Journal of Computer and Mathematical Sciences, *Vol.10(12),1654-1664 December 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Total Edge Fibonacci Irregular Labeling for Fan, Wheel and Umbrella Graph

S. Amutha* and M. Uma Devi[1]

Assistant Professors, Department of Mathematics,
Government Arts & Science College, Sivakasi – 626 124, INDIA.
email:*amudha.veerasami@gmail.com, [1]moorthy.uma17@gmail.com.

(Received on: December 5, 2019)

ABSTRACT

A total edge Fibonacci irregular labeling $f : V(G) \cup E(G) \to \{1, 2, ..., k\}$ of a graph G = (V, E) is a labeling of vertices and edges of G in such a way that for any different edges xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Fibonacci numbers. The total edge Fibonacci irregularity strength, tefs(G), is defined as the minimum k for which G has a total edge Fibonacci irregular labeling. If a graph has a total edge Fibonacci irregular labeling, then it is called a total edge Fibonacci irregular graph.

In this paper, we determined the exact value of the total edge Fibonacci irregularity strength of Fan graph F_n , double Fan graph DF_n, umbrella graph U(n, m) and Wheel graph W_n.

Keywords: Total Edge Fibonacci irregular labeling, Fan graph, Umbrella graph, Wheel graph.

1. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol $|V(G)|$ and $|E(G)|$ denotes the number of vertices and edges in a graph G. A graph labeling is an assignment of integers to the vertices or edges or both. A total vertex irregular labeling on a graph G with p vertices and q edges is an assignment of integer labels to both vertices and edges so that the weights calculated at vertices are distinct. The weight of a vertex v in G is defined as the sum of the label of v and the labels of all the edges incident with v, that is $wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$.

The total vertex irregularity strength of G, denoted by tvs(G), is the minimum value of the largest label over all such irregular assignments.

For a graph G = (V, E), define a labeling $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., k\}$ to be an edge irregular total k-labeling of the graph G if for every two different edges xy and $x'y'$ of G the edge weights $wt(xy) \neq wt(x'y')$. The total edge irregularity strength, tes(G), is defined as the minimum k for which G has an edge irregular total k-labeling.

The notion of a total vertex irregular labeling and total edge irregular labeling are introduced by Baca *et al*.[3]

Myself * and K.M. Kathiresan introduced the notion of total edge Fibonacci irregular labeling and we proved that graphs like P_n, C_n and book with (3 and 4 sides) are total edge Fibonacci irregular graphs.

In[4], the authors proved the star graph, bistar and subdivision of bistar graphs are total edge Fibonacci irregular graphs.

In this paper, we prove Fan graph F_n, double Fan graph DF_n, umbrella graph U(n, m) and Wheel graph W_n are total edge Fibonacci irregular graphs.

Definition 1.1 A total edge Fibonacci irregular labeling $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., k\}$ of a graph G = (V, E) is a labeling of vertices and edges of G in such a way that for any different edges xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Fibonacci numbers, where the Fibonacci series is $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$,…. The total edge Fibonacci irregularity strength, tefs(G), is defined as the minimum k for which G has a total edge Fibonacci irregular labeling. If a graph has a total edge Fibonacci irregular labeling, then it is called a total edge Fibonacci irregular graph.

Note that if f is a total edge Fibonacci irregular labeling of G = (V, E) with $|V(G)| = p$ and $|E(G)| = q$ then $F_4(=3) \leq wt(xy) \leq F_{q+3}$ which implies that $tefs \geq \left\lceil \frac{F_{q+3}}{3} \right\rceil$

Definition 1.2 A Fan graph F_n is defined as the graph $K_1 + P_n$, where K_1 is the empty graph on one vertex and P_n , $n \geq 2$ is the path graph on n vertices.

Definition 1.3 The double Fan graph DF_n consists of two fan graph that have a common path. In other words, $DF_n = P_n + \overline{K_2}$, $n \geq 2$

Definition 1.4 The Wheel graph W_n, $n \geq 3$ is n+1 vertices graph obtained by connecting all the vertices $\{v_1, v_2, ..., v_n\}$ of C_n to the center vertex u.

