)$$

$$td_G^+(uv) = \sum_{\substack{uw\in E\\ w\neq v}} \mu_B^+(uw) + \sum_{\substack{wv\in E\\ w\neq u}} \mu_B^+(wv) + \mu_B^+(uv) = d_G^+(uv) + \mu_B^+(uv).$$

Hence, $\quad td_G(uv) = [td_G^-(uv), td_G^+(uv)] = d_G(uv) + [\mu_B^-, \mu_B^+](uv)$.

3. SOME THEOREMS RELATED TO DEGREE AND TOTAL DEGREE OF VERTICES AND EDGES OF IVFGs

Throughout this section $G = (A, B)$ denotes an IVFG of a graph $G^* = (V, E)$ where $A = [\mu_A^-(x), \mu_A^+(x)]$ and
B= $[\mu_B^-(x), \mu_B^+(x)]$ are IVFSs on a non-empty finite set V and $E \subseteq V \times V$ respectively. The proofs of the next three propositions are straight forward, and so are omitted.

3.1. Proposition. $\sum_{u\in V} d_G(u) = 2S(G)$ ■

3.2. Proposition. $\sum_{u\in V} td_G(u) = 2S(G) + O(G)$ ■

3.3. Proposition. $\sum_{uv\in E} d_G(uv) = \sum_{uv\in E} d_G^*(uv)[\mu_B^-(uv), \mu_B^+(uv)]$ ■

3.4. Theorem. Let $G = (A, B)$ be an IVFG on a k–regular graph $G^* = (V, E)$.Then $\sum_{uv\in E} d_G(uv) = 2(k-1)S(G)$.

Proof: By proposition 3.3, $\sum_{uv\in E} d_G(uv) = \sum_{uv\in E} d_G^*(uv)[\mu_B^-(uv), \mu_B^+(uv)]$.
Since G^* is k – regular, $d_G^*(uv) = d_G^*(u) + d_G^*(v) - 2 = k + k - 2 = 2k - 2 = 2(k-1)$
$\therefore \sum_{uv\in E} d_G(uv) = \sum_{uv\in E} 2(k-1)[\mu_B^-(uv), \mu_B^+(uv)] = 2(k-1)\sum_{uv\in E}[\mu_B^-(uv), \mu_B^+(uv)]$
$= 2(k-1)[\sum_{uv\in E} \mu_B^-(uv), \sum_{uv\in E} \mu_B^+(uv)] = 2(k-1)S(G)$ ■

3.5. Corollary. Let $G = (A, B)$ be an IVFG on a fuzzy cycle $G^* = (V, E)$. Then $\sum_{u\in V} d_G(u) = \sum_{uv\in E} d_G(uv)$.

Proof: From proposition 3.1, $\sum_{u\in V} d_G(u) = 2S(G)$
Since G^* is a fuzzy cycle, G^* is k – regular with $k = 2$.
$\therefore$ From theorem 3.4, $\sum_{uv\in E} d_G(uv) = 2(k-1)S(G) = 2(2-1)S(G) = 2S(G)$
Hence $\sum_{u\in V} d_G(u) = \sum_{uv\in E} d_G(uv)$ ■

3.6. Theorem. Let $G = (A, B)$ be an IVFG on a k –regular graph $G^* = (V, E)$. Then $\sum_{uv\in E} td_G(uv) = \sum_{uv\in E} d_G^*(uv)[\mu_B^-, \mu_B^+](uv) + S(G)$.

Proof: The size of G, $S(G) = \left[\sum_{\substack{uv\in E\\ u\neq v}} \mu_B^-(uv), \sum_{\substack{uv\in E\\ u\neq v}} \mu_B^+(uv)\right]$.

$\sum_{uv\in E} td_G(uv) = \sum_{uv\in E}[td_G^-(uv), td_G^+(uv)] = \sum_{uv\in E}[d_G^-(uv) + \mu_B^-(uv), d_G^+(uv) + \mu_B^+(uv)]$

$= \sum_{uv\in E}[d_G^-(uv), d_G^+(uv)] + \sum_{uv\in E}[\mu_B^-(uv), \mu_B^+(uv)]$

$= \sum_{uv\in E} d_G(uv) + [\sum_{uv\in E}\mu_B^-(uv), \sum_{uv\in E}\mu_B^+(uv)]$

$= \sum_{uv\in E} d_G^*(uv)[\mu_B^-, \mu_B^+](uv) + S(G)$, by proposition 3.3 and definition 2.10 ■

4. REGULAR AND TOTALLY REGULAR IVFGs

4.1. Definition (Regular interval valued fuzzy graph(RIVFG)) [7]. If $d^-(u) = k_1, d^+(u) = k_2$ for all $u \in V$ and k_1, k_2 are real numbers, then the graph G is called $[\boldsymbol{k_1}, \boldsymbol{k_2}]$ ***- regular IVFG*** or ***regular IVFG of degree*** $[\boldsymbol{k_1}, \boldsymbol{k_2}]$.

