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# On the Variants of Domination Number of Certain Graphs

**S. Sarah Surya and Fathima Sana Afrin. S**

Department of Mathematics,
Stella Maris College (Autonomous), Chennai-600 086, INDIA.
email: fathimaafrin77@gmail.com

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**ABSTRACT**

Domination in graphs has applications to several fields. A *Dominating set* for a graph $G = (V, E)$ is a subset $S$ of $V$ such that every vertex not in $S$ is adjacent to at least one member of *S*. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for *G*. In this paper, we determine some parameters of domination namely, the total domination number and independent domination number for certain graphs like the Mongolian tent, Frock graph, Extended fully connected cubic network and we also obtain the domination number for the generalized web graph $W(t, 3), W(t, 4)$where $t \geq 4$.

**Keywords:** Domination number, total domination number, independent domination number, Mongolian tent, Frock graph, Extended fully connected cubic network, generalized web graph.

## 1. INTRODUCTION

Domination in graphs has a wide range of applications in many fields like engineering, physical, social and biological sciences, linguistics and so on. Mathematical study of domination in graphs began around 1960 and there are some references to domination-related problems about 100 years. In 1862, De Jaenisch attempted to determine the minimum number of queens required to cover a$n * n$ chess board. In 1892, W. W. Rouse Ball reported three basic types of problems that chess players studied during this time.

The study of domination in graphs was developed in the late 1950's and 1960's, beginning with Claude Berge[3] in 1958. Berge wrote a book on graph theory, in which he introduced the "coefficient of external stability," which is now known as the domination number of a graph. Oystein Ore[1] introduced the terms "dominating set" and "domination number" in his book on graph theory which was published in 1962. The problems described above were studied in more detail around 1964 by brothers A.M. Yaglom and I.M. Yaglom[8]. Their studies resulted in solutions to some of these problems for rooks, knights, kings, and bishops. A decade later, Cockayne and Hedetniemi[6] published a survey paper, in which the notation $\gamma(G)$ was first used to represent the domination number of a graph $G$.

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced[7].

## 2. PRELIMINARIES

**Definition 2.1** A *Dominating set* for an undirected graph $G = (V, E)$ is a subset $S$ of $V$ such that every vertex not in $S$ is adjacent to at least one member of $S$. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for $G$.

**Definition 2.2** Let $G = (V, E)$ be a graph with vertex set$V$, edge set $E$ and no isolated vertex. A set $S$ of vertices in a graph $G$ is a *total dominating set* of $G$,if every vertex of $G$ is adjacent to some vertex in $S$. The total domination number of $G$ denoted by $\gamma_t(G)$is the minimum cardinality of a total dominating set.

**Definition 2.3** Let $G$ be a graph. A set of vertices $I$ is called an *independent set* if no two vertices in $I$ are adjacent.

**Definition 2.4** A dominating set of $G$ which is also independent set is called an *independent dominating set* or *kernel*. The cardinality of the smallest independent dominating set is denoted by $\gamma_i(G)$ and we call this as independent number of $G$.

## 3. DOMINATION, TOTAL DOMINATION AND INDEPENDENT DOMINATION OF MONGOLIAN TENT

Lee[4] defines a *Mongolian tent*$M_{(m,n)}$ as a graph obtained from $P_m \times P_n$, $n$ odd,by adding a new vertex $\{v\}$ above the grid and joining every other vertex of the top row of $P_m \times P_n$ to the new vertex $\{v\}$(i.e.)$\{P_m \times P_n\} \cup \{v\}$.Let $p = n^2 + 1$ be the number of vertices and $q = n(2n - 1)$ be the number of edges of $M_{(m.n)}$. See Figure 1.

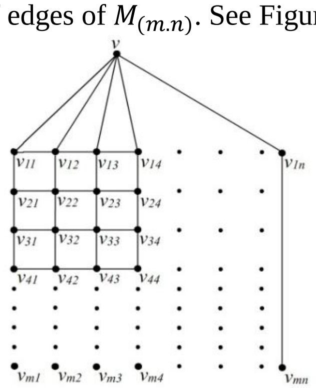



**Figure 1: Mongolian tent $M_{(m,n)}$**

**Theorem2.1.** The domination number for Mongolian tent $M_{(m.n)}$ is given by,

$$\gamma\left(M_{(m,n)}\right) \leq \begin{cases} 1\ , when\ m = 1, nodd \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, whenm = 2, nodd \\ 1 + \left(\left\lfloor \frac{m-1}{2} \right\rfloor \left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right)\right) + \left(\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{3} \right\rfloor\right), whenm \geq 3, nodd \end{cases}$$

