Some Properties of Maximal Rw-Open Sets and in Topological Spaces

R. S. Wali¹ and Prabhavati S. Mandalgeri²

¹Department of Mathematics,
Bhandari Rathi College, Gulelagud, Karnataka State, INDIA.
²Department of Mathematics,
K.L.E’S, S. K. Arts College & H. S. K. Science Institute,
Hubballi, Karnataka State, INDIA.

(Received on: September 11, 2015)

ABSTRACT

In this paper, a new class of topological spaces called $T_{rw-min}$ and $T_{rw-max}$ space and study their relations with topological spaces. Also some properties of the maximal rw-open sets and minimal rw-closed sets have been studied.

Keywords Minimal open set, Maximal closed set, Maximal rw-open set, Minimal rw-closed set.

1. INTRODUCTION

In the year 2001 and 2003, F. Nakaoka and N.oda¹,²,³ introduced and studied minimal open (resp. minimal closed) sets which are sub classes of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2006, S S Benchalli and R.S Wali⁴ introduced and studied rw-closed sets, rw-open sets, maximal rw-open sets, minimal rw-closed sets, maximal rw-closed sets, minimal rw-open sets in topological spaces.

Definition 1.1:¹ A proper non-empty open subset $U$ of a topological space $X$ is said to be minimal open set if any open set which is contained in $U$ is $\emptyset$ or $U$.

Definition 1.2:² A proper non-empty open subset $U$ of a topological space $X$ is said to be maximal open set if any open set which contains $U$ is either $X$ or $U$.

Definition 1.3:³ A proper non-empty closed subset $F$ of a topological space $X$ is said to be minimal closed set if any closed set which is contained in $F$ is $\emptyset$ or $F$.

Definition 1.4:³ A proper non-empty closed subset $F$ of a topological space $X$ is said to be maximal closed set if any closed set which contains $F$ is either $X$ or $F$. 
Definition 1.5: A subset A of (X, τ) is called rw-closed set if Cl(A) ⊆ U whenever A ⊆ U and U is regular semi-open in X.

Definition 1.6: A subset A in (X, τ) is called rw-open set in X if A is rw-closed set in X.

Definition 1.7: A proper non-empty rw-closed subset U of X is said to be maximal rw-closed set if any rw-closed set which contains F is either X or F.

Definition 1.8: A proper non-empty rw-open subset F of X is said to be Minimal rw-open set if any rw-open set which is contained in U is φ or U.

Definition 1.9: A proper non-empty rw-closed subset U of X is said to be maximal rw-open set if any rw-open set which contains F is either X or F.

Definition 1.10: A proper non-empty rw-open subset F of X is said to be Minimal rw-closed set if any rw-closed set which is contained in U is φ or U.

Lemma 1.11: A proper non-empty subset F of X is Minimal rw-open set iff X-F is a maximal rw-closed set.

Lemma 1.12: A proper non-empty subset F of X is Minimal rw-closed set iff X-F is a maximal rw-open set.

Definition 1.13: A topological space X is said to be T_min space if every nonempty proper open subset of X is minimal open set.

Definition 1.14: A topological space X is said to be T_max space if every nonempty proper open subset of X is maximal open set.

Definition 1.15: A topological space (X, τ) is called the
(i) door space if every subset of (X, τ) is either open or closed in (X, τ).
(ii) T₃ space if every g-closed subset of (X, τ) is closed in (X, τ).
(iii) α-space if every α-closed subset of (X, τ) is closed in (X, τ).
(iv) T₄ space if every w-closed subset of (X, τ) is closed in (X, τ).
(v) T₅ space if every rw-closed subset of (X, τ) is closed in (X, τ).
(vi) T₁ (or Frechet) space iff each point in X, singleton set {x} is closed in (X, τ) or for every pair of distinct points x, y in X there is an open set U such that x∈U and y∈V.
(vii) T₀ space if
(viii) T₂ (or Hausdorff) space iff for every pair of distinct points x, y in X there exist disjoint open sets U, V such that x∈U and y∈V.