4.2. Definition (Totally regular interval valued fuzzy graph(TRIVFG)) [7]. If $td^-(u) = k_1^{'}$, $td^+(u) = k_2^{'}$ for all $u \in V$ and $k_1^{'}, k_2^{'}$ are real numbers, then the graph G is called $[\boldsymbol{k_1^{'}}, \boldsymbol{k_2^{'}}]$ - ***totally regular IVFG*** or ***totally regular IVFG of degree*** $[\boldsymbol{k_1^{'}}, \boldsymbol{k_2^{'}}]$.

4.3. Example. Any connected IVFG with two vertices is regular.

4.4. Remark. In crisp graph theory, any complete graph is regular. But this result does not carry over to IVFGs. That is, a complete IVFG (CIVFG) need not be regular.

4.5. Example. The following diagram presents a CIVFG which is not regular.

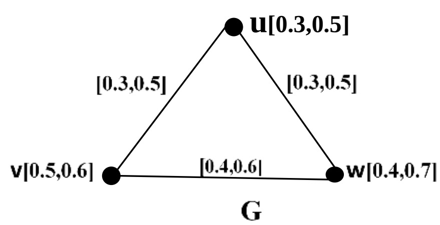


Fig 4.1: An example to show that a CIVFG need not be regular

Clearly G is a CIVFG. But $d(u) = [0.6,1.0]$, $d(v) = [0.7,1.1]$ and $d(w) = [0.7,1.1]$ so that $d(u) \neq d(v)$.
$\therefore G$ is not regular.

4.6 Remark. G is a RIVFG need not imply that G^* is regular. Also G^* is regular does not imply that G is a RIVFG.

In this context, we define partially regular and full regular IVFGs.

4.7. Definition (Partially regular IVFG (PRIVFG)). If the underlying graph G^* is regular, then G is said to be a ***partially regular IVFG.***

4.8. Definition (Fully regular IVFG (FRIVFG)). If G is both regular and partially regular, then G is said to be a ***fully regular IVFG.***

4.9. Theorem. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$ such that $B = [\mu_B^-, \mu_B^+]$ is a constant function. Then G is a RIVFG if and only if G is a PRIVFG.

Proof: Let $B = [\mu_B^-, \mu_B^+] = [c_1, c_2]$ where c_1 and c_2 are constants. Also let G be a $[k_1, k_2] -$ RIVFG. Then

G is a RIVFG $\Leftrightarrow d_G(u) = d_G(w)\ \forall\, u, w \in V$

$\Leftrightarrow [\sum_{uv\in E} \mu_B^-(u), \sum_{uv\in E} \mu_B^+(u)] = [\sum_{wv\in E} \mu_B^-(w), \sum_{wv\in E} \mu_B^+(w)]$

$\Leftrightarrow [\sum_{uv\in E} c_1, \sum_{uv\in E} c_2] = [\sum_{wv\in E} c_1, \sum_{wv\in E} c_2]$

$\Leftrightarrow \sum_{uv\in E} c_1 = \sum_{wv\in E} c_1 \ and\ \sum_{uv\in E} c_2 = \sum_{wv\in E} c_2$

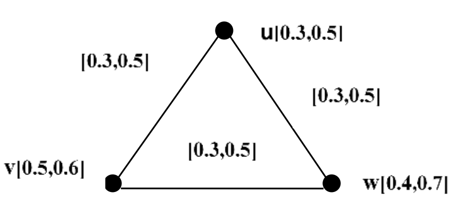
$\Leftrightarrow$ Number of edges adjacent to u = Number of edges adjacent to w

$\Leftrightarrow d_{G^*}(u) = d_{G^*}(w)\ \forall\, u, w \in V$

$\Leftrightarrow$ G is a PRIVFG ■

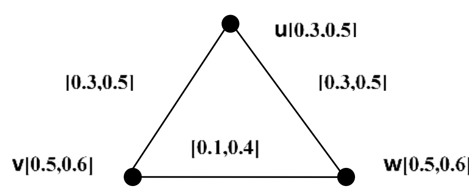
4.10. Remark. RIVFGs and TRIVFGs are independent concepts. This can be observed which is clear from the following examples.