**Proof:**

The case when $m = 1, 2,\ n$ −odd is obvious. Let us consider the Mongolian tent$M_{(m.n)}$ , $m \geq 3, n = 3{,}5, ....$Let $v_{ij}$ , $i = 1, 2, \dots, m, j = 1, 2, 3, \dots, n$. denote the vertices of $P_m \times P_n$. The dominating vertices of $M_{(m.n)}$ is then determined by the following procedure:

Obviously the vertex $\{v\}$ is in the dominating set of $M_{(m,n)}$. See Figure 2.We now consider two cases in order to determine the other dominating vertices in $P_m \times P_n$.

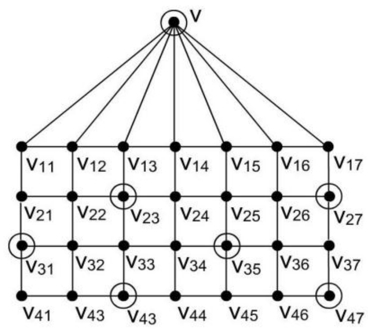



**Figure 2:**$\boldsymbol{M_{(4,7)}}$

**Case (i):** When $i = 2k + 1$, where $k = 1{,}2, .., \left\lfloor \frac{m-1}{2} \right\rfloor$, and $j = 4l + 1$,where $l = 0{,}1, .., \left\lfloor \frac{n}{4} \right\rfloor$, the vertices $v_{ij}$forms the dominating set.

**Case (ii):** When $i = 2p$, where $p = 1, 2, \dots, \left\lfloor \frac{m}{2} \right\rfloor$ and $j = 4h - 1$, where $h = 1, 2, .., \left\lfloor \frac{n}{3} \right\rfloor$, the vertices $v_{ij}$ forms the dominating set.

The vertex $\{v\}$together with the vertices $v_{ij}$ defined in cases (i) and (ii) constitute the total number of vertices in the dominating set. Therefore the domination number is given by,

$$\gamma\left(M_{(m,n)}\right) \leq \begin{cases} 1\ , whenm = 1, nodd \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, whenm = 2, nodd \\ 1 + \left(\left\lfloor \frac{m-1}{2} \right\rfloor \left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right)\right) + \left(\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{3} \right\rfloor\right)\ , whenm \geq 3, nodd \end{cases}$$

**Theorem 2.2.** The independent domination number is given by

$$\gamma_i\left(M_{(m,n)}\right) \leq \begin{cases} 1\ whenm = 1, nodd \\ \left\lfloor \frac{n}{2} \right\rfloor + 1\ m = 2, nodd \\ 1 + \left(\left\lfloor \frac{m-1}{2} \right\rfloor \left(\left\lfloor \frac{n}{4} \right\rfloor + 1\right)\right) + \left(\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{3} \right\rfloor\right)\ , whenm \geq 3, nodd \end{cases}$$

**Proof:**
The minimum dominating setis always less than or equal to the size of minimum maximal independent set. For a Mongolian tent $M_{(m.n)}$ , $m \geq 3, n = 3,5, \ldots$ the independent domination number is equal to the domination number i.e., $\gamma_i(M_{(m,n)}) = \gamma(M_{(m,n)})$.
**Theorem2.3.** The total domination number for Mongolian tent $M_{(m.n)}$ is given by
$\gamma_t(M_{(m,n)}) \leq m\left\lfloor\frac{n}{2}\right\rfloor,\ m \geq 2,\ n = 3,5, \ldots,.$
**Proof:**
Let $v_{ij}$ , $i = 1, 2, \ldots, m,\ j = 1, 2, 3, \ldots, n$denote the vertices of the path of $P_m \times P_n$. The vertex $\{v\}$ is in the dominating set of$M_{(m.n)}$. The dominating vertices is determined by $v_{ij}, i = 1,2,3, \ldots, m, j = 2s, s = 1,2, \ldots, \left\lfloor\frac{n}{2}\right\rfloor$. The vertices $v_{ij}$forms the dominating set. Therefore the total domination number is given $\gamma_t(M_{(m,n)}) \leq m\left\lfloor\frac{n}{2}\right\rfloor m \geq 2, n = 3,5, \ldots$

## 4. DOMINATION, TOTAL DOMINATION AND INDEPENDENT DOMINATION OF FROCK GRAPH

Frock graph[9] is obtained from a 3-cycle $u_1, u_2, u_3$ by adjoining $u_3$ with a fan graph $F_n$ in such a way that $u_3$ is an apex of $F_n$. We denote the graph by $FG_n,\ n \geq 5$. A Frock graph has $p = (n + 3)$ vertices and $q = (2n + 2)$ edges. See figure 3.