Definition 1.5[6] An umbrella graph U(n, m) is the graph obtained by identifying the end vertex of path P_m with a central vertex of a Fan graph F_n

Definition 1.6 The graph $F(n, m_1, m_2, ..., m_{n-1})$ is obtained by adjoining m_1 pendent edges with centre vertex u and by adjoining $m_2, ..., m_{n-1}$ pendent edges with the vertices of path $v_1, v_2, ..., v_{n-2}$ respectively in Fan graph $F_n = K_1 + P_n$, $n \geq 2$.

Example 1.1

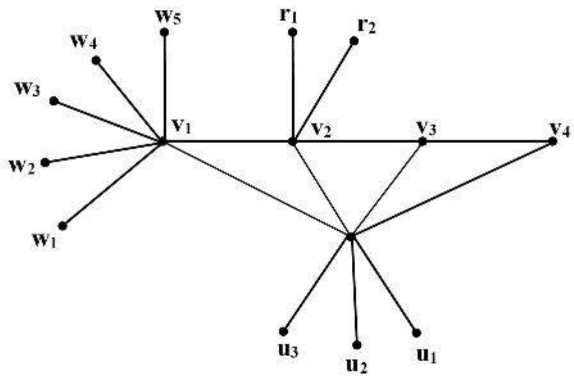

Figure 1.1: $F(4, 3, 5, 2)$

2. TOTAL EDGE FIBONACCI IRREGULAR LABELING

Theorem 2.1 The Fan graph F_n has a total edge Fibonacci irregular labeling for every $n \geq 3$ and $tefs(F_n) = F_{q+1} + 1$

Proof: Let G = F_n

Here $V(G) = \{v_1, v_2, ..., v_n, u\}$ and

$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{uv_i / 1 \leq i \leq n\}$

Therefore, $|V(G)| = n+1$, $|E(G)| = 2n - 1 = q$

Define $f : V(G) \cup E(G) \rightarrow \{1, 2, ..., F_{q+1} + 1\}$ as follows.

$f(u) = 1$

$f(v_i) = 1$, $1 \leq i \leq n-2$

$f(v_{n-1}) = F_q - 2$

$f(v_n) = F_{q+1} + 1$

$f(uv_i) = F_{3+i} - 2, 1 \leq i \leq n-2$

$f(uv_{n-1}) = F_{q-1} + 1$

$f(uv_n) = F_q - 2$

$f(v_i v_{i+1}) = F_{n+1+i} - 2, 1 \leq i \leq n-3$

$f(v_{n-2} v_{n-1}) = 1$

$f(v_{n-1} v_n) = F_{q+1} + 1$

By this labeling

$wt(uv_i) = F_{3+i}, \ 1 \le i \le n - 2$

Therefore, $wt(uv_i)$ for $1 \le i \le n - 2$ are $F_4, F_5, ..., F_{n+1}$

$wt(v_i v_{i+1}) = F_{n+1+i}, \ 1 \le i \le n - 3$

Therefore, $wt(v_i v_{i+1})$ for $1 \le i \le n - 3$ are $F_{n+2}, F_{n+3}, ..., F_{2n-2} (= F_{q-1})$

$wt(v_{n-2} v_{n-1}) = F_q$

$wt(uv_{n-1}) = F_{q+1}$

$wt(uv_n) = F_{q+2}$

$wt(v_{n-1} v_n) = F_{q+3}$

Thus the weights of all the edges in G = F_n for $n \ge 3$ are $F_4, F_5, ..., F_{q+3}$ respectively and

$tefs(F_n) = F_{q+1} + 1$

Example 2.1.