4.11. Example



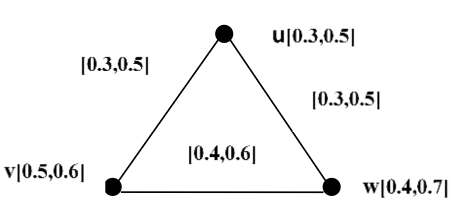
G_1

Fig 4.2: An example to show that a RIVFG need not be a TRIVFG



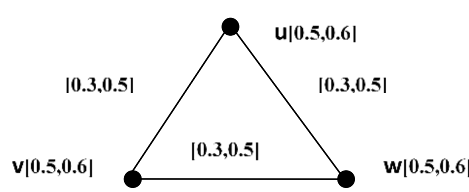
G_2

Fig 4.3: An example to show that a TRIVFG need not be a RIVFG



G_1

Fig 4.4: Example of an IVFG that is neither a RIVFG nor a TRIVFG



G_2

Fig 4.5: Example of an IVFG that is both a RIVFG and a TRIVFG

Here in Fig 4.2, $d(u) = d(v) = d(w) = [0.6,1.0]$. $\therefore$ $\boldsymbol{G_1}$ is a RIVFG. But, $td(u) = [0.9,1.5], td(v) = [1.1,1.6]$, $td(w) = [1.0,1.7]$ so that $td(u) \neq td(v) \neq td(w)$. Hence $\boldsymbol{G_1}$ is not a TRIVFG.

In Fig 4.3, $td(u) = td(v) = td(w) = [0.9,1.5]$. $\therefore$ $\boldsymbol{G_2}$ is a TRIVFG. But, $d(u) = [0.6,1.0], d(v) = [0.4,0.9]$, $d(w) = [0.4,0.9]$ so that $d(u) \neq d(v)$ Hence $\boldsymbol{G_2}$ is not a RIVFG.

In Fig 4.4, $d(u) = [0.6,1.0], d(v) = [0.7,1.1]$, $d(w) = [0.7,1.1]$ so that $d(u) \neq d(v)$. Hence $\boldsymbol{G_3}$ is not a RIVFG. Also, $td(u) = [0.9,1.5], td(v) = [1.2,1.7]$, $td(w) = [1.1,1.8]$ so that $td(u) \neq td(v) \neq td(w)$. Hence $\boldsymbol{G_3}$ is not a TRIVFG. Thus $\boldsymbol{G_3}$ is neither a RIVFG nor a TRIVFG.

In Fig 4.5, $d(u) = d(v) = d(w) = [0.6,1.0]$ and $td(u) = td(v) = td(w) = [1.1,1.6]$. $\therefore$ $\boldsymbol{G_4}$ is both a RIVFG and a TRIVFG.

The following theorem provides a necessary and sufficient condition under which a RIVFG and a TRIVFG are equivalent.

4.12. Theorem [2]. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$. Then $A = [\mu_A^-, \mu_A^+]$ is a constant function if and only if the following are equivalent:

1. G is a RIVFG.
2. G is a TRIVFG ■

1.14. Theorem. If an IVFG G is both regular and totally regular, then $A = [\mu_A^-, \mu_A^+]$ is a constant function.

Proof: Let G be a $[k_1, k_2]$ − RIVFG and a $[k_1', k_2']$ - TRIVFG.

Hence $d(u) = [k_1, k_2]$ and $td(u) = [k_1', k_2']$ $\forall\, u \in V$

Now, $td(u) = [k_1', k_2']$ $\forall\, u \in V$

$\Longrightarrow d(u) + [\mu_A^-, \mu_A^+](u) = [k_1', k_2']$ $\forall\, u \in V$

$\Longrightarrow [k_1, k_2] + [\mu_A^-, \mu_A^+](u) = [k_1', k_2']$ $\forall\, u \in V$

$\Longrightarrow [\mu_A^-, \mu_A^+](u) = [k_1', k_2'] - [k_1, k_2]$ $\forall\, u \in V$

$\Longrightarrow [\mu_A^-, \mu_A^+](u) = [k_1' - k_1, k_2' - k_2]$, $a\ constant$ $\forall\, u \in V$

Hence $A = [\mu_A^-, \mu_A^+]$ is a constant function ■

The following example shows that the converse of the above theorem is not true.

4.14. Example

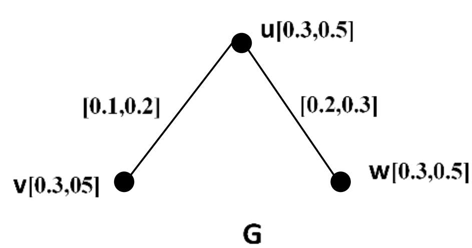


Fig 4.6: Example to show that converse of theorem 4.13 is not true

In Fig 4.6, A= $[\mu_A^-, \mu_A^+]$ is a constant function. But $d(u) = [0.3,0.5], d(v) = [0.1,0.2]$, $d(w) = [0.2,0.3]$ so that $d(u) \neq d(v) \neq d(w)$. Hence $\boldsymbol{G}$ is not a RIVFG. Also, $td(u) = [0.6,1.0], td(v) = [0.4,0.7]$, $td(w) = [0.5,0.8]$ so that $td(u) \neq td(v) \neq td(w)$. Hence $\boldsymbol{G}$ is not a TRIVFG. Thus $\boldsymbol{G}$ is neither a RIVFG nor a TRIVFG.
Theorem 4.15 and Theorem 4.17 provide a characterization of a RIVFG $G = (A, B)$ such that $G^* = (V, E)$ is a cycle.

