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Estimation of Stress Strength Reliability in s-out-of-k System for a Two Parameter Inverse Chen Distribution

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ABSTRACT

The problem of estimation of reliability of s out of k system is considered under stress-strength set up. The system with k components is studied in which each component is experiencing a common random stress. The estimation of reliability of the system is studied assuming strengths and a stress to follow inverse Chen distribution. The maximum likelihood estimator for system reliability is derived. Also, approximate Bayes estimator is derived using Lindley's approximation method. The estimation of reliability are compared by using mean squares error criteria.

Keywords: Inverse Chen distribution, s out of k system reliability, Maximum likelihood estimator, Bayes Estimator, Lindley's approximation.

1. INTRODUCTION

The researchers are interested in study of stress-strength model due to its practical importance in the field of science and technology. The assessment of reliability of a component in terms of random variable Y representing stress experienced by the component and X representing strength of a component available to overcome the possible stress. If stress exceeds the strength, then the system will fail. This setup is called "stress-strength" setup. Birnbaum (1956) introduced the idea of stress-strength reliability which is further developed by Birnbaum and McCarty (1958). Many authors studied the estimation of reliability of s out of k system in stress-strength setup along with estimation of reliability for other systems. The application of s out of k system can be seen in many real life situations, particularly in industry and military (refer Kuo and Zuo (2003)). The researchers like Rao and Kantam (2010), Rao (2014), Kizilaslan and Nadar (2015) studied the problem of estimating stress-strength reliability of an s out of k system when the underlying distributions respectively are log-

logistic, Rayleigh distribution, Weibull etc. Pandit and Kantu (2013) considered estimation of multicomponent stress-strength reliability for parallel and series systems when strength and stress variables follow exponential distribution. This paper considers stress strength reliability of s out of k system when the underlying distribution are inverse Chen. We present below the reliability function and probability density function of two parameter inverse Chen distribution which was introduced by Srivastava and Srivastava (2014).

The reliability function is given by

$$R(x) = 1 - e^{\gamma\left(1-e^{x^{-\beta}}\right)}, x > 0, \gamma, \beta > 0$$

and the probability density function is

$$f(x) = \gamma\beta\, x^{-(\beta+1)} e^{\left[x^{-\beta} + \gamma\left(1-e^{x^{-\beta}}\right)\right]}, x > 0, \gamma, \beta > 0$$

Let the random variables Y; $X_1, X_2, \ldots, X_k$ be independent, with G(y) be the cumulative distribution function of Y and F(x) be the common cummulative distribution function of $X_1, X_2, \ldots, X_k$. The reliability in an s out of k stress-strength model developed by Bhattacharyya and Johnson (1974) is

$$R_{s,k} = P\left(\text{atleast s of the}\left(X_1, X_2, \ldots, X_k\right) \text{exceed Y}\right)$$

$$= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} \left[1 - F(y)\right]^{i} \left[F(y)\right]^{k-i} dG(y) \qquad (1)$$

In section 2, reliability of the system is derived. Section 3 deals with Maximum likelihood estimation of $R_{s;k}$. In section 4, Bayes estimators of $R_{s;k}$ is obtained using Lindley's approximation. Section 5 is devoted to simulation study in which the comparison of estimators of reliability is studied. The conclusions are given in section 6.

2. THE STRESS-STRENGTH RELIABILITY FOR S OUT OF K SYSTEM

In this section, s out of k system reliability is considered when $X_1, X_2, \ldots, X_k$ follow inverse Chen distribution with parameters (γ,β) and Y follow inverse Chen distribution with parameters (δ, β).

The system reliability is given by

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_{0}^{\infty} \delta\beta \left(e^{-\gamma\left(1-e^{x^{-\beta}}\right)} \right)^{k-i} \left(1 - e^{-\gamma\left(1-e^{x^{-\beta}}\right)} \right)^{i} e^{\left[y^{-\beta} + \delta\left(1-e^{y^{-\beta}}\right)\right]} dy$$

$$= \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^{j} \frac{\delta}{\left[\gamma(k+j-i)+\delta\right]} \qquad (2)$$

3. MAXIMUM LIKELIHOOD ESTIMATION OF $R_{s;k}$

Let (X_1, X_2, ..., Xn) and (Y_1, Y_2, ...,Y_m) be two random samples of size n, m from ICD(γ, β) and ICD(δ, β) respectively. The likelihood function of γ, δ and β is

$$L_s(¥, ¢, \ )= ¥^n ¢^m \ ^{n+m}\left[\prod_{i=1}^{n} x_i \prod_{j=1}^{m} y_j\right]^{-(\ +1)} e^{\sum_{i=1}^{n} x_i^{-} + \sum_{j=1}^{m} y_j^{-}} e^{¥\left(1-e^{x^{-}}\right)+¢\left(1-e^{y^{-}}\right)}$$