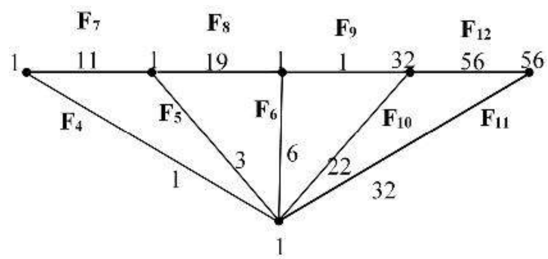

Figure 2.1: Total edge Fibonacci irregular labeling of F_5

Theorem 2.2 The Wheel graph W_n has a total edge Fibonacci irregular labeling for every $n \ge 3$ and $tefs(W_n) = F_{q+2} + 4$

Proof: Let G = W_n

Here $V(G) = \{v_1, v_2, ..., v_n, u\}$ and

$E(G) = \{v_i v_{i+1} / 1 \le i \le n - 1\} \cup \{v_n v_1\} \cup \{uv_i / 1 \le i \le n\}$

Therefore, $|V(G)| = n + 1, \ |E(G)| = 2n = q$

Define $f : V(G) \cup E(G) \to \{1, 2, ..., F_{q+2} + 4\}$ as follows.

$f(u) = 1$

$f(v_i) = 1 \ , \ 1 \le i \le n - 2$

$f(v_{n-1}) = F_q - 2$

$f(v_n) = F_{q-1} - 2$

$f(uv_i) = F_{3+i} - 2, 1 \le i \le n - 2$

$f(uv_{n-1}) = F_{q+1} + 1$

$f(uv_{n}) = F_{q} + 1$

$f(v_i v_{i+1}) = F_{n+1+i} - 2, 1 \le i \le n-3$

$f(v_{n-2} v_{n-1}) = 1$

$f(v_{n-1} v_{n}) = F_{q+2} + 4$

$f(v_n v_1) = 1$

By this labeling

$wt(uv_i) = F_{3+i}, \ 1 \le i \le n-2$

Therefore, $wt(uv_i)$ for $1 \le i \le n-2$ are $F_4, F_5, ..., F_{n+1}$

$wt(v_i v_{i+1}) = F_{n+1+i}, \ 1 \le i \le n-3$

Therefore, $wt(v_i v_{i+1})$ for $1 \le i \le n-3$ are $F_{n+2}, F_{n+3}, ..., F_{2n-2} (= F_{q-2})$

$wt(v_n v_1) = F_{q-1}$

$wt(v_{n-2} v_{n-1}) = F_q$

$wt(uv_n) = F_{q+1}$

$wt(uv_{n-1}) = F_{q+2}$

$wt(v_{n-1} v_n) = F_{q+3}$

Thus the weights of all the edges in $G = W_n$ for $n \ge 3$ are $F_4, F_5, ..., F_{q+3}$ respectively and also

$tefs(W_n) = F_{q+2} + 4$

Example 2.2.

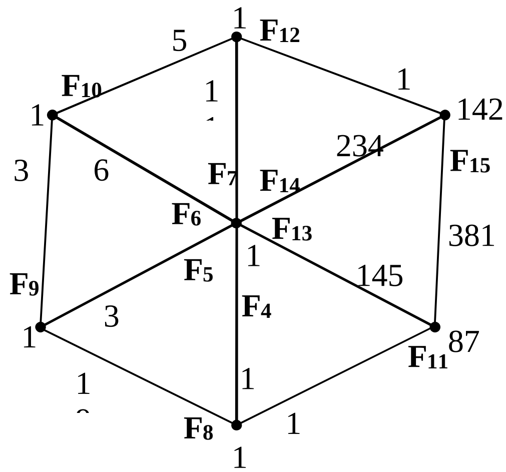

Figure 2.2: Total edge Fibonacci irregular labeling of W_6

Theorem 2.3 The Double Fan graph DF_n has a total edge Fibonacci irregular labeling for every $n \geq 3$ and $tefs(DF_n) = F_{q+2} - F_{q-2} + 4$

Proof: Let G = DF_n

Here $V(G) = \{v_1, v_2, ..., v_n, u, w\}$ and

$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{uv_i / 1 \leq i \leq n\} \cup \{wv_i / 1 \leq i \leq n\}$

Therefore, $|V(G)| = n + 2$, $|E(G)| = 3n - 1 = q$

Define $f : V(G) \cup E(G) \to \{1, 2, ..., F_{q+2} - F_{q-2} + 4\}$ as follows.