4.15. Theorem [2]. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$ such that $G^* = (V, E)$ is an odd cycle. Then G is a RIVFG if and only if B= $[\mu_B^-, \mu_B^+]$ is a constant function ■
The following example shows that the above theorem does not hold for TRIVFGs.

4.16. Example.

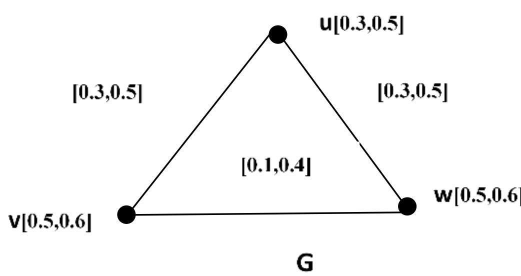


Fig 4.7: Example to show that theorem 4.15 does not hold for TRIVFGs

Here, G is an IVFG on G^* which is an odd cycle. And $td(u) = td(v) = td(w) = [0.9,1.5]$. $\therefore G$ is a TRIVFG. But B= $[\mu_B^-, \mu_B^+]$ is not a constant function.

4.17. Theorem [7]. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$ such that $G^* = (V, E)$ is an even cycle. Then G is a RIVFG if and only if either B= $[\mu_B^-, \mu_B^+]$ is a constant function or alternate edges have same membership values ■
The following example shows that the above theorem also does not hold for TRIVFGs.

4.18. Example. Here G is an IVFG defined on G^* which is an even cycle.:

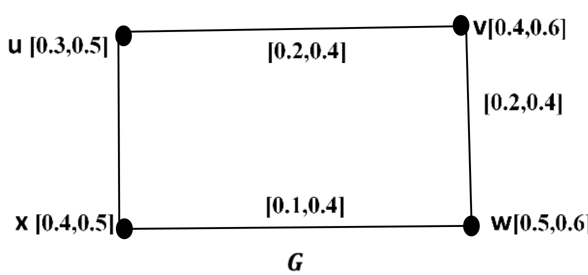


Fig 4.8: Example to show that theorem 4.17 does not hold for TRIVFGs

$td(u) = td(v) = td(w) = td(x) = [0.8,1.4]$. $\therefore G$ is a TRIVFG. But $B = [\mu_B^-, \mu_B^+]$ is neither a constant function nor alternate edges have same membership values.

5. EDGE REGULAR AND TOTALLY EDGE REGULAR IVFGs

5.1. Definition (Edge regular IVFG (ERIVFG))[10]. Let $G = (A, B)$ be an IVFG. If $d^-(uv) = c_1, d^+(uv) = c_2$ for all $uv \in E$ and c_1, c_2 are real numbers, then G is called $[c_1, c_2]$ – ***edge regular IVFG or a an edge regular IVFG of edge degree*** $[c_1, c_2]$.

5.2. Definition (Totally edge regular IVFG (TERIVFG))[10]. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$. If each edge in G has the same total degree $[c_1', c_2']$, then G is called ***a totally edge regular IVFG of total edge degree*** $[c_1', c_2']$ ***or*** $[c_1', c_2']$ ***- totally edge regular IVFG.***

5.3. Remark. In crisp graph theory, any complete graph is edge regular. But this result does not carry over to IVFGs. A CIVFG need not be edge regular.

5.4. Example

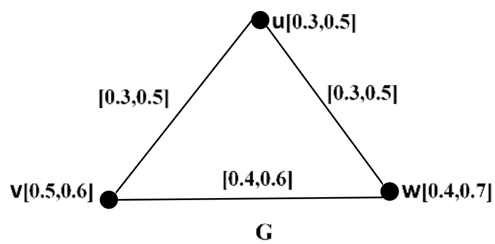


Fig 5.1: An example to show that CIVFG need not be edge regular

Clearly G is a CIVFG. But $d(uv) = [0.7,1.1]$, $d(uw) = [0.7,1.1]$ and $d(vw) = [0.6,1.0]$ so that $d(uv) \neq d(vw)$. $\therefore G$ is not an ERIVFG.

5.5. Remark. In general, there does not exist any relationship between edge regular and totally edge regular IVFGs. The following examples support this observation.