2. SOME PROPERTIES OF MAXIMAL RW-OPEN SETS.

Theorem 2.1: (i) Let U be a maximal rw-open set and W be a rw-open set then U∩W= φ or U⊂W.
(ii) Let U and V be maximal rw-open sets then U∩V= φ or U=V.
Proof: (i) Let U be a maximal $\mathcal{R}$-open set and W be a $\mathcal{R}$-open set. If $U \cap W = \emptyset$, then there is nothing to prove but if $U \cap W \neq \emptyset$ then we have to prove that $U \subset W$. Suppose $U \cap W \neq \emptyset$ then $U \cap W \subset U$ and $U \cap W$ is $\mathcal{R}$-open as the finite intersection of $\mathcal{R}$-open sets is a $\mathcal{R}$-open set. Since U is a maximal $\mathcal{R}$-open set, we have $U \cap W = U$ therefore $U \subset W$.

(ii) Let U and V be maximal $\mathcal{R}$-open sets. Suppose $U \cap V \neq \emptyset$ then we see that $U \subset V$ and $V \subset U$ by (i) Therefore $U = V$.

Theorem 2.2: Let U be a maximal $\mathcal{R}$-open set. If x is an element of U then $U \subset W$ for any open neighbourhood W of x.

Proof: Let U be a maximal $\mathcal{R}$-open set and x be an element of U. Suppose there exists an open neighbourhood W of x such that $U \not\subset W$ then $U \cap W$ is a $\mathcal{R}$-open set such that $U \cap W \subset U$ and $U \cap W \neq \emptyset$. Since U is a maximal $\mathcal{R}$-open set, we have $U \cap W = U$ that is $U \subset W$. This contradicts our assumption that $U \not\subset W$. Therefore $U \subset W$ for any open neighbourhood W of x.

Theorem 2.3: Let U be a maximal $\mathcal{R}$-open set, if x is an element of U then $U \subset W$ for any $\mathcal{R}$-open set W containing x.

Proof: Let U be a maximal $\mathcal{R}$-open set containing an element x. Suppose there exists an $\mathcal{R}$-open set W containing x such that $U \not\subset W$ then $U \cap W$ is an $\mathcal{R}$-open set such that $U \cap W \subset U$ and $U \cap W \neq \emptyset$. Since U is a maximal $\mathcal{R}$-open set, we have $U \cap W = U$ that is $U \subset W$. This contradicts our assumption that $U \not\subset W$. Therefore $U \subset W$ for any $\mathcal{R}$-open set W containing x.

Theorem 2.4: Let U be a maximal $\mathcal{R}$-open set then $U = \cap \{W : W \text{ is any } \mathcal{R}$-open set containing x\} for any element x of U.

Proof: By theorem 2.7 and from the fact that U is a $\mathcal{R}$-open set containing x, we have $U \subset \cap \{W : W \text{ is any } \mathcal{R}$-open set containing x\} which implies the result.

Theorem 2.5: Let U be a non-empty $\mathcal{R}$-open set then the following three conditions are equivalent.

(i) U is a maximal $\mathcal{R}$-open set
(ii) $U \subset \mathcal{R}$-cl(S) for any non-empty subset S of U.
(iii) $\mathcal{R}$-cl(U) = $\mathcal{R}$-cl(S) for any non-empty subset S of U.

Proof: (i) => (ii) Let U be a maximal $\mathcal{R}$-open set and S be a non-empty subset of U. Let $x \in U$ by theorem 2.2 for any $\mathcal{R}$-open set W containing x, $S \subset U \subset W$ which implies $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since S is non-empty therefore $S \cap W \neq \emptyset$. Since W is any $\mathcal{R}$-open set containing x by one of the theorems, we know that, for an $x \in X$, $x \in \mathcal{R}$-cl(A) iff $V \cap A \neq \emptyset$ for any every $\mathcal{R}$-open set V containing x, that is $x \in U$ implies $x \in \mathcal{R}$-cl(S) which implies $U \subset \mathcal{R}$-cl(S) for any non-empty subset S of U.

