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Multi-Hypergraph Grammar

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ABSTRACT

This study deals with the concept of hypergraph grammar which results in multi-hypergraph language. Few examples have been illustrated for regular multi-hypergraphs. The necessary condition for the two given multi-hypergraphs to be isomorphic is discussed. An example is given for the case that the condition is not sufficient.

Keywords: Hypergraph grammar, regular multihypergraph, regular multi- hypergraph language.

1. INTRODUCTION

Graphs are frequently used in various fields of Computer Science and Artificial Intelligence for representing knowledge of complex structures. This study deals with the concept of hypergraph grammar which is similar to string generating grammar whereas it generates multi-hypergraph instead of words. Hypergraph grammar provides a rule based mechanisms for generating, manipulating and analysing the graphs[4]. D. Caucal focused on providing some of the basic tools to reason out the deterministic graph grammar and on structural study of their generated graphs[1,2]. It further suitably defines that hypergraph grammar $R = (G_0, P)$ is an ordered pair where G_0 is an initial graph and P is a set of collection of rules of the form $X \to H$ or $H \to H'$. Here X is a hyperarc, H and H' are the multi - hypergraphs respectively. It also concerns that the rules of the grammar are deterministic and context free. Deterministic means that there is only one rule for every non-terminal. It also discusses about regular multi-hypergraph of a given grammar and the languages generated by the grammar too. It proves the necessary condition for the two given multi-hypergraphs to be isomorphic and gives an example for that the condition need not be sufficient.

2. BASIC DEFINITIONS

In this section, we have reviewed some fundamental concepts related to hypergraph grammar.

A finite set E of symbols is an alphabet of letters. E^* is the set of words over E. Any word $u \in E^n$ is of length $|u| = n$ is also represented by a mapping from $\{1, 2 \dots \dots n\}$ into E.

Definition 2.1. A multi-subset M of E is a mapping from E into $\mathbb{N}$ where for any $e \in E$, the integer M(e) is its multiplicity the number of ocurrences of e in M. It is also represented by the functional subset $\{ (e, M(e)) \mid e \in E \wedge M(e) \neq 0 \}$of $E \times \mathbb{N}_+$.If $(e, m), (e, n) \in M$ then$m = n$.The cardinality of M is $|M| = \sum_{e \in E} M(e)$. M is said to be finite if its support $\widehat{M} = \{ e \in E \mid M(e) \neq 0\}$ is finite.

Definition 2.2. Let F be a set of symbols called labels ranked by a mapping $\rho : F \to \mathbb{N}$ associating to each label f its arity and such that $F_n = \{f \in F \mid \rho(f) = n\}$ is countable $\forall n \geq 0$.

Definition 2.3. A simple, oriented and labeled hypergraph G is a subset of $\bigcup_{n\geq 0} F_n V^n$ where V is an arbitrary set such that its vertex set $V_G = \{v \in V \mid FV^*vV^* \cap G \neq \emptyset\}$ is finite or countable. Its label set $F_G = \{f \in F \mid FV^* \cap G \neq \emptyset\}$ is finite. Any $f v_1 v_2 \dots v_{\rho(f)}$ is a hyperarc labeled by f and of successive vertices $v_1, v_2, v_3, \dots\dots\dots\dots v_{\rho(f)}$, If $\rho(f) \geq 2$ then it depicts an arrow labeled f and successively linking $v_1, v_2, \dots . v_{\rho(f)}$. If $\rho(f) = 1$ then it depicts a label of f on vertex v_1 and f is called a color of v_1. If $\rho(f) = 0$ then it depicts an isolated label f called a constant.

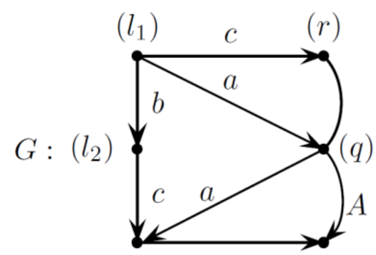


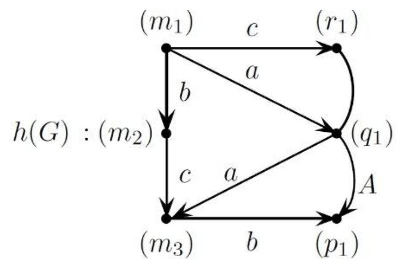
Figure 2.3

$G = \{ bl_1l_2,\ cl_2l_3,\ cl_1r,\ al_1q,\ aql_3,\ bl_3p,\ Arqp \}$; $V_G = \{ l_1,\ l_2,\ l_3,\ p,\ q,\ r \}$;
$F_G = \{ a,\ b,\ c,\ A \}$; $F_3 = \{ A \}$; $F_2 = \{ a,\ b,\ c \}$

Definition 2.4. The transformation of a hypergraph G by a function h from V_G into a set V is the hypergraph $h(G) = \{ f h(v_1) h(v_2) \dots\dots \mid f v_1 v_2 \cdots v_{\rho(f)} \in G \}$.

