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On Pre-generalized c*-closed Sets in Topological Spaces

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ABSTRACT

The aim of this paper is to introduce the notion of pre-generalized c*-closed sets in topological spaces and study their basic properties.

Keywords: c*-open sets, gc*-closed sets and pgc*-closed sets.

1. INTRODUCTION

In 1963, Norman Levine introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. Bhattacharya and Lahiri introduced and study semi-generalized closed (briefly, sg-closed) sets in 1987. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In the year 1996, Andrijevic introduced and studied b-open sets. Gnanambal introduced generalized pre-regular closed (briefly gpr-closed) sets in 1997. In this paper we introduce pre-generalized c*-closed sets in topological spaces and study its basic properties.

Section 2 deals with the preliminary concepts. In section 3, pre generalized c*-closed sets are introduced and their basic properties are discussed.

2. PRELIMINARIES

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and bcl(A) denotes the b-closure of A. Further $X\backslash A$ denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition: 2.1 A subset A of a topological space X is called

i. a semi-open set[7] if A⊆cl(int(A)) and a semi-closed set if int(cl(A))⊆A.
ii. a pre-open set[13] if A⊆int(cl(A)) and a pre-closed set if cl(int(A))⊆A.
iii. a regular-open set[15] if A=int(cl(A)) and a regular-closed set if A=cl(int(A)).
iv. a γ-open set[9] (b-open set[1]) if A⊆cl(int(A))∪int(cl(A)) and a γ-closed set (b-closed set) if int(cl(A))∩cl(int(A))⊆A.
v. a π-open set[19] if A is the finite union of regular-open sets and the complement of π-open set is said to be π-closed.

Definition: 2.2[10] A subset A of a topological space X is said to be a c*-open set if int(cl(A))⊆A⊆cl(int(A)).

Definition: 2.3 A subset A of a topological space X is called

i. a generalized closed set (briefly, g-closed)[8] if cl(A)⊆H whenever A⊆H and H is open in X.
ii. a regular-generalized closed set (briefly, rg-closed)[14] if cl(A)⊆H whenever A⊆H and H is regular-open in X.
iii. a generalized pre-regular closed set (briefly, gpr-closed)[5] if pcl(A)⊆H whenever A⊆H and H is regular-open in X.
iv. a regular generalized b-closed set (briefly, rgb-closed)[12] if bcl(A)⊆H whenever A⊆H and H is regular-open in X.
v. a regular weakly generalized closed set (briefly, rwg-closed)[17] if cl(int(A))⊆H whenever A⊆H and H is regular-open in X.
vi. a semi-generalized b-closed set (briefly, sgb-closed)[6] if bcl(A)⊆H whenever A⊆H and H is semi-open in X.
vii. a weakly closed (briefly, w-closed) set[16] (equivalently, ĝ-closed set[20]) if cl(A)⊆H whenever A⊆H and H is semi-open in X.
viii. a semi-generalized closed set (briefly, sg-closed)[3] if scl(A)⊆H whenever A⊆H and H is semi-open in X.
ix. a generalized semi-closed (briefly, gs-closed) set[2] if scl(A)⊆H whenever A⊆H and H is open in X.
x. a (gs)*-closed set[4] if cl(A)⊆H whenever A⊆H and H is gs-open in X.

The complements of the above mentioned closed sets are their respectively open sets.

Definition: 2.4[10] A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if cl(A)⊆H whenever A⊆H and H is c*-open. The complement of the gc*-closed set is gc*-open[11].

3. PRE-GENERALIZED C*-CLOSED SETS

In this section we introduce pre-generalized c*-closed sets in topological spaces. Also, we discuss about some of their basic properties.

Definition: 3.1 A subset A of a space X is said to be pre-generalized c*-closed (briefly, pgc*-closed) if pcl(A)⊆H whenever A⊆H and H is c*-open.

Example: 3.2 Let X={a,b,c} with topology τ={ϕ,{b},{c},{b,c},X}. Then the pgc*-closed sets are ϕ,{a},{a,b},{a,c},{b,c},X.

Proposition: 3.3 Let X be a topological space. Then every w-closed (equivalently, ĝ-closed) set is pgc*-closed.

Proof: Let A be a w-closed set. Let H be a c*-open set containing A. Since every c*-open set is semi-open, we have H is semi-open. Then cl(A)⊆H. Since pcl(A)⊆cl(A), we have pcl(A)⊆H. Therefore, A is pgc*-closed.

The converse of the Proposition 3.3 need not be true as seen from the following example.

Example: 3.4 Let X={a,b,c,d,e} with topology τ={ϕ,{a,b},{c,d},{a,b,c,d},X}. Then, the subset {a,b,c} is pgc*-closed but not w-closed.

