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Generalized Model for the Spread of Carrier-Dependent Infectious Diseases with Logistic Growth

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ABSTRACT

The present paper is an extension of the work of Ghosh *et al.*[15] by generalizing the birth and death rates. It is assumed that the density of carrier population increases with the increase in the cumulative density of discharges by the human population in to the environment. The growth of carrier population obeys simple logistic law. The generalization of the model is done for the following two cases: (i) the rate of cumulative environmental discharges is constant and (ii) the rate of cumulative environmental discharges is a function of total population density.

Keywords: Epidemic model, infectious diseases, carriers, environmental discharge, logistic growth.

1. INTRODUCTION

Infectious diseases in the environment are spread by direct contact between susceptible and infective. Some of these infectious diseases in the environment are transmitted to the human population by insects or other carriers (vectors). The infectious diseases spread by carriers in the environment include flies, ticks, mites and snails[2,3,4,7]. For example, air-born carriers or bacteria spread diseases such as tuberculosis and measles; while water-born carriers or bacteria are responsible for the spread of dysentery, diarrhoea, etc.[9, 20]. The modelling and analysis of infectious diseases have been done by many workers, see for example[1, 6, 11, 12, 17, 21]. In particular Hethcote[8] discussed an epidemic model in which carrier population is assumed to be constant. But in general the size of the carrier population varies and depends on the natural conditions of the environment as well as on various human related factors. Various kinds of household and other wastes, discharged into the environment is residential areas of population, provide very conducive environment for the population growth of some of these carriers. This enhances the chance of carrying more bacteria from infective to the susceptible leading to fast spread of carrier dependent infectious diseases. Ghosh *et al.*[14] studied the spread of carrier dependent infectious diseases with environmental effects using variable carrier population. The density of carrier population further increases as the human population density increases. Gao and Hethcote[13] analyzed an infectious diseases model with logistic population growth. Zhou and Hethcote[10] have studied few models of infectious diseases using various kinds of demographics. The present paper is the generalization of Ghosh *et al.*[15], in which they have studied the effect of variable carrier population caused by environmental discharges on the spread of infectious diseases.

2. GENERALIZED MODEL

Let us consider an SIS model in which the growth of human population is logistic. The disease is assumed to spread by infective as well as by carriers (vectors) in the environment. The total population density $N(t)$ is divided into two classes; susceptible $S(t)$ and infective $I(t)$. It is assumed that all susceptible living in the habitat are affected by a carrier population of density $C(t)$, which grows logistically with given intrinsic growth rate and carrying capacity. The growth rate of its density is further assumed to increase with the increase in the cumulative density of discharges by the human population into the environment. The birth as well as death rate are density dependent in such a manner that the birth rate decreases and death rate increases as the population density increases towards its carrying capacity[13].

The density dependent population growth is given by the logistic equation

$$\frac{dN}{dt} = r\left[1 - \frac{N}{K}\right]N$$

where $N(t)$ is the population size at any time t, r is the positive rate constant and K is the carrying capacity of the environment[5,13].

The generalization of the model framed by Ghosh *et al.*[15] is done by using the generalized birth and death rates represented by $\left[b - ar\left(\frac{N}{K}\right)^n\right]$ and $\left[d + (1-a)r\left(\frac{N}{K}\right)^n\right]$, respectively for $0 < a < 1$ and the parameter, $n \geq 1$. This generalization of birth and death rates have been already used by Sing *et al.*[18] in which they have generalized the model of Ghosh *et al.*[16].

When $a = 1$ the model could be called a simple generalized logistic birth model as all of its restricted growth is due to a decreasing birth and the death rate is constant. Similarly, when $a = 0$, it could be called a generalized logistic death model as all of the restricted growth is due to an increasing death rate and birth rate constant.

