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New Methodology for Solving a Maximization Assignment Problem

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ABSTRACT

Amongst the humongous possible known solution to address the maximization assignment problem (MAP), herewith we posit a new method. To start with assignment matrix is defined, this matrix is then reduced so as to form a matrix with each column to contain one zero at least. This method employs a much slimmer way of approaching the problem there by making it preferred option on addressing assignment problem. Few examples at the end of this article would further enhance the understanding of this methodology.

Keywords: Assignment problems, cost matrix, maximum profit, optimization.

1. INTRODUCTION

For a defined number of jobs an equivalent number of persons are assigned, this forms the basis of assignment problem where in if a lesser number of persons accomplish the job at hand is considered optimal. It is also highly recommended and considered best way of producing good efficiency as the workforce employed is less thereby increasing the profitability. Not always there is a case of balanced situations where number of persons equal number of jobs. Sometimes when there isn't parity between the both then the case is considered unbalanced. To affect the balancing act dummy jobs/ persons are considered before progressing on the assignment problem.[1,2,3]

Assignment is used in
1) Assigning sales people to sales territories,
2) Nurses to patient.

The primordial application in decision making was from the onset of assignment problems. There are varied methods prior to this which deals with to address and find efficient ways for assignment problem. Hungarian Method being the most prominent of all in recent times is used to in assignment problems.

From the time in the beginning of applications of assignment problem in the theory of decision making. This is the earliest of all.

This article begins with theoretical framework (in section II)containing definition model and assumptions. Then algorithm is based out (in section III) followed by numerical examples (in section IV) and finally comes the conclusion (in section V) with explanation on results.

2. THE THEOTIRICAL FRAMEWORK

The data matrix for this

Machines / Jobs	M_1	M_2	……………………….	M_n	
J_1	C_{11}	C_{12}		C_{1n}	
J_2	C_{21}	C_{22}		C_{2n}	1
⋮	⋮	⋮			1
J_n	C_{n1}	C_{n2}	……………………….	C_{nn}	
1	1		……………………………..	1	1

Let X_{ij} denote the assignement of job i to machine j such that

$$X_{ij} = \begin{cases} 1, & if\ the\ i^{th}\ job\ is\ assigned\ to\ j^{th}\ machine \\ 0, & if\ the\ i^{th}\ job\ is\ not\ assigned\ to\ j^{th}\ machine \end{cases}$$

Then the mathematical model of the assignment problem can be stated as the objective function is to,

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$

Subject to the constraints,

$$\sum_{j=1}^{n} X_{ij} = 1, \boldsymbol{for\ all\ i}.$$

$$\sum_{i=1}^{n} X_{ij} = 1, \boldsymbol{for\ all\ j}.$$

Where,$X_{ij} = 0$,if the i^{th} job is not allocated to the j^{th} machine.

Where, $X_{ij} = 1$, if the i^{th} job is allocated to the j^{th} machine and C_{ij} represents the cost of assignment of the job i to machine j [1,2,3].

3. ALGORITHM

Step 1
A matrix of assignment problem is constructed considering row as job and column as machine. For any unbalanced matrix structure, balancing is done.
Step 2
New matrix is constructed from the already present cost matrix. This is obtained by subtracting each column by the maximum cost of its columns.
Step 3
For the column in this matrix, zero position of $(i, j)^{th}$are considered. Identify the zero which has unique position, and allocation is done to that position. Delete the corresponding row and column after allocation. This process is done till all the jobs get assigned.
Step 4
If there is no distinct row for the column, identify which columns have same row. Find the value of next successor of zero. Allocation is done for the column which has maximum value.
Step 5
If there is a tie in maximum value, find the value of next to next successor of zero, allocation is done for the maximum value.
Step 6
After allocation, the reduced matrix should satisfy to have each row to have zero at least, if not the column that doesn't have zero is obtained by subtracting minimum elements from all of its elements.
Step 7
Repeat from step 3 to step 6 until all the jobs get assigned.
Step 8
Finally calculate the cost by the given below expression provided all the jobs are found to be assigned.

$$\textbf{Total Cost} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$$

4. NUMERICAL EXAMPLES

Example.4.1

Five different machines can do any of the five required jobs, with different profits resulting from each assignment as shown below.
As shown below:

Jobs \ Machines	M_1	M_2	M_3	M_4	M_5
J_1	30	37	40	28	40
J_2	40	24	27	21	36
J_3	40	32	33	30	35
J_4	25	38	40	36	36
J_5	29	62	41	44	39

Find the optimum assignment of jobs to machines and corresponding profit.

