JOURNAL OF COMPUTER AND MATHEMATICAL SCIENCES
An International Open Free Access, Peer Reviewed Research Journal
www.compmath-journal.org

ISSN 0976-5727 (Print)
ISSN 2319-8133 (Online)
Abbr:J.Comp.&Math.Sci.
2014, Vol.5(3): Pg.304-308

Hyper-Wiener Index of Multi-Thorn Even Cyclic Graphs Using Cut-Method

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(Received on: June 21, 2014)

ABSTRACT

Let G be the graph. The Wiener Index W(G) is the sum of all distances between vertices of G, where as the Hyper-Wiener index WW(G) is defined as WW(G)= $\frac{1}{2}$ W(G)+ $\frac{1}{2}$ $\sum d^2(u,v)$. In this paper we prove results on Hyper-Wiener Index of multi-thorn even cyclic graph and thorn cyclic graph using Cut method.

Keywords: Hyper-Wiener Index, multi-thorn even cyclic graph, thorn ring and Cut method.

INTRODUCTION

We have three methods for calculation of the Hyper-Wiener Index of molecular graphs.

(i) Distance Formula:
$WW(G)=\frac{1}{2}(\sum d(u,v) + \sum d^2(u,v))$

(ii) Cut Method:

(iii) The Method of Hosaya Polynomials:

Cut Method: The cut method is based on the results from Klavzar, Gutman and Mohar and the calculation of the hyper-Wiener index works for all partial cubes. A graph is a partial cube if it is isomorphic to an isometric subgraph of a hypercube.

Let G be a benzenoid graph on vertices. Then an elementary cut C divides G into two components, say $G_1(C)$ and $G_2(C)$. Let $n_1(C)$ and $n_2(C)$ be the number of vertices of $G_1(C)$ and $G_2(C)$, respectively. Then Wiener index of G can be calculated as:

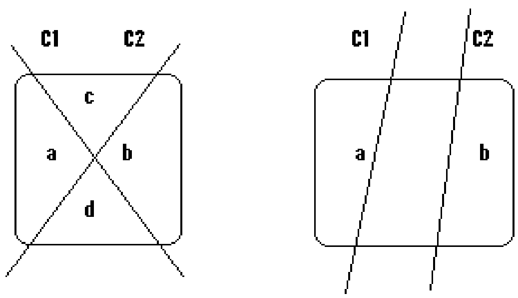
$$W(G)=\sum_i n_1(C_i)n_2(C_i) \qquad (1)$$

where the summation goes over all elementary cuts of G.

The hyper-Wiener index of a benzenoid graph G can be written as:

$$WW(G)=W(G)+WW^*(G) \qquad (2)$$

Where $WW^*(G)$ consists of a summation over all pairs of elementary cuts. Let C_1 and C_2 be two elementary cuts of a benzenoid graph G. There are two different cases, as shown in figure (a). With **a, b, c** and **d** we denote the number of vertices in the corresponding parts of G. Then the contribution of the pair C_1, C_2 to $WW^*(G)$ is **ab+cd** in the first case and **ab** in the second one.



Figure(a)

Definitions

Multi-Thorn Cyclic Graph: Let G be the graph containing n vertices. If S-number of pendent vertices are attached to each vertex of cycle Cm is called as Multi-Thorn Cyclic Graph.

Multi-Thorn Even Cyclic Graph: Let G be the graph containing n vertices. If S-number of pendent vertices are attached to each vertex of even cycle Cm is called as Multi-Thorn Cyclic Graph.

MAIN RESULT

The following results based on the cut method of Hyper-Wiener index.

Lemma 1: In any multi-thorn even cyclic graph there are $\left(\frac{r}{2}+P\right)$ number of elementary cuts present.

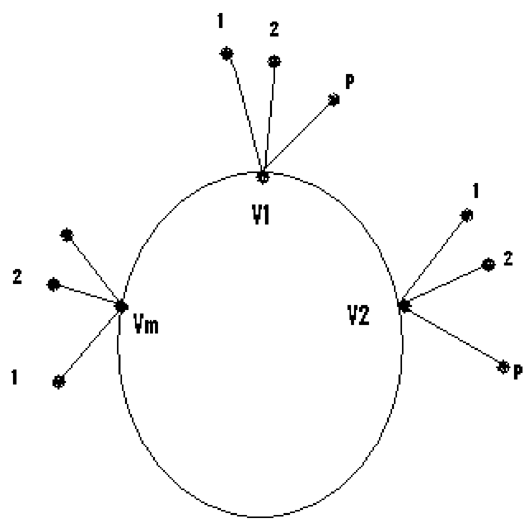
Lemma 2: In any multi-thorn even cyclic graph there are $\frac{1}{2}\left(\frac{r}{2}+P\right)\left(\frac{r}{2}+P-1\right)$ number of distinct pair cuts in $WW^*(G)$.