The generalized mathematical model can be represented by the following set of equations

$$\frac{dS}{dt} - \left[b - ar\left(\frac{N}{K}\right)^n\right]N - \left[d + (1-a)r\left(\frac{N}{K}\right)^n\right] - \beta SI - \lambda SC + \nu I$$

$$\frac{dI}{dt} = \beta SI + \lambda SC - \left[\nu + \alpha + d + (1-a)r\left(\frac{N}{K}\right)^n\right]I$$

$$\frac{dN}{dt} = r\left[1 - \left(\frac{N}{K}\right)^n\right]N - \alpha I$$

$$\frac{dC}{dt} = sC\left(1 - \frac{C}{L}\right) - \delta C + s_1 EC$$

$$\frac{dE}{dt} = Q(N) - \delta_0 E \qquad (1)$$

$S + I = N, \quad s > \delta, \quad 0 \le \alpha \le 1, \quad n \ge 1$

with initial conditions:

$S(0) > 0, \quad I(0) > 0, \quad N(0) > 0, \quad C(0) > 0 \ and \ E(0) > 0$

Here $E(t)$ is the cumulative density of environmental discharges conducive to the growth of carrier population; b and d are natural birth and death rates; $r = b - b > 0$ is the growth rate constant; K is the carrying capacity of the human population density in the natural environment; β and λ are the transmission coefficients due to infective and carrier population respectively; α is disease related death rate constant and ν is the recovery rate constant i.e. the rate at which individual recovers and moves to the susceptible class again from the infective class. The constant L is the carrying capacity of the carrier population in the natural environment; s is its intrinsic growth rate; δ is the death rate of carriers due to control measures, where $s > \delta$; s_1 the per capita growth rate coefficient of the carrier population due to the cumulative environmental discharge rate $Q(N)$, which is human population density dependent (an increasing function of N) and δ_0 is the depletion rate coefficient of the environmental discharges. In writing the model (1), we use the term transmission coefficient in the sense as used by Anderson and May[19], which means that new cases of disease occurs at the rates βXY and λXC due to interaction of susceptible with infective and carriers respectively. It is easy to note that the above model is well-posed in the region of attraction A_1 given by

$$A_1 = \{(Y, N, C, E): 0 \le Y \le N \le K, 0 \le C \le L, 0 \le E \le \tfrac{Q(K)}{\delta_0}\}.$$

The model (1) is analyzed for the following two cases:

i. The rate of cumulative environmental discharge Q is a constant ,and
ii. The rate of cumulative environmental discharge Q is a function of the population density.

3. CASE I: when Q is constant Q_a

Since S + I =N, it is sufficient to consider the following equivalent system of (1)

$$\frac{dI}{dt} = \beta\ SI + \alpha\ SC - [\nu + \alpha + d + (1-\alpha)\ r(N/K)^{\mathsf{T}} n]I$$

$$\frac{dN}{dt} = r\left[1 - \left(\frac{N}{K}\right)^n\right]N - \alpha I$$

$$\frac{dC}{dt} = sC\left(1 - \frac{C}{L}\right) - \delta C + s_1 EC$$

$$\frac{dE}{dt} = Q_a - \delta_0 E \qquad (2)$$

From the last two equations of system (2) we note that

$$\lim_{t\to\infty} \sup E(t) = \frac{Q_a}{\delta_0} \text{ and } \lim_{t\to\infty} \sup C(t) = \frac{L}{s}\left(s - \delta + s_1 \frac{Q_a}{\delta_0}\right) = C_m > 0$$

In order to study the behavior of the system (2) it is reasonable to consider the following equations of system (2):

$$\frac{dI}{dt} = \beta I(N-I) + \lambda C_m(N-I) - \left\{\nu + \alpha + d + (1-\alpha) r\left(\frac{N}{K}\right)^n\right\} I$$

$$\frac{dN}{dt} = r\left[1 - \left(\frac{N}{K}\right)^n\right]N - \alpha I \qquad (3)$$

where C_m increases as the discharge rate Q_a increases.

3.1. Stability Analysis: Let's consider the system (3) as:

$$F_1 = \beta I(N-I) + \lambda C_m(N-I) - \left\{\nu + \alpha + d + (1-\alpha) r\left(\frac{N}{K}\right)^n\right\} I$$

$F_2 = r\left[1-\left(\frac{N}{K}\right)^n\right]N - \alpha I$

The result of the equilibrium analysis is stated in the following theorem.

3.1.1. Theorem: There exists following two equilibrium points of system (3)
(i) $E_1(0,0)$ and (ii) $E_2(\hat{I},\hat{N})$,which exists if $v+\alpha+d > \left\{\frac{(\alpha-r)}{r}\right\}\lambda C_m$.