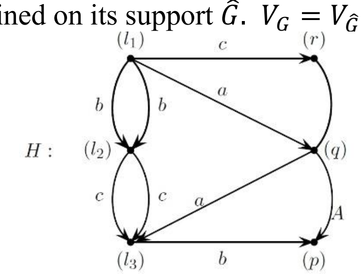
For the graph given in figure 2.3, $h(G)$ is shown below if the transformation h is defined as

h(1_1) = m_1 ; $h(l_1) = l_1$; $h(l_2) = m_2$; $h(l_3) = m_3$; $h(p) = p_1$; $h(q) = q_1$; $h(r) = r_1$



Definition 2.5. An isomorphism h from a hypergraph G to a hypergraph H is a bijection from V_G to V_H such that $h(G) = H$ and write $G \sim H$.

Definition 2.6. A multi - hypergraph G is a multi - subset of $\bigcup_{n\geq 0} F_n\, V^n$ where V is an arbitrary set. Each arc $X \in G$ depicted $G(X)$ times. The vertex set V_G and the label set F_G of a multi – hypergraph G are the sets defined on its support $\widehat{G}$. $V_G = V_{\widehat{G}}$ and $F_G = F_{\widehat{G}}$.



Definition 2.7. The transformation of any multi - hypergraph G by any function h from $\mathrm{V_G}$ into a set is extended as $\big(\mathrm{h(G)}\big)\,(\mathrm{X}) = \sum_{\mathrm{h(Y)=X}} \mathrm{G\,(Y)}$ for any hyperarc X, assuming that the sum is finite.

Definition 2.8. A hypergraph grammar is a finite set of rules of the form $f x_1 x_2 x_3 \cdots x_{\rho(f)} \to H$ where $f x_1 x_2 x_3 \cdots x_{\rho(f)}$ is a hyperarc joining pair wise distinct vertices $x_1 \neq x_2 \neq x_3 \neq \cdots x_{\rho(f)}$ and H is a finite multi - hypergraph. The labels of the left hand sides of the rules of the grammar are the non-terminals of R and denoted by $N_R = \{f \in F \mid fX \in Dom(R)\}$. The labels of R which are not non-terminals are the terminals of R and denoted by $T_R = \{\, f \in F - N_R \mid \exists P \in Im(R)\,, fX \in P\,\}$. $F_R = N_R \cup T_R$ be tbe labels of R and $\rho(R) = \, Max\,\{\, \rho(A) \mid A \in N_R\}$ be the arity of R.

Definition 2.9. A deterministic hyper graph grammar means that there is only one rule for every non-terminal. For any rule (X , H), we say $V_X \cap V_H$ are tbe inputs of H and $\{\, V_Y \mid Y \in H \wedge Y(1) \in N_R\}$ are the outputs of H.

Notation 2.11. We write a hyperarc as the word fY, where f is its label and Y its vertex word. It can also be represented as X where the first letter $X(1)$ is its label and for $1 \leq i \leq |X|$, the i[th] letter X(i) is its$(i-1)^{th}$ vertex. We use upper case letters $A, B \cdots$ for non-terminals and lower case letters $a, b, c \cdots$ for terminals. A hypergraph grammar R is said to be graph grammar if the terminals are of arity 1 or 2. For any rule R_1 of the grammar R, $Dom(R_1)$ and $Im(R_1)$ depicts the left and the right hand side of the rule R_1 respectively.

2.1 DERIVATION

In this section, we define hypergraph grammar suitably and propose two methods in the derivation of the hypergraph grammar R.

Definition 2.10. A hypergraph grammar $R = (G_0, P)$ is an ordered pair where where G_0 is the base graph and P is the finite set of rules. Each rule of P is of the form $fx_1x_2x_3 \cdots x_{\rho(f)} \rightarrow H$ or $H \rightarrow H'$ where $fx_1x_2x_3 \cdots x_{\rho(f)}$ is a hyperarc joining pairwise distinct vertices $x_1 \neq x_2 \neq x_3 \neq \cdots x_{\rho(f)}$ and *H, H'* are finite multi- hypergraphs. The labels of the left hand sides of the rules of the grammar are the non-terminals of R and denoted by $\mathrm{N_R} = \{\mathrm{f} \in \mathrm{F} \mid \mathrm{fX} \in \mathrm{Dom(R)}\}$. The labels of R which are not non-terminals are the terminals of R and denoted by $T_R = \{f \in F - N_R \mid \exists P \in Im(R), fX \in P\}$. $F_R = N_R \cup T_R$ be tbe labels of R and $\rho(R) = Max\{\rho(A) \mid A \in N_R\}$ be the arity of R.

