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A Transient Thermoelastic Analysis in a Semi-infinite Cylinder with a Sectional Heat Supply on a Surface

R. Pakade[1], V. Varghese[2], T. Dhakate[3] and Namdeo K.[1]

[1]Department of Mathematics,
Priyadarshini College of Engineering, Nagpur, INDIA.
[2]Department of Mathematics,
Smt. Sushilabai Bharti Science College, Arni, Yavatmal, INDIA.
[3]Department of Mathematics,
Mahatma Gandhi Science College, Armori, Gadchiroli, INDIA.
email: rnp812@gmail.com.

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ABSTRACT

This paper is concerned with the thermal stress of the axisymmetric problem of a semi-infinite hollow cylinder on consideration of an internal heat source that is generated according to the linear function of the temperature. In addition, assuming that an end face of the cylinder is heated with sectional heat supply having radiation-type boundary conditions. The solutions obtained for the temperature distribution of the cylinder is analysed based on the theory of integral transformations technique. Thereafter, the thermal displacements and its associated thermal stress distributions are determined with the aid of thermoelastic potential functions method and Love's displacement function method. As an illustration, numerical calculations are carried out for the temperature distributions, the displacement, and the thermal stress distributions are shown in figures and examined precisely.

Keywords: Heat conduction, thermal stress, semi-infinite cylinder, potential functions.

1. INTRODUCTION

Due to the wider use of industrial and building materials, interest in thermal stress problems has increased significantly due to their basic geometry, characterized by heat exchangers and cylindrical ribs of the brake disc rotor. Therefore, many theoretical studies on them so far reported. However, in order to simplify the analysis, almost all studies were carried out under the assumption that the upper and lower surfaces of the cylindrical structure were isolated or scattered with a uniform heat transfer coefficient over the entire surface. For example, Noda *et al*.[1] investigated a circular plate and discussed the problem of non-stationary thermal elastoplastic bending, using the deformation increment theorem. Khobragade *et al*.[2] studied the distributed heating of thin circular plates is studied by the finite Hankel and Fourier transforms of Dirichlet boundary conditions. Varghese *et al*.[10] analysed the elastic response due to the partial heating distribution of the hollow cylindrical structure has been studied using the transformation method. Nasser[8,9] studied the problems associated with heat sources in generalized thermoelasticity in thick plate profiles. Kulkarni *et al*.[3] determine the quasi-static thermal stress in a thick annular disk that is exposed to any initial temperature on the top surface and zero temperature on the bottom surface. Most papers[3,8,9] published by different authors do not consider the problem of thermoelasticity of any thick plate with radiation-type boundary conditions, where the source is generated in accordance with a linear function of temperature, which also satisfies the time-dependent heat conduction equation. When any temperature or height shift is reached, one can notice the importance of the above transformation[5,6,10] relative to the previously used or published method of integral transformation[2,3] with the outer boundary of the third type of radiation. conditions and internal surfaces with independent radiation constants. Based on the prior literature on the circular profiles, the authors found that scare of analytical procedures were available for semi-infinite hollow cylinder that consider the internal sources of heat generated in the body and are particularly subject to local heating.

2. FORMULATION OF THE PROBLEM

It is assumed that a semi-infinite cylinder is occupying the space $D:\{(r,z)\in R^2:$ $a\le r\le b,\ 0\le z<\infty\}$ under unsteady-state temperature field due to internal heat source within it. Let the cylinder in which internal sources are generated according to a linear function of the temperature are subjected to sectional heating over the upper curved surface at $(z=0)$.

2.1 Temperature distribution

The equation for heat conduction is $^{-}(r,z,t),$ the temperature, in cylindrical coordinates, is

$$\frac{\partial^{-}}{\partial t}=©\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial^{-}}{\partial r}\right)+\frac{\partial^{2-}}{\partial z^{2}}\right]+\Theta(r,z,t,^{-}) \tag{1}$$

in which thermal diffusivity $© = ª / °C,$ ª being the conductivity of the material, ° is the density and C is the calorific capacity, and $\Theta(r,z,t,^{-})$ is the source function

Putting

$$\Theta(r,z,t,^{-})=\Phi(r,z,t)+\cdot\ (t)^{-} \tag{2}$$

and

$$T = ¯ \exp[-\int_0^t · (¸)\, d¸], \quad ¡ = \Phi \exp[-\int_0^t · (¸)\, d¸] \tag{3}$$

where $T(r,z,t)$ is the temperature of the plate at a point (r,z) in time t, $¡(r,z,t)$ is the energy generation, $\Phi(¶, z, t)$ is a function of coordinates and the time, but $·(t)$ is a function of the time only.