Proof: The existence of E_1 (0, 0) is obvious. The existence of E_2 is shown as fallows. Setting the derivatives of (3) equal to zero, we get

$$\beta I^2 - \beta IN - \lambda C_m N + \lambda C_m I + \left(v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right)I = 0 \quad (4)$$

$$r\left[1-\left(\frac{N}{K}\right)^n\right]N - \alpha I = 0 \quad (5)$$

from equation (5) we have

$$I = \frac{r}{\alpha}\left[1-\left(\frac{N}{K}\right)^n\right]N \quad (6)$$

also equation (4) implies

$$\beta I^{\prime}2 - [\,\beta N - \lambda C_i m + (v+\alpha+d-(1-a)\ r(N/K)^{\prime}n\,)I - \lambda C_i m\,N = 0 \quad (7)$$

Clearly in $N-I$ plane (6) is an ellipse passing through (0, 0) and (K, 0) with vertex given by

$$\left(\frac{k}{2};\frac{rK}{4\alpha}\right)$$

The variation matrix of system (3) is given by

$$M = \begin{bmatrix} m_{11} & m_{12} \\ -\alpha & r\left[1-(n+1)\left(\frac{N}{K}\right)^n\right] \end{bmatrix}$$

Where,

$$m_{11} = \beta N - 2\beta I - \lambda C_m - \left(v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right)$$

$$m_{12} - \beta I + \lambda Cm - (1-a)\,rn\left(\frac{N}{K}\right)^{n-1}\left(\frac{1}{K}\right)$$

The variation matrix M_1 at $E_1(0,0)$ is give by

$$M_1 = \begin{bmatrix} -\lambda C_m - (v+\alpha+d) & \lambda C_m \\ -\alpha & r \end{bmatrix}$$

its characteristic equation is given by

$$[\{(-\lambda C_m - (v+\alpha+d)) - \psi\}(r-\psi) + \lambda\alpha C_m] = 0$$

or $\qquad \psi^2 + \psi x + \lambda\alpha C_m - rx = 0$

where $\qquad x = \lambda C_m + (v+\alpha+d)$

so the equilibrium is stable if

$$\lambda\alpha C_m > rx$$

i.e. if $\qquad \left(\frac{\alpha-r}{r}\right)\lambda C_m > v+\alpha+d$

clearly for $\alpha < r$ the system is unstable.

The variation matrix M_2 at $E_2(\hat{I},\hat{N})$ is given by

$$M_2 = \begin{bmatrix} m_{11} & m_{12} \\ -\alpha & r\left[1-(n+1)\left(\frac{\hat{N}}{K}\right)^n\right] \end{bmatrix}$$

its characteristic equation is given by

$$(m_{11}-\psi)\left\{r\left[1-(n+1)\left(\frac{\hat{N}}{K}\right)^n\right]-\psi\right\} + \alpha m_{12} = 0$$

where,

$$m_{11} - \beta\hat{N} - 2\beta\hat{I} - \lambda\hat{C}_m - \left(v+\alpha+d+(1-a)\,r\left(\frac{\hat{N}}{K}\right)^n\right)$$

$$m_{12} = \beta\hat{I} + \lambda\hat{C}_m - (1-a)\,rn\left(\frac{\hat{N}}{K}\right)^{n-1}\left(\frac{1}{K}\right)$$

$\Rightarrow \qquad (m_{11}-\psi)(x-\psi) + \alpha m_{12} = 0$

where, $\qquad x = r\left[1-(n+1)\left(\frac{\hat{N}}{K}\right)^n\right]$

$\Rightarrow \qquad \psi^2 + a_1\psi + a_0 - 0$

where,

$a_1 = 2\beta\hat{I} + \lambda\hat{C}_m - \beta\hat{N} + \left(v+\alpha+d+(1-a)\,r\left(\frac{\hat{N}}{K}\right)^n\right) - r\left[1-(n+1)\left(\frac{\hat{N}}{K}\right)^n\right]$

$a_0 = \left\{\beta\hat{N} - 2\beta\hat{I} - \lambda\hat{C}_m - \left(v+\alpha+d+(1-a)\,r\left(\frac{\hat{N}}{K}\right)^n\right)\right\}\left\{r\left[1-(n+1)\left(\frac{\hat{N}}{K}\right)^n\right]\right\} +$

$\alpha\left\{\beta\hat{I}+\lambda\hat{C}_m-(1-\text{a})\,rn\left(\frac{\hat{N}}{K}\right)^{n-1}\left(\frac{1}{K}\right)\right\}$

Hence by Routh-Hurwitz stability criteria, the above system is locally asymptotically stability if $a_1 > 0$ and $a_0 > 0$. Otherwise the system is unstable.