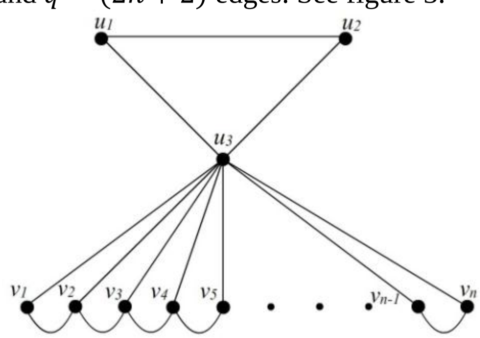



**Figure 3: Frock graph $FG_n$**

**Theorem 3.1.** The domination number, independent domination number and total domination number of Frock graph $FG_n, n \geq 5$ is given by
$\gamma(FG_n) = 1$.

**Proof:**
The domination number for Frock graph is obtained as follows: when $n = 5, FG_5$ has 3-cycle $u_1, u_2, u_3$ by adjoining $u_3$ with a fan graph $F_5$ in such a way that $u_3$ is adjacent to every other vertices in $F_5$. Hence the vertex $u_3$is the dominating vertex in every case of $FG_n,\ n \geq 5$. See Figure 4. Thus the domination number of a Frock graph is 1.

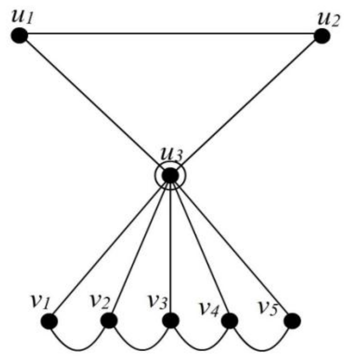



**Figure 4: $FG_5$**

The independent domination number and total domination number for frock graph follows from the above theorem.

## 5. DOMINATION, TOTAL DOMINATION AND INDEPENDENT DOMINATION OF EXTENDED FULLY CONNECTED CUBIC NETWORK

Let $Z_8 = \{0,1,2,3,4,5,6,7\}$. For $m \geq 1$ and $\in Z_8$ , let $a^m = aa..a\ (mtimes)$. For $n \geq 1$, an*n*-level $FCCN$, denoted $FCCN_n$, is a graph defined recursively as follows:

1. $FCCN_1$ is a graph with $V(FCCN_1) = Z_8$ and $E(FCCN_1) = \{(0,1),(0,2),(1,3),(2,3),(4,5)(5,7),(6,7),(0,4),(1,5),(2,6),(3,7)\}$.
2. When $n \geq 2$, $FCCN_n$ is built from eight vertex- disjoint copies of $FCCN_{n-1}$ by adding 28 edges. Specifically if , for $0 \leq k \leq 7$, we let $kFCCN_{n-1}$ denote a copy of $FCCN_{n-1}$ with each vertex being prefixed with $k$, then $FCCN_n$ is defined by $V(FCCN_n) = \bigcup_{k=0}^{7} V(kFCCN_{n-1})$, $E(FCCN_n) = (\bigcup_{k=0}^{7} V(kFCCN_{n-1})) \cup \{(pq^{n-1}, qp^{n-1}): 0 \leq p < q \leq 7\}$. For $0 \leq k \leq 7$, $kFCCN_{n-1}$ is called an $(n-1)$ $-$level sub-$FCCN$ of $FCCN_n$, or simply a sub-$FCCN$ of $FCCN_n$, if there is no ambiguity.
3. Given an $FCCN_n$,$n \geq 2$. A boundary vertex is a vertex of the form $k^n$. An intercubic edge is an edge of the form $(pq^{n-1}, qp^{n-1})$. In essence, each vertex of an $FCCN$ has four links, with each boundary vertex having one I\O channel link that is not counted in the vertex degree. Obviously, $kFCCN_{n-1}$ has 7 inter cubic vertices and 1 boundary vertex for $0 \leq k \leq 7$ and $n \geq 2$.