Method 1: A multi – hypergraph *M* derives *N* written $M \xrightarrow[R,\ X]{} N$ if we choose a non-terminal hyperarc X in M where $\mathrm{X} = \mathrm{As_1s_2s_3 \cdots s_{\rho(A)}}$ and a rule $X' \rightarrow H$ in R where $\mathrm{X'} = \mathrm{Ax_1x_2x_3 \cdots x_{\rho(A)}}$ in R such that *N* can be obtained by replacing X by H in *M.* Thus, $\mathrm{N} = (\mathrm{M} - \mathrm{X}) + \mathrm{h(H)}$ for some function *h*, mapping each $\mathrm{x_i}$ to $\mathrm{s_i}$ by and other vertices of *H* are injectively to vertices outside of *M.*

$\mathrm{N(Y)} = \mathrm{M(Y)} + \big(\mathrm{h(H)}\big)(\mathrm{Y})$ if $\mathrm{Y} \neq \mathrm{X}$

$\mathrm{N(X)} = (\mathrm{M(X)} - 1) + \big(\mathrm{h(H)}\big)(\mathrm{X})$

Method 2: The derivation $\xrightarrow[R,\ X]{}$ of a hyperarc *X* is extended in an obvious way to the derivation of any multi- subset *E* of non-terminal hyperarcs. The complete derivation $\underset{R}{\Rightarrow}$ is the derivation according to the multi-subset of all non-terminal hyperarcs. $M \underset{R}{\Rightarrow} N$ if $\mathrm{M} \xrightarrow[\mathrm{R,E}]{} \mathrm{N}$ Where E is the multi-subset of all non-terminal hyperarcs of M. Here, $M \xrightarrow[R,E]{} N$ means that *N* is obtained from *M* replacing each non-terminal hyperarc in *E* sequentially by its corresponding multi-hypergraph. Suppose that, $E = \{X_1,\ M(X_1),\ (X_2, M(X_2)),\ \cdots\ (X_n, M(X_n))\}$ be a multi-subset of all non-terminal hyperarcs of *M*. Then, the k-th occurrence of $X_i$be replaced by $g_{\{ijk\}}(Hj)$ where *Hj* being a multi-hypergraph of a rule $Yj \rightarrow Hj$ of R and $g_{\{ijk\}}$ being a transformation such that $g_{\{ijk\}}(Yj) = Xi \quad \forall k = 1, 2, 3, \ldots M(Xi)$.

$$N = (M - E) + \bigcup_{\{i,j,k\ \in M_1\}} g_{\{ijk\}}\left(H_j\right)$$

where $M_1 = \{(i,j,k) \mid \exists Y_j \rightarrow H_j \in R,\ X_i \in M,\ X_i(1) \in N_R$ and $g_{\{ijk\}}(Y_j) = X_i\}$. If the n

consecutive $(n \neq 1)$ sequence of parallel rewriting yields multi-hypergraph. G_1, G_2, $G_3 \cdots G_n$ from an initial multi - hypergraph G_0 using a hypergraph grammar R then it would be *written* $G_0 \Rightarrow G_1 \Rightarrow G_2 \Rightarrow G_3 \ldots \Rightarrow G_n$. Here G_n is said to be derivable from G_0. i.e., $G_0 \Rightarrow^* G_n$.

Notation2.13 For a given hypergraph M, $[M] = M \cap T_R V_M^*$ designates the simple set of terminal hyperarcs of M.

Notation 2.14. For a given multi-hypergraph., $[\![\boldsymbol{M}]\!] = M \cap T_R V_M^*$ designates the multi - set of terminal hyperarcs of M.

3. REGULAR MULTI - HYPERGRAPH

In this section, we define regular multi - hypergraph, undirected regular multi - hypergraph and the hypergraph languages generated by the above grammar.

Definition 3.1. A multi - hypergraph G is generated by a hypergraph grammar R from a multi-hypergraph H if G is isomorphic to a hypergraph in the following set called multi-hypergraph language.

$$\mathrm{R(H)} = \left\{ \bigcup_{n \geq 0} [\![\mathrm{H_n}]\!] \mid \mathrm{H_0 = H} \wedge \forall \mathrm{n} \geq 0 \ \mathrm{H_n} \Rightarrow \mathrm{H_{n+1}} \right\}$$

Definition 3.2. A regular multi - hypergraph is a multi - hypergraph generated by a deterministic hypergraph grammar from a finite multi - hypergraph.

Note 3.3. For any multi- hypergraph H, $\ll H \gg$ designates the multiset of undirected terminal Hyperarcs and $|| H ||$ designates the simple set of undirected terminal hyperarcs.

Definition 3.5. An undirected multi - hypergraph G is generated by a hypergraph grammar R from a multi - hypergraph H if G is isomorphic to a hypergraph in the following set called undirected multi - hypergraph language.

$$\overline{\mathrm{R}}(\mathrm{H}) = \left\{ \bigcup_{n \geq 0} \ll \mathrm{H_n} \gg \mid \mathrm{H_0 = H} \wedge \forall \mathrm{n} \geq 0 \ \mathrm{H_n} \Rightarrow \mathrm{H_{n+1}} \right\}$$