5.6. Example

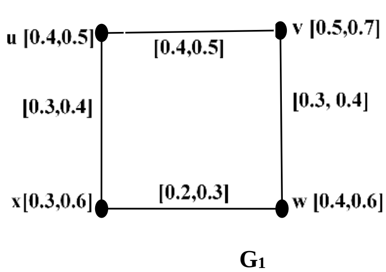


Fig 5.2: Example to show that an ERIVFG need not be a TERIVFG

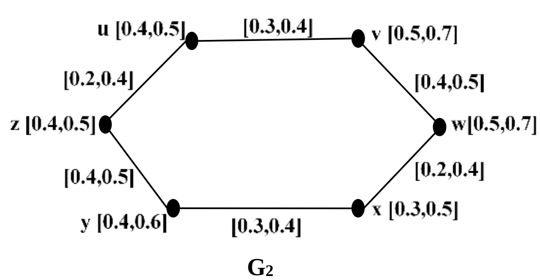
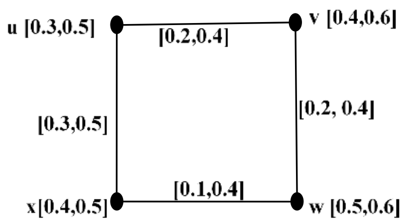
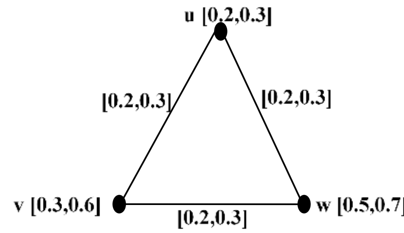


Fig 5.3: Example to show that a TERIVFG need not be an ERIVFG



G3

Fig 5.4: Example of an IVFG that is neither an ERIVFG nor a TERIVFG



G4

Fig 5.5: Example of an IVFG that is both an ERIVFG and TERIVFG

Here in Fig 5.2, $d(uv) = d(vw) = d(wx) = d(xu) = [0.6,0.8]$. $\therefore$ $\boldsymbol{G_1}$ is an ERIVFG. But, $td(uv) = [1.0,1.3], td(vw) = [0.9,1.2], td(wx) = [0.8,1.1], td(xu) = [0.9,1.2]$ *so that* $td(uv) \neq td(vw) \neq td(wx)$. Hence $\boldsymbol{G_1}$ is not a TERIVFG.

In Fig 5.3, $td(uv) = td(vw) = td(wx) = td(xy) = td(yz) = td(zu) = [0.9,1.3]$. $\therefore$ $\boldsymbol{G_2}$ is a TERIVFG. But, $d(uv) = [0.6,0.9], d(vw) = [0.5,0.8], d(wx) = [0.7,0.9], d(xy) = [0.6,0.9], d(yz) = [0.5,0.8], d(zu) = [0.7,0.9]$ so that $d(uv) \neq d(vw) \neq d(wx)$ Hence $\boldsymbol{G_2}$ is not an ERIVFG.

In Fig 5.4, $d(uv) = [0.5,0.9], d(vw) = [0.3,0.8], d(wx) = [0.5,0.9], d(xu) = [0.3,0.8]$ so that $d(uv) \neq d(vw)$. Hence $\boldsymbol{G_3}$ is not an ERIVFG. Also, $td(uv) = [0.7,1.3]$, td$(vw) = [0.5,1.2]$, $td(w) = [0.6,1.3], td(xu) = [0.6,1.3]$ so that $td(uv) \neq td(vw) \neq td(wx)$. Hence $\boldsymbol{G_3}$ is not a TERIVFG. Thus $\boldsymbol{G_3}$ is neither an ERIVFG nor a TERIVFG. In Fig 5.5, $d(uv) = d(vw) = d(uw) = [0.4,0.6]$ *and* $td(uv) = td(vw) = td(uw) = [0.6,0.9]$. $\therefore$ $\boldsymbol{G_4}$ is both an ERIVFG and a TERIVFG.

Now we proceed to obtain a necessary and sufficient condition under which an ERIVFG and a TERIVFG are equivalent.

5.7. Theorem. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$. Then $\mathcal{B} = [\mu_B^-, \mu_B^+]$ is a constant function if and only if the following are equivalent:

1. G is an ERIVFG.
2. G is a TERIVFG.

Proof: Suppose that $\mathcal{B} = [\mu_B^-, \mu_B^+]$ is a constant function. Let $[\mu_B^-, \mu_B^+](uv) = [c_1, c_2]\ \forall\, uv \in E$ where c_1 and c_2 are constants.

To prove (1) $\Longrightarrow$ (2) : Assume that G is a $[k_1, k_2]$ −ERIVFG.