$f(u) = 1$

$f(w) = 1$

$f(v_i) = 1$, $1 \leq i \leq n-2$

$f(v_{n-1}) = F_{q-2} - 2$

$f(v_n) = F_{q+1} - 2$

$f(uv_i) = F_{3+i} - 2, 1 \leq i \leq n-2$

$f(uv_{n-1}) = F_{q-1} + 1$

$f(uv_n) = F_q + 1$

$f(wv_i) = F_{n+1+i} - 2, 1 \leq i \leq n-2$

$f(wv_{n-1}) = F_{q-3} + 1$

$f(wv_n) = 1$

$f(v_i v_{i+1}) = F_{2n-1+i} - 2, 1 \leq i \leq n-3$

$f(v_{n-2} v_{n-1}) = 1$

$f(v_{n-1} v_n) = F_{q+2} - F_{q-2} + 4$

By this labeling

$wt(uv_i) = F_{3+i}$, $1 \leq i \leq n-2$

Therefore, $wt(uv_i)$ for $1 \leq i \leq n-2$ are $F_4, F_5, ..., F_{n+1}$

$wt(wv_i) = F_{n+1+i}$, $1 \leq i \leq n-2$

Therefore, $wt(wv_i)$ for $1 \leq i \leq n-2$ are $F_{n+2}, F_{n+3}, ..., F_{2n-1}$

$wt(v_i v_{i+1}) = F_{2n-1+i}$, $1 \leq i \leq n-3$

Therefore, $wt(v_i v_{i+1})$ for $1 \leq i \leq n-3$ are $F_{2n}, F_{2n+1}, ..., F_{3n-4}$ $(= F_{q-3})$

$wt(v_{n-2} v_{n-1}) = F_{q-2}$

$wt(wv_{n-1}) = F_{q-1}$

$wt(uv_{n-1}) = F_q$

$wt(wv_n) = F_{q+1}$

$wt(uv_n) = F_{q+2}$

$wt(v_{n-1}v_n) = F_{q+3}$

Thus the weights of all the edges in G = DF_n for $n \geq 3$ are $F_4, F_5, ..., F_{q+3}$ respectively and

$tefs(DF_n) = F_{q+2} - F_{q-2} + 4$

Example 2.3.

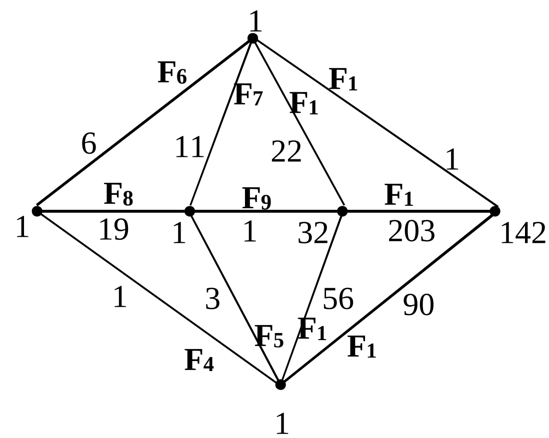

Figure 2.3: Total edge Fibonacci irregular labeling of DF4

Theorem 2.4 An umbrella graph U(n, m) has a total edge Fibonacci irregular labeling for every n,m ≥ 2and $tefs(U(n,m)) = \left\lceil \frac{F_{q+3}}{3} \right\rceil$

Proof: Let G = U(n, m)

Here $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, ..., u_m\}$ and

$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i u_1 / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq m-1\}$

Therefore, $|V(G)| = n + m$, $|E(G)| = 2n + m - 2 = q$

Define $f : V(G) \cup E(G) \to \left\{ 1, 2, ..., \left\lceil \frac{F_{q+3}}{3} \right\rceil \right\}$ as follows.