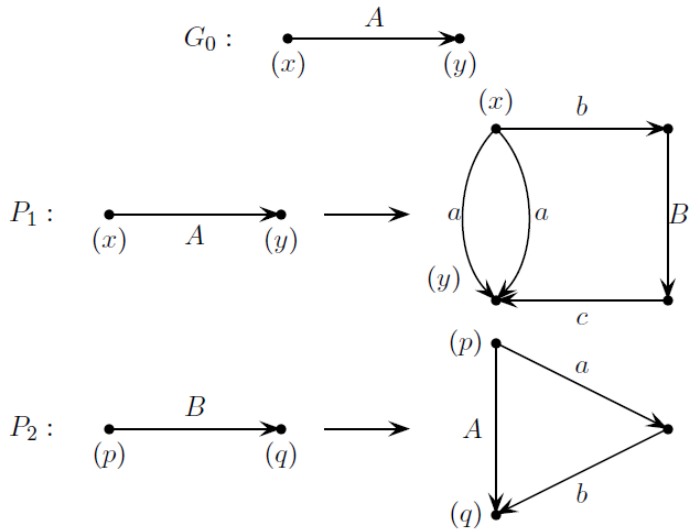
Definition 3.6. A hypergraph G is generated by a hyprgraph grammar R from a multi - hypergraph H if G is isomorphic to a hyprgraph in the following set called hypergraph language.

$$\mathrm{R^*(H)} = \left\{ \bigcup_{n \geq 0} [\mathrm{H_n}] \mid \mathrm{H_0 = H} \wedge \forall \mathrm{n} \geq 0 \ \mathrm{H_n} \Rightarrow \mathrm{H_{n+1}} \right\}$$

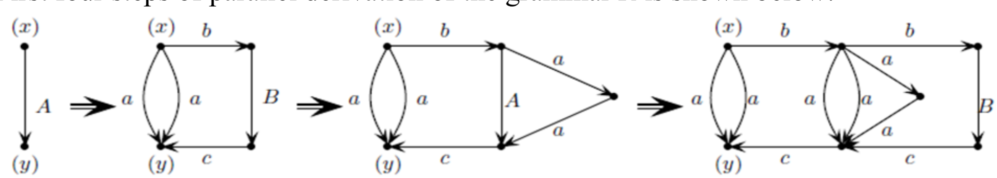
Definition 3.7. An undirected hypergraph G is generated by a hypergraph grammar R from a multi - hypergraph H if G is isomorphic to a hypergraph in the following set called undirected hypergraph language

$$R^{\star}(H) = \left\{ \bigcup_{n \geq 0} \| H_n \| \ \ |H_0 = H \wedge \forall n \geq 0 H_n \Rightarrow H_{n+1} \right\}$$

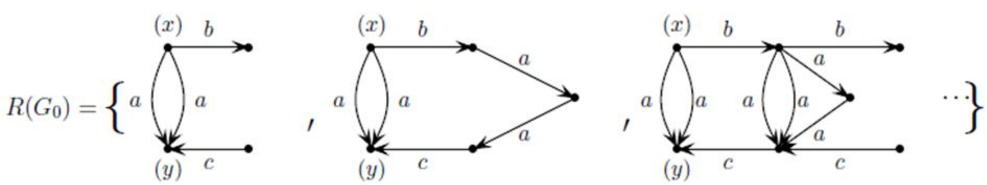
Example 3.8. Let us consider an initial graph G_0 and the rules P_1, P_2 of hypergraph grammar $R = (G_0, P)$ as in the following figures.



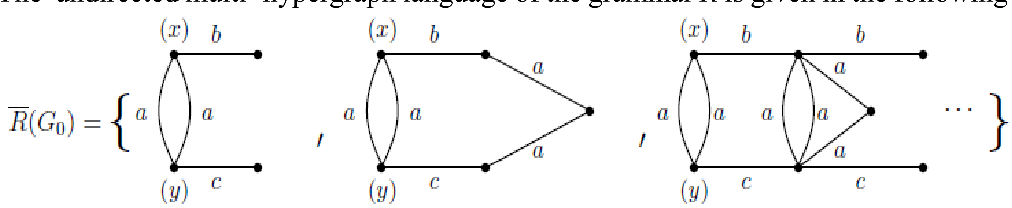
The First four steps of parallel derivation of the grammar *R* is shown below:



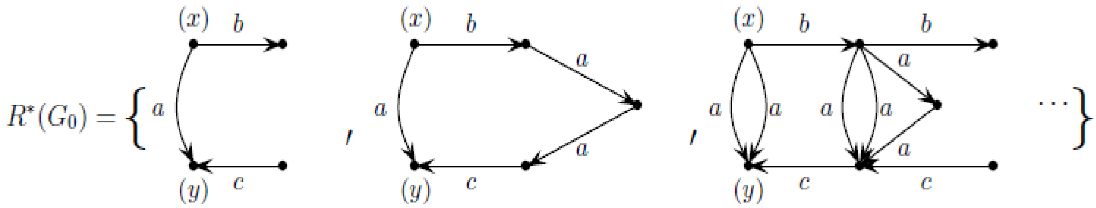
The multi-hypergraph language of the grammar *R* is given in the following figure:



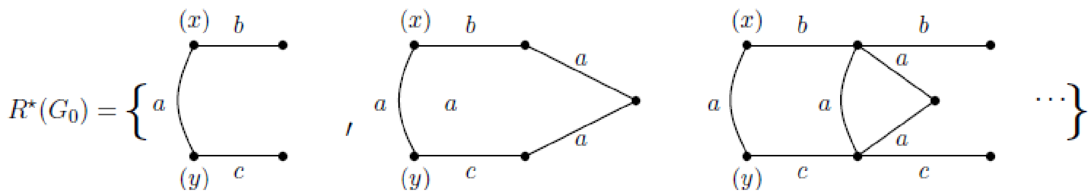
The undirected multi- hypergraph language of the grammar *R* is given in the following figure:



The hypergraph language of the grammar R is given in the following figure:



The hypergraph language of the grammar R is given in the following figure:



4. NECESSARY CONDITION FOR TWO MULTI-HYPERGRAPHS TO BE ISOMORPHIC

Theorem 4.1. If the two initial multi-hypergraph G_0 and G_0' of the grammars $R_1 = (G_0\ ,\ P)$ and $R_2 = (G_0'\ ,\ P)$ and isomorphic then $[\![G_i]\!]$ be a regular hypergraph of the grammar R_2 and each $[\![G'_i]\!]$ be of R_1 where G_i and G_i' are multi-hypergraphs generated in the i^{th} step of parallel derivation using R_1 and R_2 respectively.

Proof: The result can be proved by showing that $G_i \sim G_i'$ $\forall i$. It can be proved by the method of induction on the number of steps of derivation. The result is obviously true for $n = 0$.
Hence, $[\![G_0]\!]$ and $[\![G'_0]\!]$ are the regular hypergraphs of R_2 and R_1 respectively. By induction hypothesis, the result is true for all multi-hypergraphs that are generated in fewer than n steps. At the $(n-1)^{th}$ step of derivation, G_{n-1} is isomorphic to G_{n-1}' under the transformation h_{n-1}. The transformation $h_{n-1} : V(G_{n-1}) \to V(G_{n-1}')$ is defined as $h_{n-1}(m_i) = m_i'$. For every hyperarc X_i in G_{n-1}, we can find an arc corresponding to X_i say X_i' in G_{n-1}' of same multiplicity such that $h_{n-1}(X_i) = X_i'$.
Let G_n and G_n' be the multi-hypergraphs generated from G_{n-1} and G_{n-1}' respectively in the n^{th} step by replacing the k^{th} occurrence of their non-terminals X_i in G_{n-1} of multiplicity $G_{n-1}(X_i)$ and X_i' in G_{n-1}' of multiplicity $G_{n-1}'(X_i)$ by $f_{\{ijk\}}(H_j)$ and $f_{\{ijk\}}'(H_j)$ respectively where H_j is a multi - hypergraph of the rule $Y_j \to H_j$ of R and $f_{\{ijk\}}$, $f_{\{ijk\}}'$ are the transformations such that $f_{\{ijk\}}(Y_j) = X_i,\ f'_{\{ijk\}}(y_j) = X'_i$.

$$G_n = G_{n-1} - \bigcup_{i\,\in\,L_n} \left(X_{i,}\ G_{n-1}(X_i)\right) + \sum_{(i,j,k)\in\,M_n} f_{\{ijk\}}\left(H_j\right)$$

Where $L_n = \{i \mid X_i \in G_{n-1}, X_i(1) \in N_{R_1}\}$ and $M_n = \{(i,j,k) \mid \exists Y_j \to H_j \in R_1,\ i \in L_n,\ f_{\{ijk\}}(Y_j) = X_i\}$

$$G'_n = G'_{n-1} - \bigcup_{i \in L'_n} \left(X'_{i,}\ G'_{n-1}(X'_i)\right) + \sum_{(i,j,k) \in M_n} f'_{\{ijk\}}\ (H_j)$$

Where $L'_n = \{i \mid X'_i \in G'_{n-1}, X'_i(1) \in N_{R_2}\}$ and

$$M'_n = \{(i,j,k) \mid \exists Y_j \to H_j \in R_2,\ i \in\ f'_{\{ijk\}}(Y_j) = X_i\ \}$$

While generating G_n and G'_n non-terminal hyperarc X_i and X'_i use the same rule $Y_j \to H_j$ under the transformation $f_{\{ijk\}}$ and $f'_{\{ijk\}}$ respectively. Also, X_i and X'_i are of same multiplicity. Hence, $L_n = L'_n$ and $M_n = M'_n$. Since $f_{\{ijk\}}(Y_j) = X_i$ and $f'_{\{ijk\}}(Y_j) = X'_i$, we can say that $f_{\{ijk\}}(H_j) \sim f'_{\{ijk\}}(H_j)$. Let $G_{\{ijk\}}$ be an isomorphism existing between $f_{\{ijk\}}(H_j)$ and $f'_{\{ijk\}}(H_j)$ and we can define it as follows.

$$G_{\{ijk\}}(r_t) = \begin{cases} m'_t, & if\ r_t = m_t \\ n'_t, & if\ r_t = n_t \end{cases} \quad \text{where}$$

$$r_t = \begin{cases} m_t, & if\ r_t \in V(G_{n-1}) \cap V(G_n) \\ n_t, & if\ r_t \in V(G_n) - V(G_{n-1}) \end{cases}$$

We have to prove G_n is isomorphic to G'_n. Let us define a function $h_n : V_{G_n} \to V_{G'_n}$ as

$$h_n(l_i) = \begin{cases} h_{n-1}(m_i), & if\ l_i = m_i \\ n'_i, & if\ l_i = n_i \end{cases} \quad \text{where}$$

$$l_i = \begin{cases} m_i, & if\ l_i \in V(G_{n-1}) \cap V(G_n) \\ n_i, & if\ l_i \notin V(G_n) - V(G_{n-1}) \end{cases}$$

Let us choose an multi-hyperarc X in G_n of multiplicity $G_n(X)$.