Then, $d_G(uv) = [k_1, k_2]\ \forall\, uv \in E$.

$$\therefore\ td_G(uv) = [td_G^-(uv), td_G^+(uv)]$$
$$= [d_G^-(uv) + \mu_B^-(uv), d_G^+(uv) + \mu_B^+(uv)]$$
$$= [k_1 + c_1, k_2 + c_2]\ \forall\, uv \in E.$$

Hence G is a $[k_1 + c_1, k_2 + c_2]$ − TERIVFG.

To prove (2) $\Longrightarrow$ (1) : Now, suppose that G is a $[k_1', k_2']$ − TERIVFG.

Then, $td_G(uv) = [k_1', k_2']\ \forall\, uv \in E$.

$\Longrightarrow [d_G^-(uv) + \mu_B^-(uv), d_G^+(uv) + \mu_B^+(uv)] = [k_1', k_2'] \ \forall\, uv \in E$
$\Longrightarrow [d_G^-(uv), d_G^+(uv)] = [k_1' - \mu_B^-(uv), k_2' - \mu_B^+(uv)] \ \forall\, uv \in E$
$\Longrightarrow d_G(uv) = [k_1' - c_1, k_2' - c_2] \ \forall\, uv \in E.$
Hence G is a $[k_1' - c_1, k_2' - c_2]$ −ERIVFG. Therefore, (1) and (2) are equivalent.
Conversely, assume that (1) and (2) are equivalent. i.e. G is an ERIVFG if and only if G is a TERIVFG. We prove that $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant function.
Suppose $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is not a constant function.
Then $[\mu_B^-, \mu_B^+](uv) \neq [\mu_B^-, \mu_B^+](xy)$ for atleast one pair of edges $uv, xy \in E$.
Let G be a $[k_1, k_2]$ −ERIVFG.Then $d_G(uv) = d_G(xy) = [k_1, k_2]$
$td_G(uv) = d_G(uv) + [\mu_B^-, \mu_B^+](uv)$
$= [k_1, k_2] + [\mu_B^-, \mu_B^+](uv)$ and $td_G(xy) = d_G(xy) + [\mu_B^-, \mu_B^+](xy)$
$= [k_1, k_2] + [\mu_B^-, \mu_B^+](xy)$
Since $[\mu_B^-, \mu_B^+](uv) \neq [\mu_B^-, \mu_B^+](xy)$, we have $td_G(uv) \neq td_G(xy)$
Hence G is not a TERIVFG which is a contradiction to our assumption.
Now, let G be a TERIVFG.
Then, $td_G(uv) = td_G(xy)$
$\Longrightarrow d_G(uv) + [\mu_B^-, \mu_B^+](uv) = d_G(xy) + [\mu_B^-, \mu_B^+](xy)$
$\Longrightarrow d_G(uv) - d_G(xy) = [\mu_B^-, \mu_B^+](xy) - [\mu_B^-, \mu_B^+](uv) \neq 0 \ [Since\ [\mu_B^-, \mu_B^+](uv) \neq [\mu_B^-, \mu_B^+](xy)]$
$\therefore d_G(uv) \neq d_G(xy)$
Hence G is not a TERIVFG which is a contradiction to our assumption. Hence $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant function ■

5.8. Theorem. If an IVFG G is both edge regular and totally edge regular, then $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant function.

Proof : Let G be a $[k_1, k_2]$ −ERIVFG and $[k_1', k_2']$ -TERIVFG.
Hence $d_G(uv) = [k_1, k_2]\ and\ td_G(uv) = [k_1', k_2'] \ \forall\, uv \in E$
Now, $td_G(uv) = [k_1', k_2'] \ \forall\, uv \in E$
$\Longrightarrow d_G(uv) + [\mu_B^-, \mu_B^+](uv) = [k_1', k_2'] \ \forall\, uv \in E$
$\Longrightarrow [k_1, k_2] + [\mu_B^-, \mu_B^+](uv) = [k_1', k_2'] \ \forall\, uv \in E$
$\Longrightarrow [\mu_B^-, \mu_B^+](uv) = [k_1', k_2'] - [k_1, k_2] \ \forall\, uv \in E$
$\Longrightarrow [\mu_B^-, \mu_B^+](uv) = [k_1' - k_1, k_2' - k_2]\ , a\ constant \ \forall\, uv \in E$
Hence $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant function ■
The following example shows that converse of the above theorem is not true.