Theorem: The Hyper-Wiener Index of a even cyclic multi-thorn graph having n-vertices given by

$$WW(G)= n^3\frac{1}{8(S+1)} + P(n-1)+(S+1)^2$$

$$\sum_{i=1,\ j=\frac{n}{2(S+1)}-1,\ k=1}^{\frac{n}{2(S+1)}-1,\ 1,\ \frac{n}{2(S+1)}-1}\left(\frac{n}{2(S+1)}-k\right)(i^2+j^2)+\frac{Pnr}{4}+\frac{P(P-1)}{2}$$

where n-Number of vertices in G.
P-Total number of Pendent vertices in G.
S-Number of vertices attached to each vertex of Cm.
r-Length of the ring.



Proof: To find Hyper-Wiener index of graph, we need to find following two parts

To find W(G):

$W(G)=\sum_i n_1(C_i)\, n_2(C_i)$

$= n_1(C_1)\, n_2(C_1)+ n_1(C_2)\, n_2(C_2)+\ldots\ldots + n_1(C_i)\, n_2(C_i)$

$$=\underbrace{\frac{n}{2}\frac{n}{2}+\frac{n}{2}\frac{n}{2}+\frac{n}{2}\frac{n}{2}+\ldots+\frac{n}{2}\frac{n}{2}}_{\frac{n}{2(S+1)}\text{ times}}+\underbrace{(n-1)+(n-1)+\ldots\ldots+(n-1)}_{P\text{ times}}$$

$$W(G)= n^3\frac{1}{8(S+1)}+P(n-1) \qquad (a)$$

To find WW*(G): from the Lemma 2, in any multi-thorn even cyclic graph there are $\frac{1}{2}\left(\frac{r}{2}+P\right)\left(\frac{r}{2}+P-1\right)$ number of distinct pair cuts in WW*(G).

Therefore, $C_1C_2, C_1C_3, C_1C_4, \ldots\ldots C_{i-1}C_i$ elementary cuts present.

$WW^*(G)= C_1C_2+ C_1C_3+ C_1C_4+\ldots\ldots + C_{i-1}C_i$.

$$WW^*(G)=\left[\frac{n}{2(S+1)}-1\right]\left[(S+1)(S+1)+\frac{n}{2}-(S+1)*\frac{n}{2}-(S+1)\right]+\left[\frac{n}{2(S+1)}-2\right]\left[2(S+1)2(S+1)+\frac{n}{2}-2(S+1)*\frac{n}{2}-2(S+1)\right]+\ldots+\left[\frac{n}{2(S+1)}-\left[\frac{n}{2(S+1)}-1\right]\right]\left[\frac{n}{2}-(S+1)*\frac{n}{2}-(S+1)+(S+1)(S+1)\right]+\underbrace{\frac{n}{2}\frac{n}{2}+\frac{n}{2}\frac{n}{2}+\ldots+\frac{n}{2}\frac{n}{2}}_{\frac{rP}{2}\ times}+\underbrace{1+1+\ldots\ldots+1}_{\frac{P(P-1)}{2}\text{ times}}$$

$$=\sum_{k=1}^{\frac{n}{2(S+1)}-1}\left(\frac{n}{2(S+1)}-k\right)\left[(S+1)(S+1)+\frac{n}{2}-(S+1)*\frac{n}{2}-(S+1)\right]+\left[2(S+1)2(S+1)+\frac{n}{2}-2(S+1)*\frac{n}{2}-2(S+1)\right]+\ldots+\left[\frac{n}{2}-(S+1)*\frac{n}{2}-(S+1)+(S+1)(S+1)\right]+\frac{Pnr}{4}+\frac{P(P-1)}{2}$$

$$=\sum_{k=1}^{\frac{n}{2(S+1)}-1}\left(\frac{n}{2(S+1)}-k\right)(S+1)^2\left\{\left[1*1+\frac{n}{2(S+1)}-1*\frac{n}{2(S+1)}-1\right]+\left[2*2+\frac{n}{2(S+1)}-2*\frac{n}{2(S+1)}-2\right]+\cdots\left[\frac{n}{2(S+1)}-1*\frac{n}{2(S+1)}-1+1*1\right]\right\}+\frac{Pnr}{4}+\frac{P(P-1)}{2}.$$

$$WW^*(G) = (S+1)^2\sum_{i=1,\ j=\frac{n}{2(S+1)}-1,\ k=1}^{\frac{n}{2(S+1)}-1,\ 1,\ \frac{n}{2(S+1)}-1}\left(\frac{n}{2(S+1)}-k\right)(i^2+j^2)+\frac{Pnr}{4}+\frac{P(P-1)}{2} \qquad (b)$$

Since WW(G)=W(G)+WW*(G)

Therefore from (a) and (b)

$$WW(G)= n^3\frac{1}{8(S+1)}+P(n-1)+(S+1)^2\sum_{i=1,\ j=\frac{n}{2(S+1)}-1,\ k=1}^{\frac{n}{2(S+1)}-1,\ 1,\ \frac{n}{2(S+1)}-1}\left(\frac{n}{2(S+1)}-k\right)(i^2+j^2)+\frac{Pnr}{4}+\frac{P(P-1)}{2}$$

Corollary 1: For any ring if we introduce three pendent vertices to each vertex of even cyclic graph having n-vertices its Hyper-Wiener Index given by

$$WW(G)= n^3\frac{1}{32} + P(n-1)+(4)^2 \sum_{i=1,\ j=\frac{n}{8}-1,\ k=1}^{\frac{n}{8}-1,\ 1,\ \frac{n}{8}-1} (\frac{n}{8} - k)(i^2+j^2)+ \frac{Pnr}{4} + \frac{P(P-1)}{2}$$

Proof: Substituting S=3 in above theorem, gives the result.