4. CASE II: When Q is variable

When the cumulative rate of environment discharges Q_a is a function of total population density, we have

In this system, we consider the following equations of system (1), where $S+I=N \Rightarrow S=N-I$

$$\frac{dI}{dt}=\beta I(N-I)+\lambda C(N-I)-\left[v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right]I$$

$$\frac{dN}{dt}=r\left[1-\left(\frac{N}{K}\right)^n\right]N-\alpha I$$

$$\frac{dC}{dt}=SC\left(1-\frac{C}{L}\right)-\delta C+s_1EC$$

$$\frac{dC}{dt}=Q_0+lN-\delta_0E \tag{8}$$

4.1. Stability Analysis: In order to find equilibrium points of system (8) we consider the following equations as

$$\beta I(N-I)+\lambda C(N-I)-\left[v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right]I=0 \tag{9}$$

$$r\left[1-\left(\frac{N}{K}\right)^n\right]N-\alpha I=0 \tag{10}$$

$$SC\left(1-\frac{C}{L}\right)-\delta C+S_1EC=0 \tag{11}$$

$$Q_0+lN-\delta_0E=0 \tag{12}$$

The result of the equilibrium analysis is stated in the following theorem.

4.1.1 Theorem: After solving the above equations we will get the following equilibrium points, namely:

(i) $\overline{E_1}\left(0,0,0,\frac{Q_0}{\delta_0}\right)$,

(ii) $\overline{E_2}\left(0,K,0,\frac{Q_{0+!K}}{\delta_0}\right)$,

(iii) $\overline{E_3}\left(0,0,\overline{C},\overline{E}\right)$,

where $\bar{C}=\frac{L}{S}\left(s-\delta+s_1\frac{Q_0}{\delta_0}\right)$ and $\bar{E}=\frac{Q_{0+lN}}{\delta_0}$, ,

(iv) $\overline{E_4}(\overline{I},\overline{N},\overline{C},\overline{E})$, which exists only

if $\left(\frac{\alpha-r}{r}\right)\lambda C_m < v+\alpha+d$

i.e. $\left(\frac{\alpha-r}{r}\right)\frac{\lambda L}{S}\left(s-\delta+s_1\frac{Q_0}{\delta_0}\right)<v+\alpha+d$

i.e. $v+\alpha+d>\frac{\lambda L}{S}\left(\frac{\alpha-r}{r}\right)\left(s-\delta+s_1\frac{Q_0}{\delta_0}\right)$

Proof: let's consider the equations of system (8) as follows:

$$F_1=\ \beta IN-\beta I^2+\lambda CN-\lambda CI-\left(v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right)I$$

$$F_2=r\left[\left(1-\left(\frac{N}{K}\right)^n\right)\right]N-\alpha I$$

$$F_3=\ SC\left(1-\frac{C}{L}\right)-\delta C+S_1EC$$

$$F_4=\ Q_0+lN-\delta_0E \tag{13}$$

The variation matrix *M* at *(I, N, B, E)* corresponding to system (13) is given by

$$M=\begin{bmatrix}a_{11} & a_{12} & \lambda(N-I) & 0\\ -\alpha & r\left[1-(n+1)\left(\frac{N}{K}\right)^n\right] & 0 & 0\\ 0 & 0 & a_{33} & s_1C\\ 0 & l & 0 & -\delta_0\end{bmatrix}$$

where

$$a_{11}=\beta N-2\beta I-\lambda C-\left(v+\alpha+d+(1-a)\,r\left(\frac{N}{K}\right)^n\right)$$

$$a_{12}=\ \beta I+\lambda C-(1-a)\frac{nr}{K}\left(\frac{N}{K}\right)^{n-1}I$$

$$a_{33}=s\left(1-\frac{2C}{L}\right)-\delta+s_1E$$

The variation matrix M_I at $\overline{E_1}(0,0,0,\frac{Q_0}{\delta_0})$ is given by