The Eq. (1) reduces to the equivalent form

$$\frac{\partial T}{\partial t} = © \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + ¡(r,z,t) \tag{4}$$

subjected to the initial condition

$$T(r,z,t)\big|_{t=0} = T_0(r,z) \tag{5}$$

and boundary conditions

$$T(r,z,t) + k_1 \frac{\partial}{\partial r} T(r,z,t)\Big|_{r=a} = 0, \; T(r,z,t) + k_2 \frac{\partial}{\partial r} T(r,z,t)\Big|_{r=b} = 0 \tag{6}$$

$$T(r,z,t) + h_1 \frac{\partial}{\partial r} T(r,z,t)\Big|_{z=0} = Z(r,t) \tag{7}$$

in which $T_0(r,z)$ is the reference temperature, $Z(r,t)$ is the sectional heat supply at $z = 0$, is the radiation constant on the upper face of the cylinder and $k_i \; (i = 1,2)$ are the given surface coefficients linearly related to the corresponding heat transfer coefficients at the internal and external radial surfaces $r = a$ and $r = b$.

2.2 Thermal displacements and thermal stresses

The Navier's equations in the absence of body forces for the two-dimensional thermoelastic problem can be expressed as

$$\begin{aligned} &\nabla^2 u_r - \frac{u_r}{r} + \frac{1}{1-2³}\frac{\partial e}{\partial r} - \frac{2(1+³)}{1-2³} ~_t \frac{\partial ¯}{\partial r} = 0, \\ &\nabla^2 u_z - \frac{1}{1-2³}\frac{\partial e}{\partial z} - \frac{2(1+³)}{1-2³} ~_t \frac{\partial ¯}{\partial z} = 0 \end{aligned} \tag{8}$$

where u_r and u_z are the displacement components in the radial and axial directions, $~_t$ being the coefficient of thermal expansion and ³ is Poisson's ratio, respectively and the dilatation *e* as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$$

The displacement functions in the cylindrical coordinate system is represented by the Goodier's thermoelastic displacement potential ¤and Love's function L as

$$u_r = \frac{\partial ¤}{\partial r} - \frac{\partial^2 L}{\partial r \partial z},\ u_z = \frac{\partial ¤}{\partial z} + 2(1-³)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{9}$$

in which Goodier's potential must satisfy the equation

$$\nabla^2 ¤ = \left(\frac{1+³}{1-³}\right) \sim_t \overline{\ } \tag{10}$$

with $¤ = 0$ for $t = 0$.

and the Love's function must satisfy the equation

$$\nabla^2(\nabla^2 L) = 0 \tag{11}$$

in which

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{\ }}{\partial r}\right) + \frac{\partial^{2-}}{\partial z^2}$$

The component of the stresses is represented by the use of the Goodier's potential and Love's function as

$$\pm_{rr} = 2G\left\{\left(\frac{\partial^2 ¤}{\partial r^2} - \nabla^2 ¤\right) + \frac{\partial}{\partial z}\left(³\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\},$$

$$\pm_{—} = 2G\left\{\left(\frac{1}{r}\frac{\partial ¤}{\partial r} - \nabla^2 ¤\right) + \frac{\partial}{\partial z}\left(³\nabla^2 L - \frac{1}{r}\frac{\partial L}{\partial r}\right)\right\}, \tag{12}$$

$$\pm_{zz} = 2G\left\{\left(\frac{\partial^2 ¤}{\partial r^2} - \nabla^2 ¤\right) + \frac{\partial}{\partial z}\left((2-³)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\},$$

$$\pm_{rz} = 2G\left\{\frac{\partial^2 ¤}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-³)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}$$

in which G is the shear modulus.

The boundary condition on the traction free surface stress functions are

$$r = a, z = 0 : \pm_{zz} = \pm_{rz} = 0 \tag{13}$$

Eqs. (1) to (13) constitute the mathematical formulation of the problem.