$f(v_i) = 1 \ , \ \ 1 \le i \le n$

$f(u_i) = 1 \ , \ \ 1 \le i \le m-2$

$f(u_{m-1}) = F_{q+3} - 2\left\lceil \frac{F_{q+3}}{3} \right\rceil$

$f(u_m) = \left\lceil \frac{F_{q+3}}{3} \right\rceil$

$f(v_i v_{i+1}) = F_{3+i} - 2 , 1 \le i \le n-1$

$f(v_i u_1) = F_{n+2+i} - 2 , 1 \le i \le n$

$f(u_i u_{i+1}) = F_{2n+2+i} - 2 , 1 \le i \le m-3$

$f(u_{m-2} u_{m-1}) = 2 \left\lceil \frac{F_{q+3}}{3} \right\rceil - F_{q+1} \ -1$

$f(u_{m-1} u_m \ ) = \left\lceil \frac{F_{q+3}}{3} \right\rceil$

By this labeling

$wt(v_i v_{i+1}) = F_{3+i} , \ \ 1 \le i \le n\text{-}1$

Therefore, $wt(v_i v_{i+1})$ for $1 \le i \le n\text{-}1$ are $F_4 , F_5 , ..., F_{n+2}$

$wt(v_i u_1) = F_{n+2+i} , \ 1 \le i \le n$

Therefore, $wt(v_i u_1)$ for $1 \le i \le n$ are $F_{n+3} , F_{n+4} , ..., F_{2n+2}$

$wt(u_i u_{i+1}) = F_{2n+2+i} , \ 1 \le i \le m-3$

Therefore, $wt(u_i u_{i+1})$ for $1 \le i \le m-3$ are $F_{2n+3} , F_{2n+4} , ..., F_{2n+m-1} \left(= F_{q+1}\right)$

$wt(u_{m-2} u_{m-1}) = F_{q+2}$

$wt(u_{m-1} u_m \ ) = F_{q+3}$

Thus the weights of all the edges in G = U(n, m) for $n,m \ge 2$ are $F_4 , F_5 , ..., F_{q+3}$ respectively and $tefs(U(n, m)) = \left\lceil \frac{F_{q+3}}{3} \right\rceil$

Example 2.4.

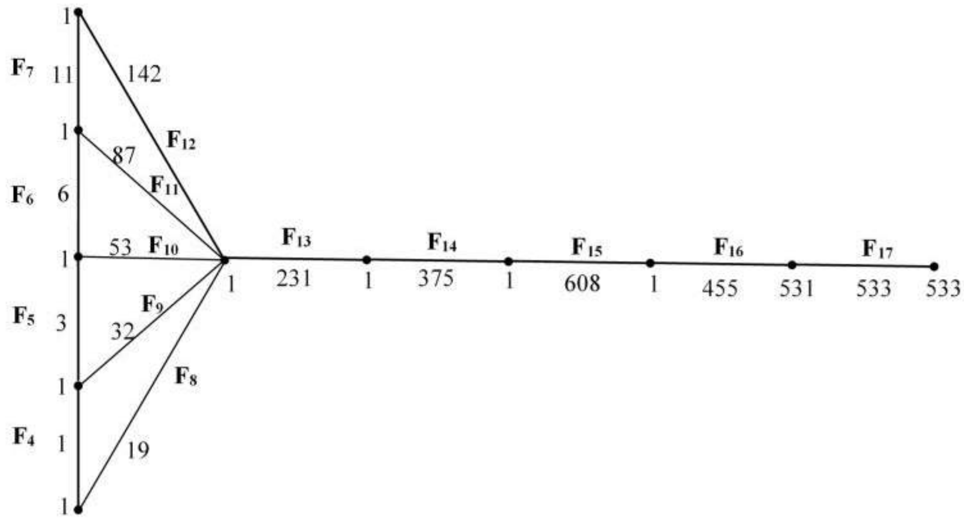

Figure 2.4: Total edge Fibonacci irregular labeling of U(5,6)