Solution

Step 1: Column reduction: Subtract from maximum each element in each column.

Jobs \ Machines	M_1	M_2	M_3	M_4	M_5
J_1	10	25	1	16	0
J_2	0	38	14	23	4
J_3	0	30	8	14	5
J_4	15	24	1	8	4
J_5	11	0	0	0	1

Locate the position of zeros.

Row	Column	successor of zero in column.
J_1 →	M_5 →	1
J_2 →	M_2 →	0
J_3 →	M_1 →	0
J_5 →	M_2 →	24*
J_5 →	M_3 →	1
J_5 →	M_4 →	8

Assign $J_5 \rightarrow M_2$

Delete the corresponding row and column.

Step 2: Reduce matrix after column reduction.

Jobs \ Machines	M_1	M_3	M_4	M_5
J_1	10	1	16	0
J_2	0	14	23	4
J_3	0	8	14	5
J_4	15	1	8	4

Subtract the maximum element from each column.

Machines / Jobs	M_1	M_3	M_4	M_5
J_1	10	1	8	0
J_2	0	13	15	4
J_3	0	7	6	5
J_4	15	0	0	4

Locate the position of zeros.

Row	Column	successor of zero in column.
J_1 →	M_3 →	0
	M_5 →	4
J_2 →	M_1 →	0
J_3 →	M_1 →	0
J_4 →	M_3 →	0
	M_4 →	6*

Assign $J_4 \rightarrow M_4$

Delete the corresponding row and column.

Machines / Jobs	M_1	M_3	M_5
J_1	10	0	0
J_2	0	13	4
J_3	0	7	5

Locate the position of zero.

Row	Column	successor of zero in column.
J_1 →	M_3 →	7*
	M_5 →	4
J_2 →	M_1 →	0
J_3 →	M_1 →	0

Assign $J_1 \rightarrow M_3$

Delete the corresponding row and column.

Step 3: Reduced matrix after column reduction:

Machines / Jobs	M_1	M_5
J_2	0	4
J_3	0	5

Subtract the minimum element in each column:

Machines / Jobs	M_1	M_5
J_2	0	0
J_3	0	1

Locate the position of zero.

Row	Column	successor of zero in column.
J_2 →	M_1 →	0
↘	M_5 →	1*
J_3 →	M_1 →	0

Assign $J_2 \rightarrow M_5$, $J_3 \rightarrow M_1$

Optimum Solution

$J_1 \rightarrow M_3$

$J_2 \rightarrow M_2$

$J_3 \rightarrow M_1$

$J_4 \rightarrow M_4$

$J_5 \rightarrow M_2$

Optimal Profit is, 40 + 36 + 40 + 36 + 62= 214

Example.4.2

Four jobs can be processed on four different machines, one job on one machines. Resulting profits vary with assignments. They are given below,

Machines / Jobs	M_1	M_2	M_3	M_4
J_1	42	35	28	21
J_2	30	25	20	15
J_3	30	25	20	15
J_4	24	20	16	12

Find the optimal assignment of jobs to machines and corresponding profit.

Solution:

Step 1: Column Reduction:

Subtract from maximum in each column, all element in that column.

Machines / Jobs	M_1	M_2	M_3	M_4
~~J_1~~	~~0~~	~~0~~	~~0~~	~~0~~
J_2	12	10	8	6
J_3	12	10	8	6
J_4	18	15	12	9

Locate the position of zeros

Row	Column	Successor of Zero in column
J_1	→ M_1	→ 12*
	→ M_2	→ 10
	→ M_3	→ 8
	→ M_4	→ 6

Assign $J_1 \rightarrow M_1$

Delete the corresponding row and column

Step 2: Reduced matrix after column reduction.

Machines / Jobs	M_2	M_3	M_4
J_2	10	8	6
J_3	10	8	6
J_4	15	12	9

Subtract minimum element from each column.

Machines / Jobs	M_2	M_3	M_4
J_2	0	0	0
J_3	0	0	0
J_4	5	4	3

Allocation of zeros.

Row	Column	Successor of zero in column	
J_2	→ M_2	→ 0	5*
	→ M_3	→ 0	4
	→ M_4	→ 0	3
J_3	→ M_2	→ 0	5
	→ M_3	→ 0	4
	→ M_4	→ 0	3

Assign $J_2 \rightarrow M_2$

Delete the corresponding row and column.