For every $\in Z_8$ , the *extended fully connected cubic network*$FCCN_n^{t}$[5] is the graph obtained from $FCCN_n$ by joining the vertices in the set $\{p^n | p \in Z_8 - \{t\}\}$ to an extra vertex $w$. $FCCN_n^i$is isomorphic to $FCCN_n^j$ for every $i, j$ in $Z_8$.See Figure 5.$FCCN_n^t$ has $8^n + 1$ vertices and $2(8^n) + 3$ edges.

We label the vertices of $FCCN_n^t$ using radix lexicographic ordering:

**Radix-lexicographic ordering:**

Label the vertices of $FCCN_n^t$, $n \geq 1$ as follows:

1. Label the vertices of $FCCN_1^t$ as 1 digit radix $Z_8$ number, say $0,1,\dots,7$ by lexicographic order.
2. Label the vertices of $FCCN_2^t$ , as 2 digit radix $Z_8$ number ,$0FCCN_1^t, 1FCCN_1^t, \dots, 7FCCN_1^t$.
3. Inductively label the vertices of $FCCN_n^t$ , as $n$ digit radix $Z_8$ number, $kFCCN_{n-1}^t$ ,$0 \leq k \leq 7$.

**Theorem 4.1.**

The domination number of Extended fully connected cubic network $FCCN_n^t, t \in Z_8$ is given by

$\gamma(FCCN_n^t) \leq 15(8^{n-2}), n \geq 2$.

**Input**:

An extended fully connected cubic network $FCCN_n^t$ with radix-lexicographic ordering

**Algorithm:**

(i) When $n = 1$, The minimum dominating setoff $FCCN_1^t$ for each $t \in Z_8$ is $D_1 = \{0,7\}$ and hence the domination number of $\gamma(FCCN_1^t)$ = 2.

(ii) When$n = 2$, the minimum dominating set of $FCCN_2^t$, $t \in Z_8$ is $D_2 = \{00,07,10,17,20,27,30,37,40,47,50,57,60,67,77\}$ . See Figure 5.3

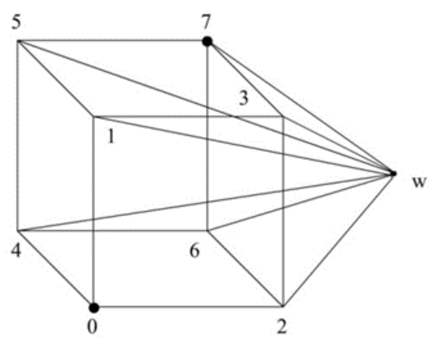
(iii) When $n = 3$ , $FCCN_3^t$,$t \in Z_8$ the minimum dominating set is $D_3 = \bigcup_{k=0}^{7} kD_2$ .

(iv) Inductively , the minimum dominating set contains the vertices of the form, $D_i = \bigcup_{k=0}^{7} kD_{i-1}$ , $3 \leq i \leq n$.

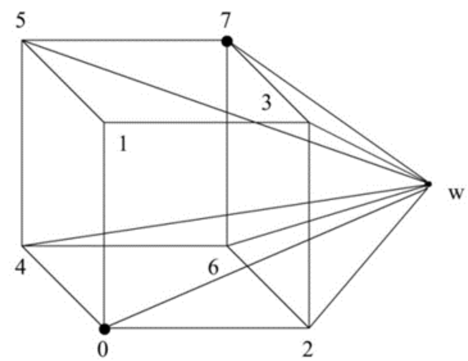
**Output:** $\gamma(FCCN_n^t) \leq 15(8^{n-2}), n \geq 2$

**Proof:**

**Case (i):** When $n = 1, t = 0$ the dominating vertices are $D_1 = \{0,7\}$ as shown in Figure 5 (a). When $n = 1, t = 1$. The dominating vertices are $D_1 = \{0,7\}$see figure 5(b)




(a)




(b)

**Figure 5: (a)Extended Fully Connected Cubic Network $FCCN_1^0$. The doted vertices are dominating vertices.Hence $\gamma(FCCN_1^0) = 2$ (b)Extended Fully Connected Cubic Network $FCCN_1^1$. The doted vertices are dominating vertices. Hence$\gamma(FCCN_1^1) = 2$**