(ii) => (iii) Let S be a non-empty subset of U, that is $S \subset U$ which implies $\mathcal{R}$-cl(S) $\subset \mathcal{R}$-cl(U) --(a)
Again from (ii) \( U \subset \text{rw-cl} (S) \) for any non-empty subset \( S \) of \( U \). Which implies \( \text{rw-cl}(U) \subset \text{rw-cl}(\text{rw-cl}(S)) = \text{rw-cl}(S) \) i.e., \( \text{rw-cl}(U) \subset \text{rw-cl}(S) \). (b) from (a) and (b), \( \text{rw-cl}(U) = \text{rw-cl}(S) \) for any non empty subset \( S \) of \( U \).

(iii) \( \Rightarrow \) (i) From (iii) we have \( \text{rw-cl}(U) = \text{rw-cl}(S) \) for any non-empty subset \( S \) of \( U \). Suppose \( U \) is not a maximal \( \text{rw} \)-open set then there exist a non-empty \( \text{rw} \)-open set \( V \) such that \( V \subset U \) and \( V \neq U \). Now there exists an element \( a \in U \) such that \( a \notin V \) which implies \( a \in V^c \) that is \( \text{rw-cl}\{\{a\}\} \subset \text{rw-cl}\{(V^c)\} = V^c \), as \( V^c \) is a \( \text{rw} \)-closed set in \( X \). It follows that \( \text{rw-cl}\{\{a\}\} \neq \text{rw-cl}(U) \). This is contradiction to fact that \( \text{rw-cl}\{\{a\}\} = \text{rw-cl}(U) \) for any non empty subset \( \{a\} \) of \( U \). Therefore \( U \) is a maximal \( \text{rw} \)-open set.

**Theorem 2.6:** Let \( V \) be a non-empty finite \( \text{rw} \)-open set, then there exists at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \).

**Proof:** Let \( V \) be a non-empty finite \( \text{rw} \)-open set. If \( V \) is a maximal \( \text{rw} \)-open set, we may set \( U = V \). If \( V \) is not a maximal \( \text{rw} \)-open set, then there exists a (finite) \( \text{rw} \)-open set \( V_1 \) such that \( \phi \neq V_1 \subset V \). If \( V_1 \) is a maximal \( \text{rw} \)-open set, we may set \( U = V_1 \). If \( V_1 \) is not a maximal \( \text{rw} \)-open set then there exists a (finite) \( \text{rw} \)-open set \( V_2 \) such that \( \phi \neq V_2 \subset V_1 \). Continuing this process we have a sequence of \( \text{rw} \)-open sets \( V_k \subset V_{k+1} \subset \cdots \subset V_1 \subset V \). Since \( V \) is a finite set, this process repeats only finitely then finally we get a maximal \( \text{rw} \)-open set \( U = V_n \) for some positive integer \( n \).

**Corollary 2.7:** Let \( X \) be a locally finite space and \( V \) be a non-empty \( \text{rw} \)-open set then there exists at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \).

**Proof:** Let \( X \) be a locally finite space and \( V \) be a non-empty \( \text{rw} \)-open set. Let \( x \in V \) since \( X \) is a locally finite space we have a finite open set \( V_x \) such that \( x \in V_x \) then \( V \cap V_x \) is a finite \( \text{rw} \)-open set. By theorem 2.5 there exist at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \cap V_x \) that is \( U \subset V \subset V_x \subset V \). Hence there exists at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \).

**Corollary 2.8:** Let \( V \) be a finite minimal open set then there exist at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \).