Step 3: Reduce matrix after column reduction

Machines / Jobs	M_3	M_4
J_3	0	0
J_4	4	3

Allocation of zero.

Row	Column	Successor of zero in Column
J_3 →	M_3 →	4*
J_3 →	M_4 →	3

Assign $J_3 \rightarrow M_3$ and $J_4 \rightarrow M_4$

Optimum solution:
Assign

$$J_1 \rightarrow M_1$$
$$J_2 \rightarrow M_2$$
$$J_3 \rightarrow M_3$$
$$J_4 \rightarrow M_4$$

Optimum solution is; 42 + 25 + 20 + 12 = 99

Example 4.3
Consider the problem of assigning five jobs to five persons. The assignment profits are given below:

Persons \ Jobs	J_1	J_2	J_3	J_4	J_5
P_1	8	4	2	6	1
P_2	0	9	5	5	4
P_3	3	8	9	2	6
P_4	4	3	1	0	3
P_5	9	5	8	9	5

Obtain the optimum assignment schedule and maximum assignment profit.

Solution:

Step 1: Column Reduction: Subtract from maximum in each column:

Persons \ Jobs	J_1	J_2	J_3	J_4	J_5
P_1	1	5	7	3	5
P_2	9	0	4	4	2
P_3	6	1	0	7	0
P_4	5	6	8	9	3
P_5	0	4	1	0	1

Locate the position of zero

Row	Column	Successor of zero in
P_2 →	J_2 →	1
P_3 →	J_3 →	1
	J_5 →	1
P_5 →	J_1 →	1
	J_4 →	3*

Assign $P_5 \rightarrow J_4$

Delete the corresponding row and column

Step 2: Reduced matrix after column reduction:

Jobs / Persons	J_1	J_2	J_3	J_5
P_1	1	5	7	5
P_2	9	0	4	2
P_3	6	1	0	0
P_4	5	6	8	3

Subtract the minimum element from each column

Jobs / Persons	J_1	J_2	J_3	J_5
P_1	0	5	7	5
P_2	8	0	4	2
P_3	5	1	0	0
P_4	4	6	8	3

Locate the position of zero.

Row	Column	Successor of zero in
P_1 →	J_1 →	4*
P_2 →	J_2 →	1
P_3 →	J_3 →	4
	J_5 →	2

Assign $P_1 \rightarrow J_1$

Delete the corresponding row and column

Step 3: Reduced matrix after column reduction

Jobs / Persons	J_2	J_3	J_5
P_2	0	4	2
P_3	1	0	0
P_4	6	8	3

Subtract the minimum element from each column:

Jobs / Persons	J_2	J_3	J_5
P_2	0	4	2
P_3	1	0	0
P_4	6	8	3

Locate the position of zero.

$P_2 \longrightarrow J_2 \longrightarrow 1$

$P_3 \longrightarrow J_3 \longrightarrow 4^*$

$\quad\;\; \searrow J_5 \longrightarrow 2$

Assign $P_3 \rightarrow J_3$

Delete corresponding row and column.

Step 4 : Reduce matrix after column reduction

Jobs / Persons	J_2	J_5
P_2	0	2
P_4	6	3

Locate the position of zeros.

Row Column Successor of zero in column

$P_2 \longrightarrow J_2 \longrightarrow 6^*$

Assign $P_2 \rightarrow J_2$and $P_4 \rightarrow J_5$

Optimum Solution:

Assign:

$P_1 \rightarrow J_1$

$P_2 \rightarrow J_2$

$P_3 \rightarrow J_3$

$P_4 \rightarrow J_5$

$P_5 \rightarrow J_4$

Optimum Solution is, 8 + 9 + 9 + 3 + 9 = 38

Comparison:

	Ex 4.1	Ex 4.2	Ex 4.3
Hungarian Method	214	99	38
Maximization Assignment Problem (MAP)	214	99	38

5. CONCLUSION

In this article a direct method was introduced for solving maximization problems. The method finds its applicability in all of assignment problems and its kind. The direct method has systematic procedure and very easy to understand. From this article it can be concluded that this method provides an optimal solution directly in few steps for the maximization assignment problem. As this method consumes less time and easier to understand and apply so it can be really helpful for decision makers. The solution thus obtained is at per with that of Hungarian method[1,2,3]. Hence, this research article proposes an equivalent method for addressing maximization assignment problem differing from other methods.[4,5,6]

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