5.9. Example

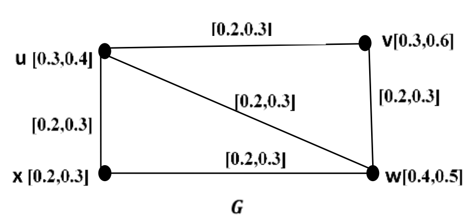


Fig 5.6: Example to show that converse of theorem 5.8. is not true

In Fig 5.6, $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant. Also $d(uv) = d(vw) = d(wx) = d(xu) =$ $[0.6,0.9]. But, d(uw) = [0.8,1.2]$ so that $d(uv) = d(vw) = d(wx) = d(xu) \neq d(uw)$. Hence $\boldsymbol{G}$ is not an ERIVFG. Again, $td(uv) = [0.8,1.2]$ and $td(uw) = [1.0,1.5]$ so that $td(uv) \neq td(uw)$. Hence $\boldsymbol{G}$ is not a TERIVFG. Thus $\boldsymbol{G}$ is neither an ERIVFG nor a TERIVFG.

5.10. Definition(*Edge regular graph (ERG)).* Let $G^* = (V, E)$ be a graph. Then G^* is said to be ***edge regular*** if each edge in G^* has the same degree.

5.11. Definition (Partially edge regular IVFG (PERIVFG)). If the underlying graph $G^*$is edge regular, then G is said to be a ***partially edge regular IVFG***.

5.12. Definition (Fully edge regular IVFG (FERIVFG)). If G is both edge regular and partially edge regular, then G is said to be a ***fully edge regular IVFG***.

5.13. Theorem. Let $G = (A, B)$ be an IVFG on a graph $G^* = (V, E)$ such that $B = [\mu_B^-, \mu_B^+]$ is a constant function. If G is a FRIVFG, then G is a FERIVFG.

Proof : Given that $B = [\mu_B^-, \mu_B^+]$ is a constant function. Then $[\mu_B^-, \mu_B^+] = [c_1, c_2]$ where c_1 and c_2 are constants. Assume that G is a FRIVFG.
Then $d_G(u) = [k_1, k_2]$ and $d_G^*(u) = r \; \forall \, u \in V$ and k_1, k_2, r are constants.
Hence $d_G^*(uv) = d_G^*(u) + d_G^*(v) - 2 = r + r - 2 = 2r - 2, a\ constant.$
Hence G^* is edge regular.
Now,

$$\begin{aligned} d_G(uv) &= [d_G^-(uv), d_G^+(uv)] \\ &= [d_G^-(u) + d_G^-(v) - 2\mu_B^-(uv), d_G^+(u) + d_G^+(v) - 2\mu_B^+(uv)] \\ &= [k_1 + k_1 - 2c_1, k_2 + k_2 - 2c_2] \\ &= [2(k_1 - c_1), 2(k_2 - c_2)], \text{ a constant.} \end{aligned}$$

Hence G is edge regular.Thus G is edge regular and G^* is edge regular. Hence G is a FERIVFG ■
The following example shows that the converse of the above theorem is not be true.

5.14. Example

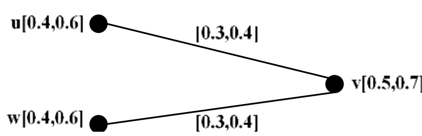


Fig 5.7: Example to show that converse of theorem 5.13. is not true

In Fig 5.7, $\mathcal{B}$= $[\mu_B^-, \mu_B^+]$ is a constant and $\boldsymbol{G}$ is full edge regular since G^* is edge regular and $d(uv) = d(vw) = [0.3,0.4]$.But, $d(u) = [0.3,0.4]\ and\ d(v) = [0.6,0.8]$ so that $d(u) \neq d(v)$. Hence $\boldsymbol{G}$ is not regular and so $\boldsymbol{G}$ is not full regular.

6. SOME PROPERTIES OF REGULAR AND EDGE REGULAR IVFGs.

6.1. Theorem [7]. The size of a $[k_1, k_2] -$ RIVFG $G = (A, B)$ on a graph $G^* = (V, E)$ is $\left[\frac{Pk_1}{2}, \frac{Pk_2}{2}\right]$ where $P = |V|$.

6.2. Theorem [7]. Let $G = (A, B)$ be a $[k_1', k_2'] -$ TRIVFG. Then $2S(G) + O(G) = P[k_1', k_2']$ where $P = |V|$.

6.3.Theorem. Let $G = (A, B)$ be a $[k_1, k_2] -$ RIVFG and $[k_1', k_2'] -$ TRIVFG. Then $O(G) = [P(k_1' - k_1), P(k_2' - k_2)]$ where $P = |V|$.

Proof : From proposition 3.1, $2S(G) = P[k_1, k_2]$ where $P = |V|$.
Again from proposition 3.2, $2S(G) + O(G) = P[k_1', k_2']$ where $P = |V|$.
That is, $P[k_1, k_2] + O(G) = P[k_1', k_2']$ where $P = |V|$.
$\therefore O(G) = [P(k_1' - k_1), P(k_2' - k_2)]$ where $P = |V|$ ■

6.4. Theorem. The size of a $[k_1, k_2] -$ ERIVFG $G = (A, B)$ on a $r -$ ERG $G^* = (V, E)$ is $\left[\frac{qk_1}{r}, \frac{qk_2}{r}\right]$ where $q = |E|$.