**Proof:** Let \( V \) be a finite minimal open set then \( V \) is a non-empty finite \( \text{rw} \)-open set, by theorem 2.5 there exist at least one (finite) maximal \( \text{rw} \)-open set \( U \) such that \( U \subset V \).

**Theorem 2.9:** Let \( U \) and \( U_{\lambda} \) be maximal \( \text{rw} \)-open sets for any element \( \lambda \) of \( \Lambda \). If \( U \subset \bigcup_{\lambda \in \Lambda} U_{\lambda} \) then there exists \( \lambda \) element \( \lambda \in \Lambda \) such that \( U = U_{\lambda} \).

**Proof:** Let \( U \subset \bigcup_{\lambda \in \Lambda} U_{\lambda} \) then \( U \cap \bigcup_{\lambda \in \Lambda} U_{\lambda} = U \) that is \( \bigcup_{\lambda \in \Lambda} (U \cap U_{\lambda}) = U \) , also by Theorem 2.1 (ii) \( U \cap U_{\lambda} = \phi \) for any \( \lambda \in \Lambda \) follows that there exist an element \( \lambda \in \Lambda \) such that \( U = U_{\lambda} \).

**Theorem 2.10:** Let \( U \) and \( U_{\lambda} \) be maximal \( \text{rw} \)-open sets for any element \( \lambda \in \Lambda \). If \( U = U_{\lambda} \) for any element \( \lambda \) of \( \Lambda \) then \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \cap U = \phi \).
Proof: Suppose that \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \cap U \neq \emptyset \) that is \( \bigcup_{\lambda \in \Lambda} (U \cap U_{\lambda}) \neq \emptyset \). Then there exists an element \( \lambda \in \Lambda \) such that \( U \cap U_{\lambda} \neq \emptyset \) by theorem 2.1 (ii) we have \( U = U_{\lambda} \), which contradicts the fact that \( U \neq U_{\lambda} \) for any \( \lambda \in \Lambda \) then \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \cap U = \emptyset \).

Theorem 2.11: Let \( U_{\lambda} \) be a maximal rw-open set for any element \( \lambda \in \Lambda \) and \( U_{\lambda} \neq U_{\mu} \) for any element \( \lambda \) and \( \mu \) of \( \Lambda \) with \( \lambda \neq \mu \). Assume that \( |\Lambda| \geq 2 \). Let \( \mu \) be any element of \( \Lambda \) then \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \cap U_{\mu} = \emptyset \).

Proof: Put \( U = U_{\mu} \) in theorem 2.9, then we have the result.

Corollary 2.12: Let \( U_{\lambda} \) be a maximal rw-open set for any element \( \lambda \in \Lambda \) and \( U_{\lambda} \neq U_{\mu} \) for any element \( \lambda \) and \( \mu \) of \( \Lambda \) with \( \lambda \neq \mu \). If \( \Gamma \) a proper non-empty subset of \( \Lambda \) then \( \bigcup_{\gamma \in \Gamma} U_{\gamma} = \emptyset \).

Theorem 2.17: Let \( U_{\lambda} \) and \( U_{\gamma} \) be maximal rw-open sets for any element \( \lambda \in \Lambda \) and \( \gamma \in \Gamma \). If there exists an element \( \gamma \) of \( \Gamma \) such that \( U_{\lambda} \neq U_{\gamma} \) then for any element \( \lambda \) of \( \Lambda \), then \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \subset \bigcup_{\lambda \in \Lambda} \bigcup_{\gamma \in \Gamma} U_{\gamma} \).