Theorem 2.5 The graph $F(n, m_1, m_2, ..., m_{n-1})$ has a total edge Fibonacci irregular labeling and $tefs(F(n, m_1, m_2, ..., m_{n-1})) = F_{q+1} + 1$

Proof: Let G = $F(n, m_1, m_2, ..., m_{n-1})$

Here $V(G) = \{v_1, v_2, ..., v_n, u, u_1, u_2, ..., u_{m_1}, w_1, w_2, ..., w_{m_2}, ..., y_1, y_2, ..., y_{m_{n-1}}\}$ and

$E(G) = \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{uv_i / 1 \le i \le n\} \cup \{uu_i / 1 \le i \le m_1\} \cup \{v_1 w_i / 1 \le i \le m_2\}$
$\cup ... \cup \{v_{n-2} y_i / 1 \le i \le m_{n-1}\}$

Therefore,

$|V(G)| = m_1 + m_2 + ... + m_{n-1} + n + 1,$ $|E(G)| = m_1 + m_2 + ... + m_{n-1} + 2n - 1 = q$

Define $f : V(G) \cup E(G) \to \{1, 2, ..., F_{q+1} + 1\}$ as follows.

$f(u) = 1$

$f(v_i) = 1$, $1 \le i \le n-2$

$f(v_{n-1}) = F_q - 2$

$f(v_n) = F_{q+1} + 1$

$f(u_i) = 1$, $1 \le i \le m_1$

$f(w_i) = 1$, $1 \le i \le m_2$

....

$f(y_i) = 1$, $1 \le i \le m_{n-1}$

$f(uv_i) = F_{3+i} - 2, 1 \le i \le n-2$
$f(uv_{n-1}) = F_{q-1} + 1$
$f(uv_n) = F_q - 2$
$f(v_i v_{i+1}) = F_{n+1+i} - 2, 1 \le i \le n-3$
$f(v_{n-2} v_{n-1}) = 1$
$f(v_{n-1} v_n) = F_{q+1} + 1$
$f(uu_i) = F_{2n-2+i} - 2 \ , \ 1 \le i \le m_1$
$f(v_1 w_i) = F_{2n-2+m_1+i} - 2 \ , \ 1 \le i \le m_2$
....
$f(v_{n-2} y_i) = F_{2n-2+m_1+m_2+...+m_{n-2}+i} - 2 \ , \ 1 \le i \le m_{n-1}$
By this labeling
$wt(uv_i) = F_{3+i}, \ 1 \le i \le n-2$
Therefore, $wt(uv_i)$ for $1 \le i \le n-2$ are $F_4, F_5, ..., F_{n+1}$
$wt(v_i v_{i+1}) = F_{n+1+i}, \ 1 \le i \le n-3$
Therefore, $wt(v_i v_{i+1})$ for $1 \le i \le n-3$ are $F_{n+2}, F_{n+3}, ..., F_{2n-2}$
$wt(uu_i) = F_{2n-2+i}, \ 1 \le i \le m_1$
Therefore, $wt(uu_i)$ for $1 \le i \le m_1$ are $F_{2n-1}, F_{2n}, ..., F_{2n-2+m_1}$
$wt(v_1 w_i) = F_{2n-2+m_1+i}, \ 1 \le i \le m_2$
Therefore, $wt(v_1 w_i)$ for $1 \le i \le m_2$ are $F_{2n-1+m_1}, F_{2n+m_1}, ..., F_{2n-2+m_1+m_2}$
……………………………………………………………
$wt(v_{n-2} y_i) = F_{2n-2+m_1+m_2+...+m_{n-2}+i}, \ 1 \le i \le m_{n-1}$
Therefore, $wt(v_{n-2} y_i)$ for $1 \le i \le m_{n-1}$ are
$F_{2n-1+m_1+m_2+...+m_{n-2}+1}, F_{2n+m_1+m_2+...+m_{n-2}+2}, ..., F_{2n-2+m_1+m_2+...+m_{n-2}+m_{n-1}} (= F_{q-1})$
$wt(v_{n-2} v_{n-1}) = F_q$