Proposition: 3.5 Let X be a topological space. Then every (gs)*-closed set is pgc*-closed.

Proof: Let A be a (gs)*-closed set. Then cl(A)⊆U whenever A⊆U and U is gs-open. Let H be a c*-open set containing A. Since every c*-open set is gs-open, we have H is gs-open. Then cl(A)⊆H. Since pcl(A)⊆cl(A), we have pcl(A)⊆H. Hence A is pgc*-closed.

The following example shows that the converse of the Proposition 3.5 is not true.

Example: 3.6 Let X={a,b,c,d} with topology τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d},X}. Then the subset {a,c} is pgc*-closed but not (gs)*-closed.

Proposition: 3.7 Let X be a topological space. Then every closed set is pgc*-closed.

Proof: Let A be a closed set. Since every closed set is w-closed, we have A is w-closed. Then, by Proposition 3.3, A is pgc*-closed.

The converse the Proposition 3.7 need not be true and is proved by the following example.

Example: 3.8 Let X={a,b,c,d} with topology τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d},X}. Then the subset {a,b} is pgc*-closed but not closed.

Proposition: 3.9 Let X be a topological space. Then every π-closed set is pgc*-closed.

Proof: Let A be a π-closed set. Then A is closed. Hence, by Proposition 3.7, A is pgc*-closed.

The converse of the Proposition 3.9 need not be true, which can be verified from the following example.

Example: 3.10 In Example 3.8, the subset {a,b,c} is pgc*-closed but not π-closed.

Proposition: 3.11 Let X be a topological space. Then every regular closed set is pgc*-closed.

Proof: Let A be a regular closed set. Since every regular closed set is closed, we have A is closed. Hence, by Proposition 3.7, A is pgc*-closed.

The converse of the Proposition 3.11 is not true as shown in the following example.

Example: 3.12 In Example 3.8, the subset {c} is pgc*-closed but not regular closed.

Proposition: 3.13 Let X be a topological space. Then every gc*-closed set is pgc*-closed.

Proof: Let A be a gc*-closed set. Let H be a c*-open set containing A. Then cl(A)⊆H. Since pcl(A)⊆cl(A), we have pcl(A)⊆H. Therefore, A is pgc*-closed.
The converse the Proposition 3.13 need not be true and is proved by the following example.

Example: 3.14 Let X={a,b,c,d} with topology τ={ϕ,{a},{b},{a,b},{a,c},{a,b,c},X}. Then the subset {c} is pgc*-closed but not gc*-closed.

Proposition: 3.15 Let X be a topological space. Then every pgc*-closed set is gpr-closed.

Proof: Let A be a pgc*-closed set. Let U be a regular open set containing A. Since every regular open set is c*-open, we have U is c*-open. Then, pcl(A)⊆U. Hence A is gpr-closed.
The converse of the Proposition 3.15 need not be true as seen from the following example.

Example: 3.16 Let X={a,b,c,d,e} with topology τ={ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e},X}. Then the subset {a,b} is gpr-closed but not pgc*-closed.

Proposition: 3.17 Let X be a topological space. Then every pgc*-closed set is rgb-closed.

Proof: Let A be a pgc*-closed set. Then pcl(A)⊆H whenever A⊆H and H is c*-open. Let U be a regular open set containing A. Then U is c*-open. This implies, pcl(A)⊆U. Since every pre-closed set is b-closed, bcl(A)⊆pcl(A). Then bcl(A)⊆U. Therefore, A is rgb-closed.
The following example shows that the converse of the Proposition 3.17 is not true.

Example: 3.18 Let X={a,b,c,d} with topology τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d},X}. Then the subset {a} is rgb-closed but not pgc*-closed.
The union of two pgc*-closed subsets of a space X need not be pgc*-closed. For example, let X={a,b,c,d,e} with topology τ={ϕ,{a,b},{c,d},{a,b,c,d},X}. Then the subsets {a} and {b} are pgc*-closed sets but their union {a,b} is not a pgc*-closed set.
The intersection of two pgc*-closed subsets of a space X need not be pgc*-closed. For example, let X={a,b,c,d,e} with topology τ={ϕ,{a,b},{c,d},{a,b,c,d},X}. Then the subsets {a,b,c} and {a,b,d} are pgc*-closed sets but their intersecion {a,b} is not pgc*-closed.

Proposition: 3.19 If a subset A of a space X is pgc*-closed, then pcl(A)\A does not contain any non-empty c*-open set in X.