Proof: Suppose that an element \( \gamma \) of \( \Gamma \) satisfies \( U_{\lambda} = U_{\gamma} \) for any element \( \lambda \) of \( \Lambda \). If \( \bigcup_{\lambda \in \Lambda} U_{\lambda} \subset \bigcup_{\gamma \in \Gamma} U_{\gamma} \) then we see \( U_{\gamma} \subset \bigcup_{\lambda \in \Lambda} U_{\lambda} \). By theorem 2.9 there exists an element \( \lambda \) of \( \Lambda \) such that \( U_{\gamma} \subset U_{\lambda} \). Since \( U_{\lambda} \neq U_{\lambda} \), which is a contradiction. It follows that \( \bigcup_{\gamma \in \Gamma} U_{\gamma} \neq \bigcup_{\lambda \in \Lambda} U_{\lambda} \).

3. SOME PROPERTIES OF MINIMAL RW-CLOSED SETS

Theorem 3.1: (i) Let \( F \) be a Minimal rw-closed set and \( W \) be a rw-closed set then \( F \cup W = X \) or \( W \subset F \).

(ii) Let \( F \) and \( S \) be Minimal rw-closed sets then \( F \cup S = X \) or \( F = S \).

Proof: (i): Let \( F \) be a Minimal rw-closed set and \( W \) be a rw-closed set if \( F \cup W = X \) then there is nothing to prove but if \( F \cup W \neq X \), then we have to prove that \( W \subset F \). Suppose \( F \cup W \neq X \) then \( F \subset F \cup W \) and \( F \cup W \) is rw-closed as the finite union of rw-closed set is a rw-closed set we have \( F \cup W = X \) or \( F \cup W = F \). Therefore \( F \cup W = F \) which implies \( W \subset F \).

(ii): Let \( F \) and \( S \) be Minimal rw-closed sets. Suppose \( F \cup S \neq X \) then we see that \( F \subset S \) and \( S \subset F \) by (i) therefore \( F = S \).
Theorem 3.2: Let $F$ be a Minimal $\text{rw}$-closed set. If $x$ is an element of $F$ then for any $\text{rw}$-closed set $S$ containing $x$, $F \cup S = X$ or $S \subseteq F$.

Proof: Proof is similar to 2.2 theorem.

Theorem 3.3: Let $F_\alpha, F_\beta, F_\gamma$ be Minimal $\text{rw}$-closed sets such that $F_\alpha \neq F_\beta$ if $F_\alpha \cap F_\beta \subseteq F_\gamma$ then either $F_\alpha = F_\gamma$ or $F_\beta = F_\gamma$.

Proof: Given that $F_\alpha \cap F_\beta \subseteq F_\gamma$, if $F_\alpha = F_\gamma$ then there is nothing to prove but if $F_\alpha \neq F_\gamma$ then we have to prove $F_\beta = F_\gamma$.

Now we have $F_\beta \cap F_\gamma = (F_\beta \cap (F_\alpha \cup F_\beta))$ (by theorem 3.1 (ii))

$= F_\beta \cap (F_\gamma \cap (F_\alpha \cup F_\beta))$

$= (F_\beta \cap F_\gamma) \cup (F_\gamma \cap F_\beta)$

$= (F_\beta \cap F_\gamma) \cup (F_\gamma \cap F_\beta)$ (by $F_\beta \cap F_\gamma \subseteq F_\beta$)

$= F_\beta \cap F_\gamma = X \cap F_\beta$

(since $F_\alpha$ and $F_\gamma$ are Minimal $\text{rw}$-closed sets by thm 3.1 (ii) $F_\alpha \cup F_\gamma = X$)

$= F_\beta$

That is $F_\beta \cap F_\gamma = F_\beta$ implies $F_\beta \subseteq F_\gamma$, since $F_\beta, F_\gamma$ are minimal $\text{rw}$-closed sets, we have $F_\beta = F_\gamma$.

Theorem 3.4: Let $F_\alpha, F_\beta, F_\gamma$ be Minimal $\text{rw}$-closed sets which are different from each other then $(F_\alpha \cap F_\beta) \not\subseteq (F_\alpha \cap F_\gamma)$.