**Case(ii):** When $n = 2, t = 0$ the dominating vertices are
$D_2 = \{00,07,10,17,20,27,30,37,40,47,50,57,60,67,77\}$ as shown in Figure 6.
When $n = 2, t = 1$ the dominating vertices are
$D_2 = \{00,07,10,17,20,27,30,37,40,47,50,57,60,67,77\}$ see fig.7

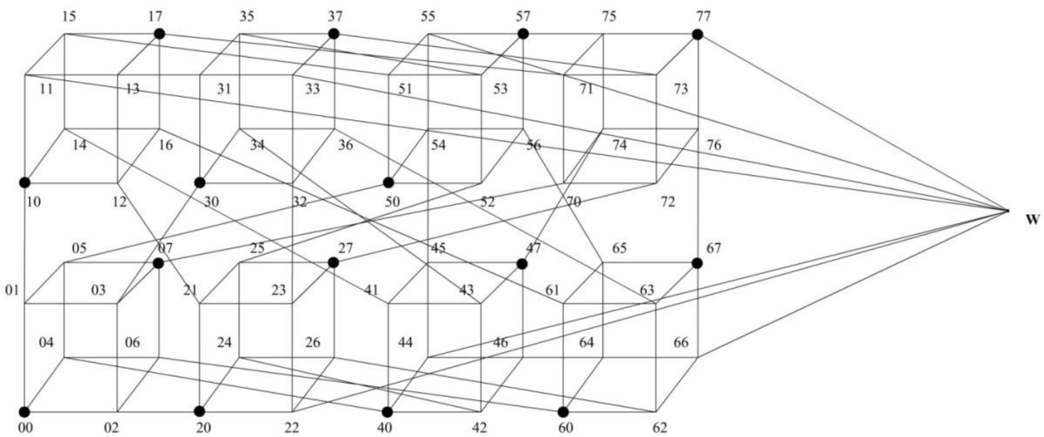



**Figure 6: Extended Fully Connected Cubic Network $FCCN_2^0$ The doted vertices are the dominating vertices. Hence $\gamma(FCCN_2^0) = 15$**

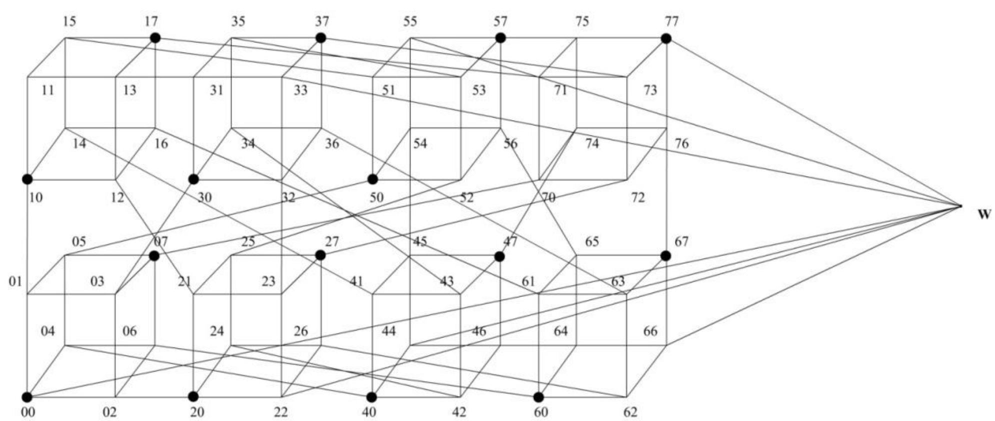



**Figure 7: Extended Fully Connected Cubic Network $FCCN_2^1$ The doted vertices are the dominating vertices. Hence $\gamma(FCCN_2^1) = 15$**

Generalizing we get $D_i = \bigcup_{k=0}^{7} kD_{i-1}, 3 \leq i \leq n$.
Hence the domination number is$\gamma(FCCN_n^t) \leq 15(8^{n-2}), n \geq 2$.

### Theorem 4.2.

The total domination number of Extended fully connected cubic network $FCCN_n^t, t \in Z_8$ is given by $\gamma(FCCN_n^t) \leq 18(8^{n-2}), n \geq 2$.