Thus, the log-likelihood function of γ, δ and β is

$$\log L_s(¥, ¢, \ ) = n\log ¥ + m \log ¢ + (n+m)\log \ -(\ +1)\left[\sum_{i=1}^{n}\log x_i + \sum_{j=1}^{m}\log y_j\right] + \sum_{i=1}^{n} x_i^{-} + \sum_{j=1}^{m} y_j^{-}$$

$$+¥\sum_{i=1}^{n}\left(1-e^{x_i^{-}}\right)+¢\sum_{j=1}^{m}\left(1-e^{y_j^{-}}\right) \quad (3)$$

where $\hat{\ }$ is the solution of the nonlinear equation of the form,

h(β) =β

A closed form solution of (3) is not possible and hence a numerical technique has been used to _nd MLE of $R_{s;k}$.(for details refer Pandit and Joshi (2018)).

The MLE of $\hat{R}_{s,k}$ is obtained by using the invariance property of MLEs as

$$\hat{R}_{s,k}^{M\,srs} = \sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^j \frac{\hat{¢}}{\left[\hat{ª}(k+j-i)+\hat{¢}\right]}$$

4. BAYES ESTIMATION OF $R_{s;k}$

The Bayes estimator is obtained for s out of k stress-strength reliability assuming that all the parameters γ, δ and β are unknown and independent random variables with gamma priors (c_i, d_i), i = 1, 2, 3.

The p.d.f of gamma random variables X with parameters (α, λ) is

$$f(x) = \frac{ª^{\sim}}{\Gamma\sim} x^{\sim -1} e^{-xª}; x>0, \sim, ª > 0$$

Thus the joint prior distribution for γ, δ and β is

$$g(¥, ¢, \ ) = g(¥)\, g(¢)\, g(\ )$$

where

$$g(¥) = \frac{d_1^{c_1}}{\Gamma c_1} ¥^{c_1-1} e^{-¥ d_1}; ¥ > 0, c_1, d_1 > 0$$

$$g(¢) = \frac{d_2^{c_2}}{\Gamma c_2} ¢^{c_2 - 1} e^{-¢ d_2} ; ¢ > 0, c_2, d_2 > 0$$

$$g(\ ) = \frac{d_3^{c_3}}{\Gamma c_3} \ ^{c_1 - 1} e^{-\ d_1} ; \ > 0, c_3, d_3 > 0$$

The corresponding joint posterior density function of γ, δ and β is

$$®(¥, ¢, \ ) = \frac{g(¥, ¢, \ ) L_s(¥, ¢, \ )}{\int_0^\infty \int_0^\infty \int_0^\infty g(¥, ¢, \ ) L_s(¥, ¢, \ ) d¥\, d¢\, d}$$

$$= \frac{A}{\int_0^\infty \int_0^\infty \int_0^\infty A\, d¥\, d¢\, d}$$

where

$$A = ¥^{n + c_1 - 1} ¢^{m + c_2 - 1} \ ^{n + m + c_3 - 1} e^{-\ \left(\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j + d_3 \right)} e^{-¥ d_1} e^{-¢ d_2}$$

$$e^{\left(\sum_{i=1}^{n} \left[x_i^{-} \ + ¥ \left(1 - e^{x_i^{-}} \right) \right] + \sum_{j=1}^{m} \left[y_j^{-} \ + ¢ \left(1 - e^{y_j^{-}} \right) \right] \right)}$$

here, the Bayes estimator of R is obtained as the posterior expectation of reliability under squared error (SE) loss function

$$\hat{R}_{s,k}^{B} = \int_0^\infty \int_0^\infty \int_0^\infty R_{s,k}\, ®(¥, ¢, \ ) d¥\, d¢\, d$$

The evaluation of posterior mean is not tractable. However, approximate posterior mean can be obtained using Lindley's approximation method.

The simplest method to approximate is Lindley's (Lindley (1980)) approximation method.

The Lindley's approximation method evaluates the ratio of the integrals as a whole and produces a single numerical result.