Proof: Let $(F_\alpha \cap F_\beta) \subseteq (F_\alpha \cap F_\gamma)$ which implies $(F_\alpha \cap F_\beta) \cup (F_\gamma \cap F_\beta) \subseteq (F_\alpha \cap F_\gamma) \cup (F_\gamma \cap F_\beta)$ which implies $(F_\alpha \cup F_\beta) \cap F_\gamma \subseteq F_\gamma \cap (F_\alpha \cup F_\beta)$ since by theorem 3.6 (ii) $F_\alpha \cap F_\gamma = X$ and $F_\alpha \cap F_\beta = X$ which implies $X \cap F_\beta \subseteq F_\gamma \cap X$ which implies $F_\beta \subseteq F_\gamma$. From the definition of Minimal $\text{rw}$-closed set it follows that $F_\beta = F_\gamma$.

This is contradiction to the fact that Let $F_\alpha, F_\beta, F_\gamma$ are different from each other. Therefore $(F_\alpha \cap F_\beta) \not\subseteq (F_\alpha \cap F_\gamma)$.

Theorem 3.5: Let $F$ be a Minimal $\text{rw}$-closed set and $x$ be an element of $F$ then $F = \cup \{S: S$ is a $\text{rw}$-closed set containing $x$ such that $F \cup S \neq X\}$.

Proof: By theorem 3.7 and from the fact that $F$ is a $\text{rw}$-closed set containing $x$ we have $FC \subseteq \{S: S$ is a $\text{rw}$-closed set containing $x$ such that $F \cup S \neq X\} \subseteq F$ therefore we have the result.

Theorem 3.6: Let $F$ be a Proper non-empty co-finite $\text{rw}$-closed subset then there exists (co-finite) Minimal $\text{rw}$-closed set $E$ such that $F \subseteq E$.

Proof: Let $F$ be a non-empty co-finite $\text{rw}$-closed set. If $F$ is a Minimal $\text{rw}$-closed set, we may set $E = F$. If $F$ is not a Minimal $\text{rw}$-closed set, then there exists a (co-finite) $\text{rw}$-closed set $F_1$ such that $F \subseteq F_1 \neq X$. If $F_1$ is a Minimal $\text{rw}$-closed set, we may set $E = F_1$. If $F_1$ is not a
Minimal rw-closed set, then there exists a (co-finite) rw-closed set sets $F_2$ such that $F \subseteq F_1 \subseteq F_2 \not= X$ continuing this process we have a sequence of rw-closed sets $F \subseteq F_1 \subseteq F_2 \subseteq \ldots \subseteq F_k \subseteq \ldots$ since $F$ is a co-finite set, this process repeats only finitely then finally we get a maximal rw-open set $E=F_n$ for some positive integer $n$.

**Theorem 3.7:** Let $F$ be a Minimal rw-closed set. If $x$ is an element of $X-F$ then $X-F \subseteq E$ for any rw-closed set containing set $E$ containing $x$.

**Proof:** Let $F$ be a Minimal rw-closed set and $x \in X-F$. $E \supseteq F$ for any rw-closed set $E$ containing $x$ then $E \cup F = X$ by theorem 3.1(ii). Therefore $X-F \subseteq E$.

4. **$T_{rw-min}$ and $T_{rw-max}$ SPACE**

**Definition 4.1:** A topological space $X$ is said to be $T_{rw-min}$ space if every nonempty proper rw-open subset of $X$ is minimal rw-open set.

**Definition 4.2:** A topological space $X$ is said to be $T_{rw-max}$ space if every nonempty proper rw-open subset of $X$ is maximal rw-open set.

**Remark 4.3:** The concepts of $T_{rw-min}$ and $T_{rw-max}$ spaces are not enough general. $T_{rw-min}$ or $T_{rw-max}$ topological space $X$ will be either indiscrete space or $\{X, \phi, A\}$ or $\{X, \phi, A, A^c\}$.