**Input:**

An extended fully connected cubic network $FCCN_n^t$ with radix-lexicographic ordering

**Algorithm:**

(i) When $n = 1$, The minimum total dominating set $FCCN_1^t$ for each $t \in Z_8$ is $TD_1 = \{w\} \cup \{i\}, i \in \{0,1,2,..7\}, i \neq t$

For example,

$t = 0, FCCN_1^0$ the dominating vertices are $TD_1 = \{w\} \cup \{1\}$

$t = 1, FCCN_1^1$ the dominating vertices are $TD_1 = \{w\} \cup \{0\}$

$t = 2, FCCN_1^2$ the dominating vertices are $TD_1 = \{w\} \cup \{3\}$

$t = 3, FCCN_1^3$ the dominating vertices are $TD_1 = \{w\} \cup \{2\}$

$t = 4, FCCN_1^4$ the dominating vertices are $TD_1 = \{w\} \cup \{5\}$

$t = 5, FCCN_1^5$ the dominating vertices are $TD_1 = \{w\} \cup \{4\}$

$t = 6, FCCN_1^6$ the dominating vertices are $TD_1 = \{w\} \cup \{7\}$

$t = 7, FCCN_1^7$ the dominating vertices are $TD_1 = \{w\} \cup \{6\}$. See Figure 5.3

Therefore the total domination number of $\gamma(FCCN_1^t)$=2.

(ii) When $n = 2$, $FCCN_2^t$ , $t \in Z_8$, the minimum total dominating set is $TD_2 = \{D_2\} \cup \{ii\} \cup \{w\}, where D_2 = \{10,01,17,71,24,42,23,32,35,53,54,45,06,60,67,76\}$ Figure 5.4.

(iii) When $n = 3$ , $FCCN_3^t, t \in Z_8$ the minimum total dominating set is $TD_3 = \bigcup_{k=0}^{7} kD_2 \cup \{iii\} \cup \{w\}$.

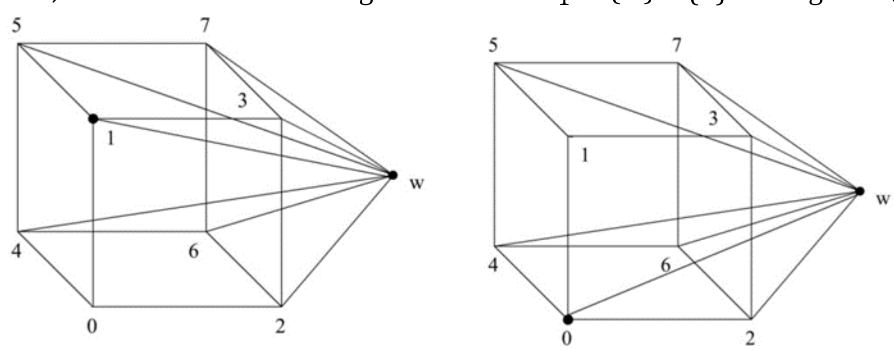
(iv) In general, the minimum dominating set contains the vertices in this manner ,$TD_i = \bigcup_{k=0}^{7} kD_{i-1} \cup \{\underbrace{ii \dots i}_{ntimes}\} \cup \{w\}, 3 \leq i \leq n$.

**Output:** $\gamma(FCCN_n^t) \leq 18(8^{n-2}), n \geq 2$

**Proof:**

**Case (i):** When $n = 1, t = 0$ the total dominating vertices are $TD_1 = \{w\} \cup \{1\}$ as shown in figure 8 (a)

When $n = 1, t = 1$ the total dominating vertices are $TD_1 = \{w\} \cup \{0\}$ See Figure 8 (b)




(a) (b)

**Figure 8: (a)Extended Fully Connected Cubic Network $FCCN_1^0$ The doted vertices are the minimum total dominating vertices. Hence $\gamma(FCCN_1^0) = 2$ (b) Extended Fully Connected Cubic Network $FCCN_1^1$ The doted vertices are the minimum total dominating vertices. Hence $\gamma(FCCN_1^1) = 2$**

**Case(ii):** When $n = 2, t = 0$ the total dominating vertices are
$TD_2 = \{10,01,17,71,24,42,23,32,35,53,54,45,06,60,67,76\} \cup \{11\} \cup \{w\}$
as shown in figure 9
When $n = 2, t = 1 TD_2 = \{10,01,17,71,24,42,23,32,35,53,54,45,06,60,67,76\} \cup \{00\} \cup \{w\}$ See Figure 10