Proof: By proposition 3.3, $\sum_{uv\in E} d_G(uv) = \sum_{uv\in E} d_G^*(uv)[\mu_B^-(uv), \mu_B^+(uv)]$.

$$\Rightarrow \sum_{uv\in E}[k_1, k_2] = \sum_{uv\in E} r[\mu_B^-(uv), \mu_B^+(uv)]$$
$$\Rightarrow \sum_{uv\in E}[k_1, k_2] = r\sum_{uv\in E}[\mu_B^-(uv), \mu_B^+(uv)]$$
$$\Rightarrow q[k_1, k_2] = rS(G), q = |E|.$$
$$\Rightarrow [qk_1, qk_2] = rS(G), q = |E|.$$
$$\Rightarrow S(G) = \left[\frac{qk_1}{r}, \frac{qk_2}{r}\right], q = |E|.$$

6.5. Theorem Let $G = (A, B)$ be a $[c_1, c_2] -$ totally edge regular and $c -$ partially edge regular IVFG. Then $S(G) = \left[\frac{qc_1}{c+1}, \frac{qc_2}{c+2}\right]$ where $q = |E|$.

Proof: The size of $S(G) = \left[\sum_{\substack{uv\in E\\u\neq v}} \mu_B^-(uv), \sum_{\substack{uv\in E\\u\neq v}} \mu_B^+(uv)\right]$.
Since G is $[c_1, c_2] -$ totally edge regular and G^* is $c -$ edge regular,
$td_G(uv) = [c_1, c_2]$ and $d_G^*(uv) = c \ \forall\, uv \in E$.
Thus by *theorem* 3.6,

$$\sum_{uv\in E} td_G(uv) = \sum_{uv\in E} c[\mu_B^-, \mu_B^+](uv) + S(G)$$
$$\Rightarrow \sum_{uv\in E}[c_1, c_2] = c[\sum_{uv\in E} \mu_B^-(uv), \sum_{uv\in E} \mu_B^+(uv)] + S(G)$$
$$\Rightarrow q[c_1, c_2] = c\, S(G) + S(G) \ where\ q = |E|$$
$$\Rightarrow [qc_1, qc_2] = (c + 1)\, S(G)$$
$$\Rightarrow S(G) = \left[\frac{qc_1}{c+1}, \frac{qc_2}{c+1}\right]$$ ■

6.6. Theorem. Let $G = (A, B)$ be a $[k_1, k_2] -$ edge regular and $[r_1, r_2] -$ totally edge regular IVFG. Then $S(G) = [q(r_1 - k_1), q(r_2 - k_2)]$ where $q = |E|$.

Proof: Let G be a $[k_1, k_2] -$ edge regular and $[r_1, r_2] -$ totally edge regular IVFG. Then, $d_G(uv) = [k_1, k_2]$ and $td_G(uv) = [r_1, r_2] \ \forall\, uv \in E$.

The size of $G, S(G) = \left[\sum_{\substack{uv\in E\\u\neq v}} \mu_B^-(uv), \sum_{\substack{uv\in E\\u\neq v}} \mu_B^+(uv)\right]$.

$\sum_{uv\in E} d_G(uv) = [\sum_{uv\in E} k_1, \sum_{uv\in E} k_2] = [qk_1, qk_2]$ *and*
$\sum_{uv\in E} td_G(uv) = [\sum_{uv\in E} r_1, \sum_{uv\in E} r_2] = [qr_1, qr_2]$ where $q = |E|$
Since $td_G(uv) = d_G(uv) + [\mu_B^-, \mu_B^+](uv)$,
$\sum_{uv\in E} td_G(uv) = \sum_{uv\in E} d_G(uv) + \sum_{uv\in E}[\mu_B^-, \mu_B^+](uv)$
$\Rightarrow \sum_{uv\in E} td_G(uv) = \sum_{uv\in E} d_G(uv) + [\sum_{uv\in E} \mu_B^-(uv), \sum_{uv\in E} \mu_B^+(uv)]$
$\Rightarrow [qr_1, qr_2] = [qk_1, qk_2] + S(G)$
$\Rightarrow S(G) = [q(r_1 - k_1), q(r_2 - k_2)]$ where $q = |E|$ ■

7. CONCLUSION

In this paper we have extended the notion of regularity, total regularity, edge regularity and total edge regularity to IVFGs. We have proved a several theorems which reveal the relationship between these concepts.

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