Case 1: Suppose that X be an hyperarc in $[\![G_{n-1}]\!]$ but not in $\cup_{(i,j,k \in M_n)} f_{\{ijk\}}(H_j)$,
Now, $X = f l_1 l_2 l_3 \cdots l_{\rho(f)}$ be a terminal hyperarc in G_{n-1} and its multiplicity $G_n(X) = G_{n-1}(X)$. Each l_i can be replaced by m_i as X is a terminal hyperarc in G_{n-1}. Since $G_{n-1} \sim G'_{n-1}$,

$$\begin{aligned} h_{n-1}(X) &= h_{n-1}(f m_1 m_2 m_3 \cdots m_{\rho(f)}) \\ &= f h_{n-1}(m_1) h_{n-1}(m_2) \cdots h_{n-1} \\ &= f m'_1 m'_2 m'_3 \cdots m'_{\rho(f)} = X' \in G'_{n-1} \end{aligned}$$

Since no $f_{\{ijk\}}(H_j)$ contains X and $f_{\{ijk\}}(X) \sim f'_{\{ijk\}}(H_j)$, $G_{\{ijk\}}(X) \notin f'_{\{ijk\}}(H_j)$.

$$\begin{aligned} G_{\{ijk\}}(X) &= G_{\{ijk\}}(f m_1 m_2 m_3 \cdots m_{\rho(f)}) \\ &= f G_{\{ijk\}}(m_1) G_{\{ijk\}}(m_2) \cdots G_{\{ijk\}}(m_{\rho(f)}) \\ &= f m'_1 m'_2 \cdots m'_{\rho(f)} = X' \notin \bigcup_{(i,j,k) \in M'_n} f'_{\{ijk\}}(H_j) \end{aligned}$$

Thus, $X' \in G'_{n-1}$ but not in $\cup f'_{\{ijk\}}(H_j)$. Hence, $X' \in G'_n$ and $G'_n(X') = G'_{n-1}(X')$. From the above, $G_n(X) = G_{n-1}(X)$ and $G'_n(X') = G'_{n-1}(X')$. Since $G_{n-1}(X) = G'_{n-1}(X')$, $G_n(X) = G'_n(X')$.

$$\begin{aligned} h_n(X) &= h_n\big(fm_1m_2\cdots m_{\rho(f)}\big) \\ &= fh_n(m_1)h_n(m_2)\cdots h_n(m_{\rho(f)}) \\ &= fh_{n-1}(m_1)h_{n-1}(m_2)\cdots h_{n-1}(m_{\rho(f)}) \\ &= fm'_1m'_2\cdots m'_{\rho(f)}) = X' \in G'_n. \end{aligned}$$

Case 2: Suppose that *X* be in $\bigcup_{(i,j,k\,\in\,M_n)} f_{\{ijk\}}(H_j)$ but not in $[\![G_{n-1}]\!]$.

In this case, X be either a terminal or a non-terminal hyperarc and so each l_i be either m_i or n_i and it is written as (m_i/n_i). Since $X \in \cup f_{\{ijk\}}(H_j)$, $X \in f_{\{ijk\}}(H_j)$ for some $(i,j,k) \in M_n$. Since $f_{\{ijk\}}(H_j)$ is isomorphic to $f'_{\{ijk\}}(H_j)$ under the isomorphism G_{ijk}, for every arc in $f_{\{ijk\}}(H_j)$ we can find an arc in $f'_{\{ijk\}}(H_j)$ of same multiplicity.

Let $T_n = \{(i,j,k) \in M_n | X \in f_{\{ijk\}}(H_j)\}$

$$\begin{aligned} G_{\{ijk\}}(X) &= G_{\{ijk\}}\big((f(m_1/n_1)(m_2/n_2)\cdots(m_{\rho(f)}/n_{\rho(f)})\big)) \\ &= fG_{\{ijk\}}(m_1/n_1)G_{\{ijk\}}(m_2/n_2)\cdots G_{\{ijk\}}(m_{\rho(f)}/n_{\rho(f)}) \\ X' &= f(m'_1/n'_1)(m'_2/n'_2)\cdots(m'_{\rho(f)}/n'_{\rho(f)}) \end{aligned}$$

Also, $(f_{\{ijk\}}(H_j))(X) = (f'_{\{ijk\}}(H_j))(X')\forall(i,j,k) \in T_n$.