Log of joint prior probability density function is

$$° = \log g(¥, ¢, \ ) = c_1 \log d_1 - \log \Gamma(c_1) + (c_1 - 1) \log ¥ - ¥ d_1$$
$$+ c_2 \log d_2 - \log \Gamma(c_2) + (c_2 - 1) \log ¢ - ¢ d_2$$
$$+ c_3 \log d_3 - \log \Gamma(c_3) + (c_3 - 1) \log \ - \ d_3$$

Then

$$^\circ{}_1 = \frac{c_1 - 1}{\text{¥}} - d_1,\ ^\circ{}_2 = \frac{c_2 - 1}{\text{¢}} - d_2 \text{ and } ^\circ{}_3 = \frac{c_3 - 1}{} - d_3$$

also the values of Lij can be obtained as follows for i, j=1,2.

$$L_{11} = -\frac{n}{\text{¥}^2}, L_{22} = -\frac{m}{\text{¢}^2}, L_{13} = L_{31} = \sum_{i=1}^{n} e^{x_i^{-}} \quad x_i^{-} \quad \log x_i,$$

$$L_{23} = L_{32} = \sum_{j=1}^{m} e^{y_j^{-}} \quad y_j^{-} \quad \log y_j$$

$$L_{33} = -\frac{n+m}{2} + \sum_{i=1}^{n} x_i^{-} \quad (\log x_i)^2 + \sum_{j=1}^{m} y_j^{-} \quad (\log y_j)^2 - \text{¥} \sum_{i=1}^{n} e^{x_i^{-}} \quad x_i^{-} \quad (\log x_i)^2 [1 + x_i^{-} \quad]$$

$$- \text{¢} \sum_{j=1}^{m} e^{y_j^{-}} \quad y_j^{-} \quad (\log y_j)^2 [1 + y_j^{-} \quad]$$

and the values of L_{ijk} for i, j, k = 1, 2, 3 (For details refer Pandit and Joshi (2018))

$$L_{111} = \frac{2n}{\text{¥}^3}, L_{222} = \frac{2m}{\text{¢}^3}, L_{133} = L_{331} = \sum_{i=1}^{n} e^{x_i^{-}} \quad x_i^{-} \quad (\log x_i)^2 [1 + x_i^{-} \quad],$$

$$L_{233} = L_{332} = \sum_{j=1}^{m} e^{y_j^{-}} \quad y_j^{-} \quad (\log y_j)^2 [1 + y_j^{-} \quad],$$

$$L_{333} = \frac{2(n+m)}{3} - \sum_{i=1}^{n} x_i^{-} \quad (\log x_i)^3 - \sum_{j=1}^{m} y_j^{-} \quad (\log y_j)^3 + \text{¥} \sum_{i=1}^{n} e^{x_i^{-}} \quad x_i^{-} \quad (\log x_i)^3 [3 + (x_i^{-} \quad)^3]$$

$$+ \text{¢} \sum_{j=1}^{m} e^{y_j^{-}} \quad y_j^{-} \quad (\log y_j)^3 [3 + (y_j^{-} \quad)^3]$$

since, u = u(γ, δ, β) = $R_{s;k}$,

$$u_1 = \frac{\partial u}{\partial \text{¥}} == \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^{j+1} \frac{\text{¢}(k+j-i)}{[\text{¥}(k+j-i) + \text{¢} \quad]^2}$$

$$u_2 = \frac{\partial u}{\partial \text{¢}} == \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^{j} \frac{\text{¥}(k+j-i)}{[\text{¥}(k+j-i) + \text{¢} \quad]^2}$$

$$u_{11} = \frac{\partial^2 u}{\partial \gamma^2} == \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i}\binom{i}{j}(-1)^j \frac{2\delta (k+j-i)^2}{[\gamma(k+j-i)+\delta]^3}$$

$$u_{12} = \frac{\partial^2 u}{\partial \gamma \partial \delta} == \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i}\binom{i}{j}(-1)^{j+1} \frac{(k+j-i)[\gamma(k+j-i)-\delta]}{[\gamma(k+j-i)+\delta]^3}$$

$$u_{22} = \frac{\partial^2 u}{\partial \delta^2} == \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i}\binom{i}{j}(-1)^j \frac{2\gamma (k+j-i)}{[\gamma(k+j-i)+\delta]^3}$$

$u_3 = \frac{\partial u}{\partial} = 0$ and $u_{13} = u_{23} = u_{31} = u_{32} = u_{33} = 0$ and

$$a_4 = (u_{11}\sigma_{11}),\ a_5 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22})$$

$$A = \sigma_{11}L_{111} + \sigma_{33}L_{331},\ B = \sigma_{22}L_{222} + \sigma_{33}L_{332} \text{ and}$$

$$C = 2\sigma_{13}L_{133} + 2\sigma_{23}L_{233} + \sigma_{33}L_{333}$$

The quantities u_i, u_{ij}, σ_{ij} and L_{ijk}, i, j, k = 1, 2, 3. are evaluated by replacing (γ, δ, β) by $(\hat{\gamma}, \hat{\delta}, \hat{\ })$. Then, the Bayes estimator of $R_{s;k}$ is

$$\hat{R}^B_{s,k} = R_{s,k} + [u_1 a_1 + u_2 a_2 + a_4 + a_5] + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12}) + B(u_1\sigma_{21} + u_2\sigma_{22}) + C(u_1\sigma_{31} + u_2\sigma_{32})]$$