Proof: Assume that A is a pgc*-closed set in X. Suppose H is a c*-open set such that H⊆ pcl(A)\A and H≠ϕ. Then, H⊆X\A. This implies, A⊆X\H. Since H is c*-open, we have X\H is c*-open. Also, since A is pgc*-closed and A⊆X\H, we have pcl(A)⊆X\H. This implies, H⊆ X\pcl(A). Then H⊆pcl(A)∩(X\pcl(A))=ϕ, which is a contradiction. Hence pcl(A)\A does not contain any non-empty c*-open set in X.

Proposition: 3.20 Let X be topological space. Then for any element $p \in X$, the set $X \backslash \{p\}$ is either pgc*-closed or c*-open.

Proof: Suppose $X \backslash \{p\}$ is not a c*-open set. Then X is the only c*-open set containing $X \backslash \{p\}$. This implies, $pcl(X \backslash \{p\}) \subseteq X$. Hence $X \backslash \{p\}$ is a pgc*-closed set in X.

Proposition: 3.21 Let A be a pgc*-closed set in a topological space X. Then A is pre-closed if and only if $pcl(A) \backslash A$ is c*-open.

Proof: Suppose A is pre-closed. Then, pcl(A)=A. This implies, $pcl(A) \backslash A = \phi$, which is c*-open. Conversely, suppose that $pcl(A) \backslash A$ is c*-open. Since A is pgc*-closed, by Proposition 3.19, $pcl(A) \backslash A = \phi$. This implies, A=pcl(A). Hence A is pre-closed.

Proposition: 3.22 Let X be a topological space. If A is pgc*-closed subset of X such that $A \subseteq B \subseteq pcl(A)$, then B is a pgc*-closed set in X.

Proof: Let H be a c*-open set containing B. Then $A \subseteq H$. Since A is pgc*-closed, we have $pcl(A) \subseteq H$. Since $B \subseteq pcl(A)$, we have $pcl(B) \subseteq H$. Therefore, B is a pgc*-closed set in X.

Proposition: 3.23 Let X be a topological space. If ϕ and X are the only c*-open sets, then all the subsets of X are pgc*-closed.

Proof: Let A be a subset of X. If $A=\phi$, then A is pgc*-closed. If $A \neq \phi$, then X is the only c*-open set containing A. This implies, $pcl(A) \subseteq X$. Hence A is pgc*-closed.
The converse of the Proposition 3.23 need not be true as seen from the following example.

Example: 3.24 Let X={a,b,c,d}. Then, clearly τ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{c,d}, {a,b,c}, {b,c,d},{a,c,d},X} is a topology on X. Here all the subsets of X are pgc*-closed but the c*-open sets are ϕ,{a},{b},{a,b},{c,d},{a,c,d},{b,c,d},X.

Proposition: 3.25 A subset A of a space X is pgc*-closed if and only if for each $A \subseteq H$ and H is c*-open, there exists a pre-closed set F such that $A \subseteq F \subseteq H$.

Proof: Suppose that A is pgc*-closed and $A \subseteq H$ and H is c*-open. Then $pcl(A) \subseteq H$. If we put F=pcl(A), then $A \subseteq F \subseteq H$. Conversely, assume that H is a c*-open set containing A. Then by hypothesis, there exists a pre-closed set F such that $A \subseteq F \subseteq H$. Since pcl(A) is the smallest pre-closed set containing A, we have $pcl(A) \subseteq F$. Then $pcl(A) \subseteq H$. Therefore, A is pgc*-closed.

Proposition: 3.26 If a subset A of a space X is pgc*-closed, then $pcl(A) \backslash A$ does not contain any non-empty regular open(resp. regular closed) set in X.

Proof: Suppose H is a regular open (resp. regular closed) set contained in $pcl(A) \backslash A$ and $H \neq \phi$. Since every regular open (resp. regular closed) set is c*-open, we have H is c*-open. Thus, H is a c*-open set contained in $pcl(A) \backslash A$. Then, by Proposition 3.19, $H=\phi$. This is a contradiction. Hence $pcl(A) \backslash A$ does not contain any non-empty regular open (resp. regular closed) set in X.

Proposition: 3.27 Let X be a topological space and A be a subset of X. If A is regular open and pgc*-closed, then A is both pre-open and pre-closed.

Proof: Assume that A is regular open and pgc*-closed. Since every regular open set is c*-open, we have pcl(A)⊆A. Then A=pcl(A). This implies, A is pre-closed. Since A is regular open, we have A is pre-open. Hence A is both pre-open and pre-closed.

CONCLUSION

In this paper we have introduced pgc*-closed sets in topological spaces and studied some of its basic properties. Also, we have studied the relationship between pgc*-closed sets with some generalized sets in topological spaces.

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