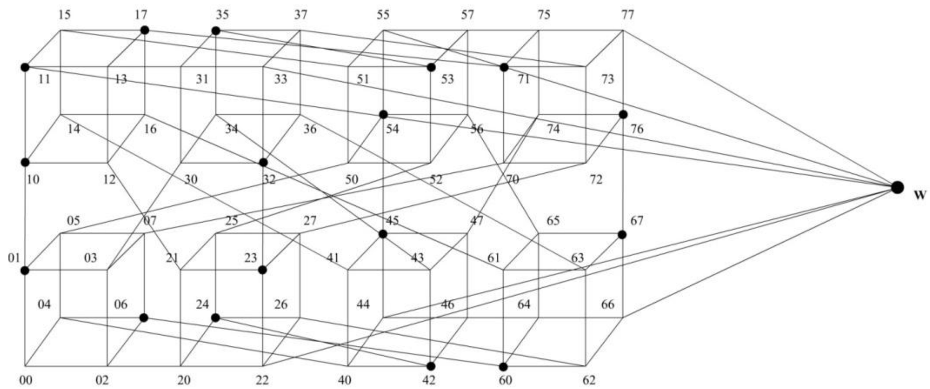


**Figure 9 : Extended Fully Connected Cubic Network $FCCN_2^1$ The doted vertices are the minimum total dominating vertices. Hence $\gamma(FCCN_2^1) = 18$**

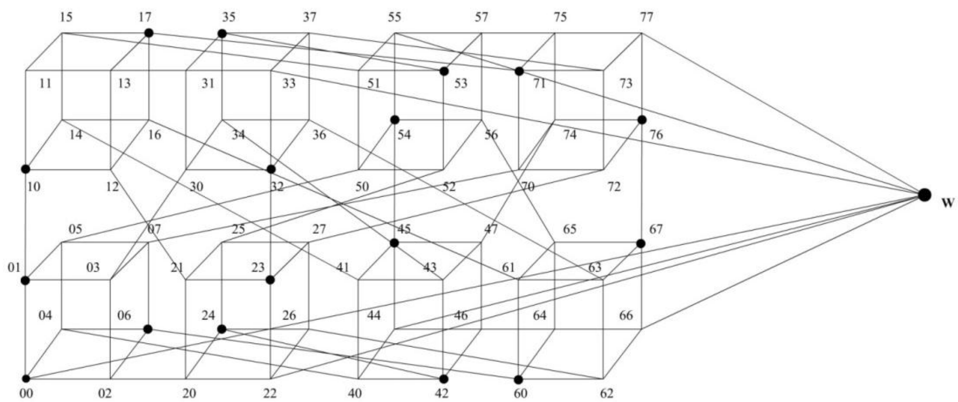


**Figure 10: Extended Fully Connected Cubic Network $FCCN_2^1$ The doted vertices are the minimum total dominating vertices. Hence $\gamma(FCCN_2^1) = 18$**

Generalizing we get $TD_i = \bigcup_{k=0}^{7} kD_{i-1} \cup \{\underbrace{ii \dots i}_{ntimes}\} \cup \{w\}, 3 \le i \le n.$ Hence the total domination number is $\gamma(FCCN_n^t) \le 18(8^{n-2}), n \ge 2.$

## 6. DOMINATION NUMBER OF GENERALIZED WEB GRAPH

A web graph is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.

The generalized web[2] $W(t, n)$ is the graph with $t$ cycles each of order $n$.

**Theorem 5.1.**

The domination number of generalized web graph $W(t, 3), t \geq 4$ is given by,

$$\gamma(W(t,3)) \leq \left\lfloor \frac{t-1}{2} \right\rfloor + \left( \left\lfloor \frac{t}{2} \right\rfloor - 1 \right) + 1 + n$$

**Proof:**

Consider web graph $W(t, 3), t \geq 4$ . let $c$ denote the centre apex vertex and the cycles are denoted as $C_1, C_2 C_3, \dots, C_t$ as seen in Figure 5. The vertices are denoted as $v_{1,1} v_{1,2}, \dots, v_{1,n}$. The next set of vertices are denoted as $v_{2,1} v_{2,2}, \dots, v_{2,n}$ respectively. Similarly, proceeding we get the vertices on the $t^{th}$cycle are denoted as$v_{t,1,} v_{t,2} v_{t,3}, \dots, v_{t,n}$ The pendant vertices are denoted as $u_{1,} u_2 u_3, \dots, u_n$. See Figure 11.