5. SIMULATION STUDY

A Simulation study is conducted to compute mean square error of the estimators by generating samples from inverse Chen distribution for comparison of estimates based on 100000 random samples of size n and m. We have evaluated empirical mean square errors for different sets of values for (γ, δ, β) for an s out of k system. For present study, the values of (γ, δ, β) are (0.5, 0.5, 1), (0.5, 0.2, 1) and (0.2, 0.3, 2). The corresponding true values of stress-strength reliability for s-out-of-k system with (s; k) = (1; 3) are 0:75; 0:8823; 0:6666 and that for (s; k) = (2, 4) are 0.6, 0.8021, 0.5250. The Bayesian estimates under squared error loss function using gamma prior are $c_1 = 7$, $c_2 = 3$, $c_3 = 1$, $d_1 = 4$, $d_2 = 2$, $d_3 = 1$ (prior1) and $c_1 = 1$, $c_2 = 1$, $c_3 = 1$, $d_1 = 1$, $d_2 = 1$, $d_3 = 1$ (prior2).

The results are shown in table 1, table 2 and table 3.

Table 1: MLEs and Bayes estimates of reliability and corresponding mean square errors

γ = 0.5, δ = 0.5, β = 1, prior1						
(s, k)	R	n = m	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{B}$)
(1, 3)	0.75	10	0.7782	0.7692	0.0101	0.0063
		15	0.7769	0.7592	0.0092	0.0087
		20	0.7696	0.7536	0.0081	0.0077
		30	0.7642	0.7521	0.0072	0.0065
		35	0.7645	0.7525	0.0064	0.0041
		40	0.7642	0.7499	0.0032	0.0028
		50	0.7502	0.7486	0.0024	0.0011
(2, 4)	0.6	10	0.6982	0.6796	0.0082	0.0069
		15	0.6949	0.6592	0.0068	0.0052
		20	0.6756	0.6483	0.0057	0.0045
		30	0.6524	0.6312	0.0049	0.0032
		35	0.6284	0.6130	0.0042	0.0030
		40	0.6120	0.6115	0.0036	0.0024
		50	0.6090	0.6045	0.0023	0.0014

Table 2: MLEs and Bayes estimates of reliability and corresponding mean square errors

γ = 0.5, δ = 0.2, β = 2, prior1						
(s, k)	R	n = m	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{B}$)
(1, 3)	0.8823	10	0.8914	0.8894	0.0097	0.0081
		15	0.8901	0.8891	0.0092	0.0087
		20	0.8895	0.8882	0.0074	0.0062
		30	0.8886	0.8867	0.0072	0.0031
		35	0.8871	0.8853	0.0059	0.0029
		40	0.8847	0.8838	0.0032	0.0018
		50	0.8835	0.8821	0.0024	0.0009
(2, 4)	0.8021	10	0.8649	0.8512	0.0097	0.0062
		15	0.8615	0.8500	0.0086	0.0051
		20	0.8502	0.8447	0.0075	0.0046
		30	0.8493	0.8322	0.0054	0.0031
		35	0.8311	0.8146	0.0044	0.0020
		40	0.8200	0.8031	0.0032	.0.0014
		50	0.8109	0.8017	0.0011	0.0008

Table 3: MLEs and Bayes estimates of reliability and corresponding mean square errors

γ = 0.2, δ = 0.3, β = 2, prior2						
(s, k)	R	n = m	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{B}$)
(1, 3)	0.6666	10	0.6684	0.6679	0.0074	0.0052
		15	0.6681	0.6670	0.0071	0.0049
		20	0.6675	0.6669	0.0058	0.0023
		30	0.6671	0.6664	0.0041	0.0012
		35	0.6668	0.6661	0.0037	0.0011
		40	0.6665	0.6660	0.0024	0.0009
		50	0.6661	0.6659	0.0019	0.0005
(2, 4)	0.5250	10	0.5419	0.5316	0.0069	0.0058
		15	0.5401	0.5300	0.0054	0.0054
		20	0.5319	0.5299	0.0048	0.0047
		30	0.5297	0.5271	0.0037	0.0016
		35	0.5284	0.5266	0.0021	0.0011
		40	0.5263	0.5254	0.0016	0.0009
		50	0.5260	0.5241	0.0014	0.0007

6. CONCLUSIONS

1. Here we have taken the underlying distribution for both stress and strength to be inverse Chen.
2. It is observed that mean square error of maximum likelihood and Bayes estimator of $R_{s,k}$ decreases as sample size increases.
3. For the priors considered here the Bayes estimator of $R_{s,k}$ under squared error loss function has smaller mean square error than maximum likelihood estimator.
4. The system considered here is more general system which includes series and parallel systems corresponds to when s = 1 and s = k. (Joshi and Pandit (2018)).

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