**Theorem 4.4:** A space $X$ is $T_{rw-min}$ if and only if it is $T_{rw-max}$.

**Proof:** Let $X$ is $T_{rw-min}$ space. Suppose that $X$ is not $T_{rw-max}$, so there is a proper rw-open subset $K$ of $X$ which is not maximal, this mean there exist a rw-open subset of $X$ with $K \subseteq H \neq \phi$. Thus we get that $H$ is not minimal which is contradict of being $X$ is $T_{rw-min}$.

Conversely, Let $X$ is $T_{rw-max}$ space. Suppose that $X$ is not $T_{rw-min}$, so there is a proper rw-open subset $K$ of $X$ which is not minimal, this mean there exist an rw-open subset of $X$ with $\phi \neq H \subseteq K$. Thus we get that $H$ is not maximal which is contradict of being $X$ is $T_{rw-max}$.

**Theorem 4.5:** A topological space $X$ is $T_{rw-min}$ space if and only if every nonempty proper rw-closed subset of $X$ is maximal rw-closed set in $X$.

**Proof:** Let $F$ be a proper rw-closed subset of $X$ and suppose $F$ is not maximal. So there exists an rw-closed subset $K$ of $X$ with $K \neq X$ such that $F \subseteq K$. Thus $X-K \subseteq X-F$. Hence $X-F$ is a proper rw-open which is not minimal and this contradicts of being $X$ is $T_{rw-min}$ space.

Conversely, Suppose $U$ is a proper rw-open subset of $X$, thus $X-U$ is a proper rw-closed subset of $X$, so $X-U$ is maximal rw-closed subset of $X$, and from Lemma 1.11$^5$ $U$ is minimal rw-open. Thus $X$ is $T_{rw-min}$ space.

**Theorem 4.6:** A topological space $X$ is $T_{rw-max}$ space if and only if every nonempty proper rw-closed subset of $X$ is minimal rw-closed set in $X$. 
Proof: let F be a proper rw-closed subset of X, suppose F is not minimal rw-closed in X, so there is a proper rw-closed subset of X such that K⊂F Thus X-F⊂X-K but X-K is proper rw-open in X. so X-F is not maximal in X. Contradiction to the fact X-F is maximal rw-open. Conversely , let U be a proper rw-open subset of X, then X-U is a proper rw-closed subset of X and so it is minimal rw-closed set. Lemma 1.12 we get that U is maximal rw-open. Thus X is T_{rw-max} space

Theorem 4.7: Every pair of different minimal rw-open sets of T_{rw-min} space are disjoint.

Proof: Let U and V be minimal rw-open subsets of T_{rw-min} space X such that U≠V to show that U∩V=ϕ suppose not i.e. U∪V≠ϕ. So U∩V⊂U and U∩ V⊂V. Since U∩V⊂U and U is minimal rw-open then U∩V=U or U∩V=ϕ. ThusU∩V=U. Now since U∩V⊂V and V is minimal rw-open then U∩V=V or U∩V=ϕ. Thus U∪ V=V. Hence we get that U=V this result contradicts the fact that U and V are different. Therefore U∪ V=ϕ.

Theorem 4.8: Union of every pair of different maximal rw-open sets in T_{rw-max} space is X.

Proof: Let U and V be maximal rw-open subsets of T_{rw-max} space X such that U≠V to show that U∪ V=X suppose not i.e. U∪ V≠X. So U⊂U∪ V and V⊂U∪ V. Since U⊂U∪ V and U is maximal rw-open then U∪ V=U or U∪ V=X. Thus U∪ V=U… (1). Now since V⊂U∪ V and V is maximal rw-open thenU∪ V=V or U∪ V=ϕ. Thus U∪ V=V… (2) Hence from (1) and (2) we get that U=V this result contradicts the fact that U and V are different. Therefore U∪ V=X.

REFERENCES

5. R. S. Wali, Minimal rw-open sets and maximal rw-closed sets in topological spaces, Ph.D thesis, KUD.