Since $X \in \bigcup_{(i,j,k)\in T_n} f_{\{ijk\}}(H_j)$ and $f_{\{ijk\}}(H_j) \sim f'_{\{ijk\}}(H_j)$, $X' \in \bigcup_{(i,j,k)\in T_n} f'_{\{ijk\}}(H_j)$

$$\begin{aligned} h_n(X) &= h_n(f(m_1/n_1)(m_2/n_2)\cdots(m_{\rho(f)}/n_{\rho(f)})) \\ &= fh_n(m_1/n_1)h_n(m_2/n_2)\cdots h_n(m_{\rho(f)}/n_{\rho(f)}) \\ &= f(m'_1/n'_1)(m'_2/n'_2)\cdots(m'_{\rho(f)}/n'_{\rho(f)}) = X' \in G'_n \end{aligned}$$

$$G_n(X) = \sum_{(i,j,k)\in T_n}\Big(f_{\{ijk\}}(H_j)\Big)(X) = \sum_{(i,j,k)\in T_n}\Big(f'_{\{ijk\}}(H_j)\Big)(H_j) = G'_n(X')$$

Case 3: Suppose that *X* be in $[\![G_{n-1}]\!] \cap \Big(\bigcup_{\{ijk\,\in\,M_n\}}\Big(f_{\{ijk\}}(H_j)\Big)(X)\Big)$

In this case, *X* be a terminal hyperarc and its multiplicity in G_n is given by

$$G_n(X) = G_{n-1}(X) + \bigcup_{\{i,j,k\,\in\,M_n\}}\Big(f_{\{ijk\}}(H_j)\Big)(X)$$

Since $X \in [\![G_{n-1}]\!]$ and $G_{n-1} \sim G'_{n-1}$, we can say that there exist a arc X' in G'_{n-1} such that $X' = h_{n-1}(X)$ and both *X, X'* are of same multiplicity.

$$\begin{aligned} X' &= h_{n-1}(X) = h_{n-1}(fm_1m_2m_3\cdots m_{\rho(f)}) \\ &= fh_{n-1}(m_1)h_{n-1}(m_2)h_{n-1}(m_3)\cdots h_{n-1}\big(m_{\rho(f)}\big) \\ &= fm'_1m'_2m'_3\cdots m'_{\rho(f)} \end{aligned}$$

Since $X \in \bigcup_{\{ijk\,\in\,M_n\}}\Big(f_{\{ijk\}}(H_j)\Big)$, $X \in f_{\{ijk\}}(H_j)$ for some $(i,j,k) \in M_n$.

Let $S_n = \{(i,j,k) \in M_n | X \in f_{\{ijk\}}(H_j)\}$

$$G_{\{ijk\}}(X) = G_{\{ijk\}}(fm_1m_2m_3\cdots m_{\rho(f)})$$
$$= fG_{\{ijk\}}(m_1)G_{\{ijk\}}(m_2)G_{\{ijk\}}(m_3)\cdots G_{\{ijk\}}(m_{\rho(f)})$$
$$= fm'_1m'_2m'_3\cdots m'_{\rho(f)} = X'$$

Also, $f_{\{ijk\}}(H_j)(X) = f'_{\{ijk\}}\big(H_j\big)(X')\ \forall(i,j,k) \in S_n$

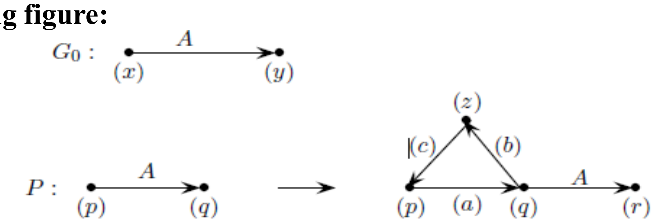
Thus, $X' \in G'_{n-1} \cap f'_{\{ijk\}}(H_j)\forall(i,j,k) \in S_n$

Since $X \in \cup_{(i,j,k)\in S_n} f_{\{ijk\}}(H_j)$ and $f_{\{ijk\}}(H_j) \sim f'_{\{ijk\}}(H_j)$, $X' \in \cup_{(i,j,k)\in S_n} f'_{\{ijk\}}(H_j)$

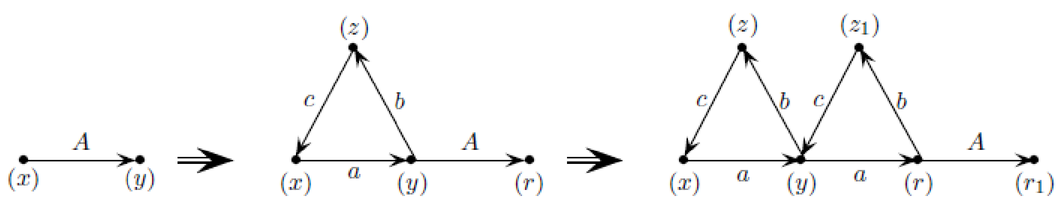
$$G_n(X) = G_{n-1}(X) + \cup_{(i,j,k)\in S_n} \ (f_{\{ijk\}}(H_j))(X)$$
$$= G'_{n-1}(X') + \cup_{(i,j,k)\in S_n} \ (f'_{\{ijk\}}(H_j))(X')$$
$$= G'_{n-1}(X') + \cup_{(i,j,k)\in S_n} \ (f'_{\{ijk\}}(H_j))(X') = G'_n(X')$$
$$h_n(X) = h_n\left(fm_1m_2m_3\cdots m_\rho(f)\right)$$
$$= fh_n(m_1)h_n(m_2)h_n(m_3)\cdots h_n(m_{\rho(f)})$$
$$= fm'_1m'_2m'_3\cdots m'_{\rho(f)} = X' \in G'_{n-1} \cap f'_{\{ijk\}}(H_j)$$