$wt(uv_{n-1}) = F_{q+1}$
$wt(uv_n) = F_{q+2}$
$wt(v_{n-1} v_n) = F_{q+3}$
Thus the weights of all the edges in G = $F(n, m_1, m_2, ..., m_{n-1})$ are $F_4, F_5, ..., F_{q+3}$ respectively and $tefs(F(n, m_1, m_2, ..., m_{n-1})) = F_{q+1} + 1$

Example 2.5.

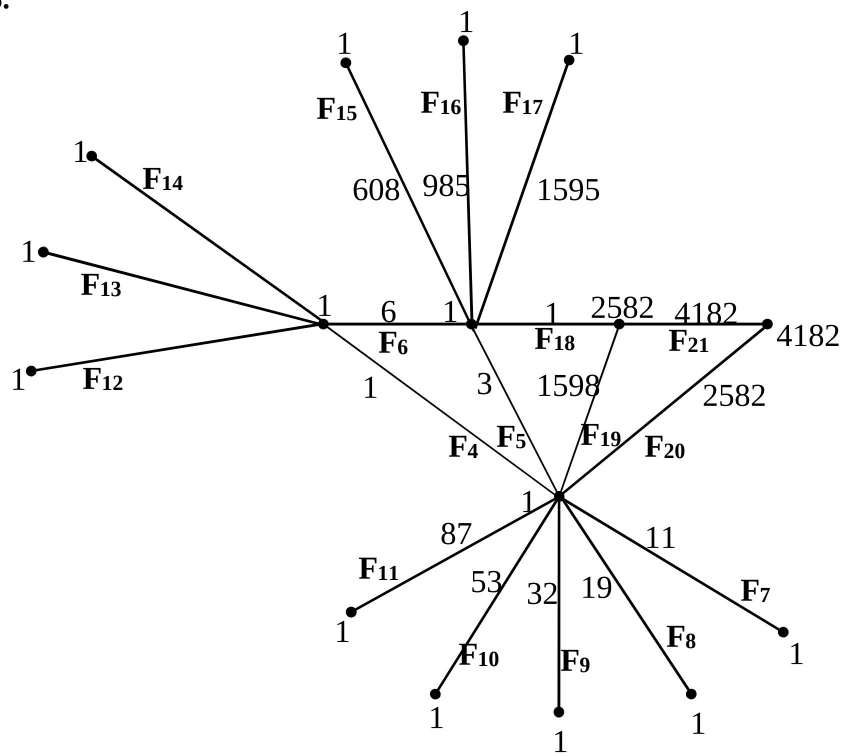

Figure 2.5: Total edge Fibonacci irregular labeling of F(4, 5, 3, 3)

REFERENCES

1. M.Baca, M.Stainslav Jendrol, Miller and Joseph Ryan, On irregular total labeling, *Discrete Math*.,307, 137-138 (2007).
2. J.A.Bondy, U.S.R. Murty, Graph Theory with Applications, MacMillan, Newyork, (1976).
3. J.A.Gallian, A dynamic survey of graph labeling, *Electron. J. Combin*. 21, #DS6 (2018).
4. F.Harary, Graph Theory, Addition-Wesley, Reading, Mass, (1972).
5. S.Karthikeyan, S.Navaneethakrishnan,R.Sridevi, Total edge Fibonacci irregular labeling of some star graphs, *Int. J. Math. Soft Comput*. 5 (1), 73 – 78 (2015).
6. G.Sankari, S.Lavanya, Odd-Even Graceful Labeling of Umbrella and tadpole graphs, *Int. J. Pure and Applied Mathematics*, 114 (6), 139 – 143 2017.