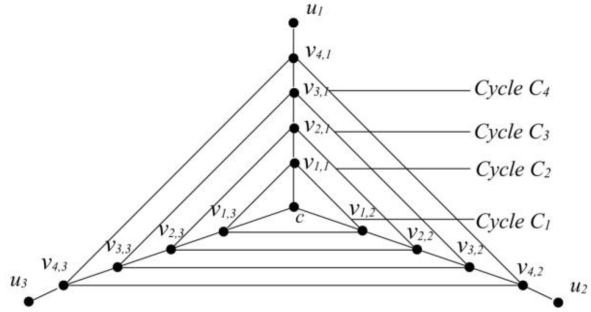



**Figure 11:$W(4, 3)$**

We know that generalized web graph contains $p = tn + 4$ vertices and $q = 3(2t + 1)$ edges. The dominating set of $W(t, 3), t \geq 4$ will contain the centre apex vertex $c$ which will dominate cycle $C_1$ and the pendant vertices $u_l$ , $l = 1,2, \dots, n$ dominate the cycle $C_t$. We determine the other dominating vertices in two cases :

**Case (i):**

When $i = 2k, k = 1,2, \dots, \left\lfloor \frac{t-1}{2} \right\rfloor$ and $= \left\lfloor \frac{n}{3} \right\rfloor$ , the vertices $v_{ij}$ forms the dominating set.

**Case(ii):**

When $i = 2h + 1, h = 1,2, \dots, \left( \left\lfloor \frac{t}{2} \right\rfloor - 1 \right)$ and $j = \left\lfloor \frac{n}{3} \right\rfloor + 2$, the vertices $v_{ij}$ forms the dominating set.

Therefore, the vertex $c$, pendant vertices $u_{1,} u_2 u_3, \dots, u_n$ together with cases (i) and (ii) constitutes the vertices in the dominating set. Hence the domination number is given by $\gamma(W(t,3)) \leq \left\lfloor \frac{t-1}{2} \right\rfloor + \left( \left\lfloor \frac{t}{2} \right\rfloor - 1 \right) + 1 + n, t \geq 4, n = 3$. See figure 12

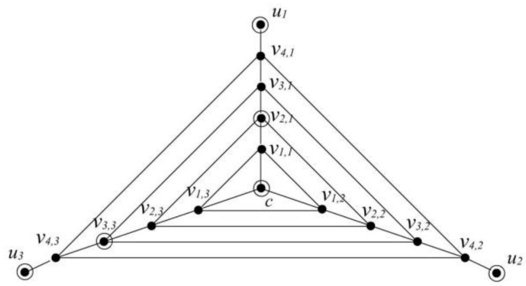


**Figure 12:** $\boldsymbol{\gamma(W(4,3)) = 6}$

**Theorem 5.2.**

The domination number of generalized web graph $W(t,4)$is given by

$$\gamma(W(t,4)) \leq \left\lfloor \frac{t-1}{2} \right\rfloor + \left( \left\lfloor \frac{t}{2} \right\rfloor - 1 \right) + 1 + n$$

**Proof :**

The web graph contains $p = tn + 5\ , t \geq 4, n = 4$ vertices and $q = 4(2t+1)$ edges. The proof is similar to the theorem 5.1.See Figure 13.

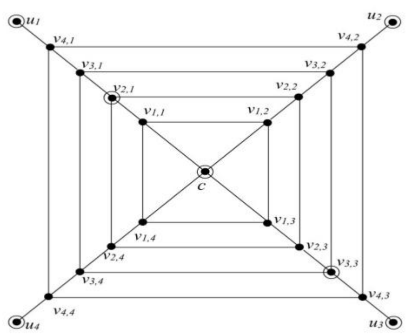


**Figure 13:** $\boldsymbol{\gamma(W(4,4)) = 7}$

**Remark:**

The minimum dominating set is always less than or equal to the size of minimum maximal independent set[10]. Therefore, the independent domination number is equal to domination number for extended fully connected cubic network and generalized web graph.

**CONCLUSION**

Domination in graphs has a wide range of applications to many fields like engineering, physical, social and biological sciences, linguistics and so on. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor muststand in order to take height measurements for an entireregion). In this paper, we have determined the domination and domination parameters for Mongolian tent, frock graph, extended fully connected cubic network and generalized web graph.

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