Thus, $G_n \sim G'_n$ and so $[\![G_n]\!] \sim [\![G'{}_i]\!]$. By the definition of graph language, each $[\![G_i]\!]$ is a regular multi-hypergraph of R_2 and each $[\![G'_i]\!]$ is of R_1.

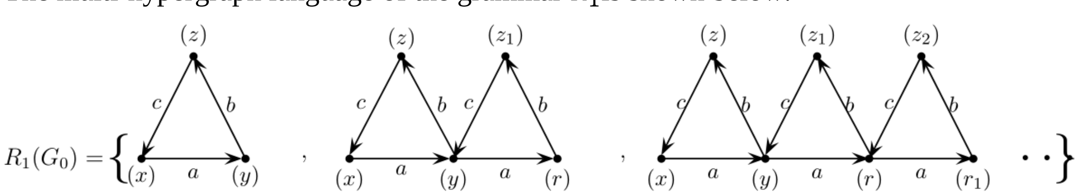
The converse of the above theorem is not true. Let us consider the grammars R_1 as in the following figure:



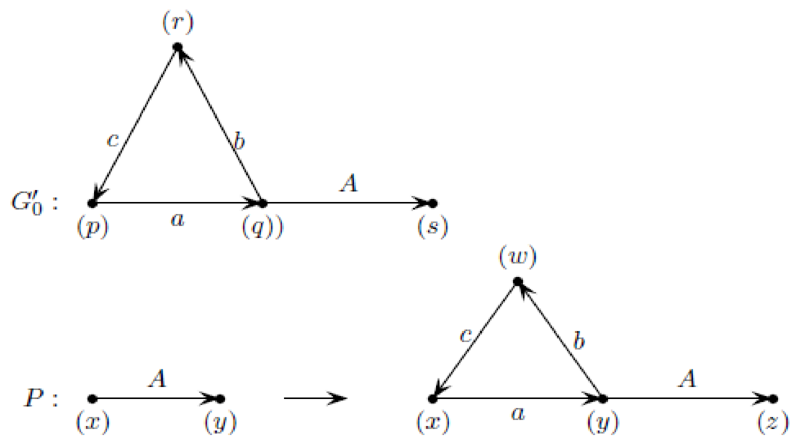
The first three steps of parallel derivation is shown below:



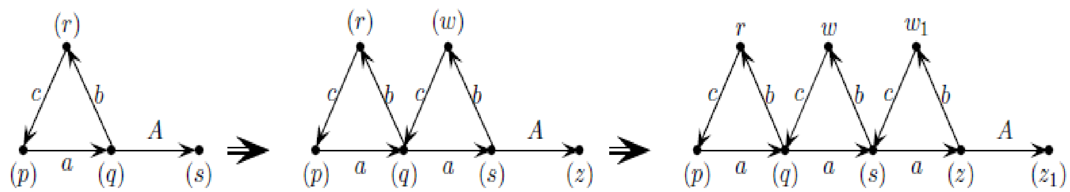
The multi-hypergraph language of the grammar $R_1$is shown below:



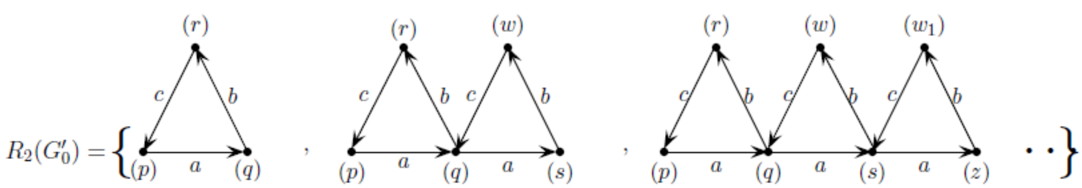
Let us consider the grammar $R_2 = (G_0', P)$ as in the following figure:



The first three steps of parallel derivation is shown below:



The multi-hypergraph language of the grammar R_2 is shown below:



The above example concludes that G_0 and G'_0 need not be isomorphic if each $[\![G_i]\!]$ and $[\![G'_i]\!]$ be regular hypergraph of $R_1$and R_2 respectively. Thus, the theorem can be used as a necessary condition for the graphs to be isomorphic.

5. CONCLUSION

The author has discussed the concept of hypergraph grammar and the languages recognized the grammar with suitable examples. The study has proved the necessary condition for the two multi-hypergraphs to be isomorphic. An example has been given for the condition is not sufficient.

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