Corollary 2: For any ring if we introduce two pendent vertices to each vertex of even cyclic graph having n-vertices its Hyper-Wiener Index given by

$$WW(G)= n^3\frac{1}{24} + P(n-1)+(3)^2 \sum_{i=1,\ j=\frac{n}{6}-1,\ k=1}^{\frac{n}{6}-1,\ 1,\ \frac{n}{6}-1} (\frac{n}{6} - k)(i^2+j^2)+ \frac{Pnr}{4} + \frac{P(P-1)}{2}$$

Proof: Substituting S=2 in above theorem, gives the result.

Corollary 3: For any ring if we introduce one pendent vertices to each vertex of even cyclic graph having n-vertices its Hyper-Wiener Index given by

$$WW(G)= n^3\frac{1}{16} + P(n-1)+(2)^2 \sum_{i=1,\ j=\frac{n}{4}-1,\ k=1}^{\frac{n}{4}-1,\ 1,\ \frac{n}{4}-1} (\frac{n}{4} - k)(i^2+j^2)+ \frac{Pnr}{4} + \frac{P(P-1)}{2}$$

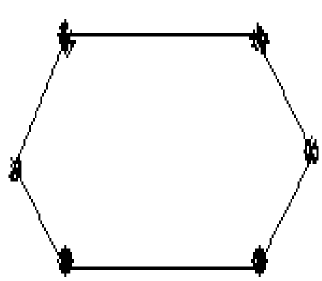
Proof: Substituting S=1 in above theorem, gives the result.

Corollary 4: Hyper-Wiener Index of even cyclic graph given by

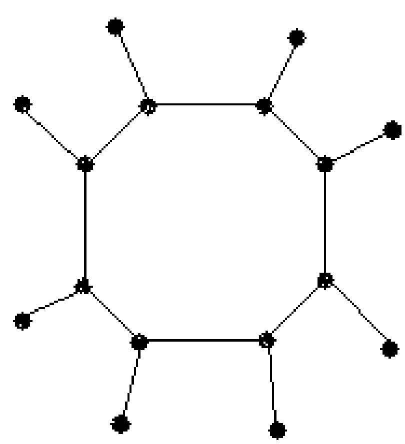
$$WW(G)= n^3\frac{1}{8} + P(n-1)+ \sum_{i=1,\ j=\frac{n}{2}-1,\ k=1}^{\frac{n}{2}-1,\ 1,\ \frac{n}{2}-1} (\frac{n}{2} - k)(i^2+j^2)+ \frac{Pnr}{4} + \frac{P(P-1)}{2}$$

Proof: Substituting S=0 in above theorem, gives the result.

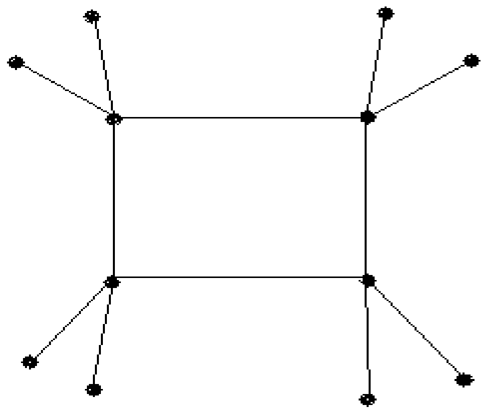
Illustrations:



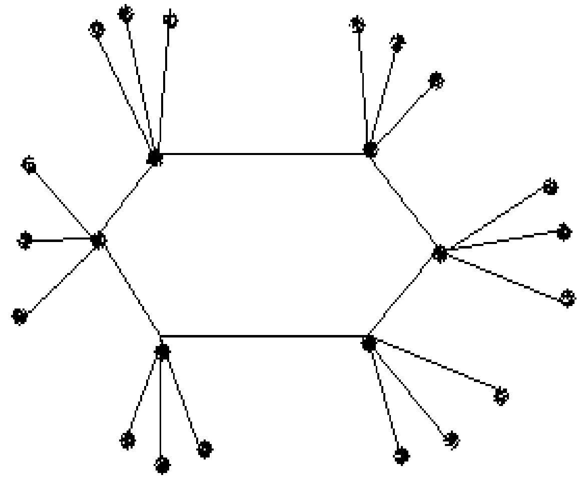
n=6, S=0, P=0, r=6, WW(G)=42



n=16, S=1, P=8, r=8, WW(G)=884



n=12, S=2, P=8, r=4, WW(G)=302



n=24, S=3, P=18, r=6, WW(G)=1887

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