$$M_1 = \begin{bmatrix} -(v+\alpha+d) & 0 & 0 & 0 \\ -\alpha & r & 0 & 0 \\ 0 & 0 & s-\delta+s_1\frac{Q_0}{\delta_0} & 0 \\ 0 & l & 0 & -\delta_0 \end{bmatrix}$$

its characteristic equation is given by:

$$[-(v+\alpha+d)-\psi][r-\psi]\left[\left(s-\delta+s_1\frac{Q_0}{\delta_0}\right)-\psi\right][-\delta_0-\psi]=0$$

which has at least one Eigen value positive. Hence the equilibrium point $\overline{E_1}(0,0,0,\frac{Q_0}{\delta_0})$ is unstable.

The variation matrix M_2 at $\overline{E_2}(0,K,0,\frac{Q_{0+Kl}}{\delta_0})$ is given by

$$M_2 = \begin{bmatrix} \beta K-[v+\alpha+d+(1-a)\,r] & 0 & \lambda K & 0 \\ -\alpha & -rn & 0 & 0 \\ 0 & 0 & s-\delta+s_1\frac{Q_{0+KL}}{\delta_0} & 0 \\ 0 & l & 0 & -\delta_0 \end{bmatrix}$$

its characteristic equation is given by:

$$\{[\beta K-[v+\alpha+d+(1-a)\,r]]-\psi\}\{-rn-\psi\}\left\{\left[s-\delta+s_1\frac{Q_{0+KL}}{\delta_0}\right]-\psi\right\}\{-\delta_0-\psi\}=0$$

which gives

$$(x-\psi)(rn+\psi)(y-\psi)(\delta_0+\psi)=0$$

or

$$\psi^4+a_3\psi^3+a_2\psi^2+a_1\psi^3+a_0=0$$

where,

$$a_3 = y-x+rn-\delta_0, \quad a_2 = xy-(rn+\delta_0)(x+y)rn\delta_0$$

$$a_1 = xy(rn+\delta_0)-\delta_0 rn(x+y), \quad a_0 = rn\delta_0 xy$$

and, $x=\beta K-(v+\alpha+d+(1-\alpha)\,), \quad y = s-\delta+s_1\frac{Q_{0+KL}}{\delta_0}$

Hence by Routh-Hurwitz stability criteria $\overline{E_2}(0,K,0,\frac{Q_{0+Kl}}{\delta_0})$ is stable if

$$a_3>0, \quad \begin{vmatrix} a_3 & a_1 \\ 1 & a_2 \end{vmatrix}>0, \quad \begin{vmatrix} a_3 & a_1 & 0 \\ 1 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{vmatrix}>0$$

otherwise the system is unstable.

The variation matrix M_3 at $\overline{E_3}(0,0,\bar{C},\bar{E})$ is given by

$$M_3 = \begin{bmatrix} \lambda\bar{C}-(v+\alpha+d) & \lambda\bar{C} & 0 & 0 \\ -\alpha & r & 0 & 0 \\ 0 & 0 & s\left(1-\frac{2\bar{C}}{L}\right)-\delta+s_1\bar{E} & s_1\bar{C} \\ 0 & l & 0 & -\delta_0 \end{bmatrix}$$

its characteristic equation is given by

$$\{[\lambda\bar{C}-(v+\alpha+d)]-\psi\}\{r-\psi\}\left\{\left[\ s\left(1-\frac{2C}{L}\right)-\delta+s_1\bar{E}\right]-\psi\right\}\{-\delta_0-\psi\}+$$
$$\alpha\lambda\bar{C}\left\{\left[\left[s\left(1-\frac{2\bar{C}}{L}\right)-\delta+s_1\bar{E}\right]-\psi\right]\right\}\{-\delta_0-\psi\}=0$$

or

$$(y-\psi)(-\delta_0-\psi)[(x-\psi)(r-\psi)+\lambda r\bar{C}]=0$$

which has at least one eigen value positive. Hence the system is unstable. i.e. $\psi=\delta_0$. .

Thus $\overline{E_3}(0,0,\bar{C},\bar{E})$ is unstable.

The variation matrix M_4 at $\bar{E}_4(\hat{I},\widehat{N},\hat{C},\hat{E})$ is given by

$$M_4 = \begin{bmatrix} a_{11} & a_{12} & \lambda(\widehat{N}-\hat{I}) & 0 \\ -\alpha & r\left[1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right] & 0 & 0 \\ 0 & 0 & a_{33} & s_1\hat{C} \\ 0 & l & 0 & -\delta_0 \end{bmatrix}$$

its characteristic equation is given by:

$$(a_{11}-\psi)\left\{r\left[1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right]-\psi\right\}\{(a_{33}-\psi)(-\delta_0-\psi)\}+\alpha a_{12}(a_{33}-\psi)(-\delta_0-\psi)=0$$

or $\quad \psi^4+a_3\psi^3+a_2\psi^2+a_1\psi^3+a_0=0$

where

$$a_3=\left[\delta_0-\left\{s\left(1-\frac{2\hat{C}}{L}\right)-\delta+s_1\hat{E}\right\}\right]-\left[\left\{\beta\widehat{N}-2\beta\hat{I}-\lambda\hat{C}-\left(v+\alpha+d+(1-a)\,r\left(\frac{\widehat{N}}{K}\right)^n\right)\right\}+r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}\right]$$

$$a_2=\left\{\beta\widehat{N}-2\beta\hat{I}-\lambda\hat{C}-\left(v+\alpha+d+(1-a)\,r\left(\frac{\widehat{N}}{K}\right)^n\right)\right\}\left[\left\{s\left(1-\frac{2\hat{C}}{L}\right)-\delta+s_1\hat{E}\right\}-\delta_0+r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}\right]+\left\{s\left(1-\frac{2\hat{C}}{L}\right)-\delta+s_1\hat{E}\right\}\left[r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}-\delta_0\right]-\delta_0 r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}$$

$$a_1=\left\{s\left(1-\frac{2\hat{C}}{L}\right)-\delta+s_1\hat{E}\right\}\left[\left\{\beta\widehat{N}-2\beta\hat{I}-\lambda\hat{C}-\left(v+\alpha+d+(1-a)\,r\left(\frac{\widehat{N}}{K}\right)^n\right)\right\}\left(\delta_0-r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}\right)+\delta_0 r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}-\alpha\left\{\beta\hat{I}+\lambda\hat{C}-(1-a)\frac{nr}{K}\left(\frac{N}{K}\right)^{n-1}\hat{I}\right\}\right]+\delta_0\left[r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}\left\{\beta\widehat{N}-2\beta\hat{I}-\lambda\hat{C}-\left(v+\alpha+d+(1-a)\,r\left(\frac{\widehat{N}}{K}\right)^n\right)\right\}+\alpha\left\{\beta\hat{I}+\lambda\hat{C}-(1-a)\frac{nr}{K}\left(\frac{N}{K}\right)^{n-1}\hat{I}\right\}\right]$$

$$a_0=\delta_0\left\{s\left(1-\frac{2\hat{C}}{L}\right)-\delta+s_1\hat{E}\right\}\left[r\left\{1-(n+1)\left(\frac{\widehat{N}}{K}\right)^n\right\}\left\{2\beta\hat{I}+\lambda\hat{C}-\beta\widehat{N}+\left(v+\alpha+d+(1-a)\,r\left(\frac{\widehat{N}}{K}\right)^n\right)\right\}+\alpha\left\{(1-a)\frac{nr}{K}\left(\frac{N}{K}\right)^{n-1}\hat{I}-\beta\hat{I}-\lambda\hat{C}\right\}\right]$$

Hence by Routh-Hurwitz stability criteria, the above system is locally asymptotically stable if the fallowing conditions are satisfied:

$a_0>0,\ a_1>0,\ a_2>0,\ a_3>0,\ a_0\,a_1-,$ $a_3>0,\ a_0a_1a_2>a_2{}^2+a_0{}^2a_3,$ otherwise the system is unstable.

5. CONCLUSION

In this paper an SIS model for carrier-dependent infectious diseases caused by direct contact of susceptible with infective as well as by carriers is proposed and analysed. We have generalized the model for both the cases: first, the cumulative rate of environmental discharges is constant and the second is cumulative rate of environmental discharge is function of total population density. The equilibrium analysis is presented for the generalised model for both cases and it is seen that the local stability of the nontrivial equilibria in both the cases is guaranteed only under certain conditions.

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