3. SOLUTION TO THE PROBLEM

3.1 Solution for the temperature distribution

In order to solve Eq. (4) under the boundary condition (6), we first introduce the unconventional integral transform[6] of order n over the variable r. Let n be the parameter of the transform, then the integral transform and its inversion theorem are written as

$$\bar{g}(n) = \int_a^b r\, g(r)\; S_0(k_1, k_2, «_n\, r)\, dr,\; g(r) = \sum_{n=1}^{\infty} \bar{g}(n)\, S_0(k_1, k_2, «_n\, r) / C_n \tag{14}$$

where $\bar{g}(n)$ is the transform of $g(r)$ with respect to the kernel $S_0(k_1, k_2, «_n\, r)$ and the eigenvalues $«_n$ are the positive roots of the characteristic equation

$J_0(k_1, «\, a)\; Y_0(k_2, «\, b) - J_0(k_2, «\, b)\; Y_0(k_1, «\, a) = 0$. The kernel function can be defined as

$$S_0(k_1, k_2, «_n\, r) = J_0(«_n r)[Y_0(k_1, «_n a) + Y_0(k_2, «_n b)] - Y_0(«_n r)[J_0(k_1, «_n a) + J_0(k_2, «_n b)]$$

with

$$\left.\begin{aligned} J_0(k_i, «\, r) &= J_0(«\, r) + k_i\, «\, J_0'(«\, r) \\ Y_0(k_i, «\, r) &= Y_0(«\, r) + k_i\, «\, Y_0'(«\, r) \end{aligned}\right\} \text{ for } i = 1, 2$$

and

$$C_n = \int_a^b r[S_0(k_1, k_2,\, «_n b)]^2\, dr$$

in which $J_0(«\, r)$ and $Y_0(«\, r)$ are Bessel functions of first and second kind of order $p = 0$. Applying the integral transform (14) to the Eq. (4) the following reduction is made

$$\frac{\partial \bar{T}}{\partial t} = ©\left[-«_n^2\, \bar{T} + \frac{\partial^2 \bar{T}}{\partial z^2} \right] + \bar{¡}(n, z, t) \tag{15}$$

Now, by using Churchill's integral transform[11] and its inversion theorem are written as

$$\bar{f}(m) = \int_0^{\infty} f(z)\, k(m, z)\, dz,\;\; f(z) = \int_0^{\infty} \bar{f}(m)\, k(m, z)\, dm \tag{16}$$

having kernel as

$$k(m, z) = \left(\frac{2}{®}\right)^{1/2} \frac{\sin(mz) + mh_1 \cos(mz)}{(1 + h_1^2 m^2)^{1/2}}$$

and with the property

$$\int_0^{\infty} \frac{\partial^2 f(z)}{\partial z^2} k(m, z)\, dz = \frac{m}{(1 + h_1^2\, m^2)^{1/2}} \left\{ f(z) + h_1 \left.\frac{\partial f(z)}{\partial z}\right|_{z=0} \right\} - m^2\, \bar{f}(m)$$

the differential equation with $\bar{T}(n, z, t)$ is transformed to

$$\frac{\partial \bar{\bar{T}}}{\partial t} = ©\,[-\Lambda_{n,m}\, \bar{\bar{T}} + \bar{E}(n, t)] + \bar{\bar{¡}}(n, m, t) \tag{17}$$

where

$$\Lambda_{n,m} = «_n^2 + m^2, \bar{E}(n,t) = \frac{m}{(1+h_1^2 m^2)^{1/2}} \bar{Z}(n,t)$$

Then, resolving the differential equation in $\bar{\bar{T}}(n,m,t)$ by Laplace's transform theorem, and anti-transforming with the corresponding inversion theorems, the following solution is arrived at

$$\text{¯}(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_0^{\infty}\left\{k(m,z)\left[\int_0^t \exp[-© \Lambda_{n,m}(t-¸)][©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸ + \bar{\bar{T}}_0(n,m)\exp[-©\Lambda_{n,m}t]\right]dm\right\} S_0(k_1,k_2,«_n r)\exp[\int_0^t ·(¸)d¸]/C_n \quad (18)$$

The function is given in Eq. (18) represents the temperature at every instant and at all points of thesemi-infinite cylinder when there are conditions of radiation type with sectional heating on the upper surface.

3.2 Solution for thermal displacements and stresses

Referring to the fundamental Eq. (1) and its solution (18) for the heat conduction problem, the solution for the displacement function is represented by the Goodier's thermoelastic displacement potential governed by (10) is represented as

$$¤(r,z,t) = \frac{K}{m^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\int_0^{\infty}\left\{k(m,z)\left[\int_0^t \exp[-© \Lambda_{n,m}(t-¸)][©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸ + \bar{\bar{T}}_0(n,m)\exp[-©\Lambda_{n,m}t]\right]dm\right\} S_0(k_1,k_2,«_n r)\exp[\int_0^t ·(¸)d¸]/C_n\Lambda_{n,m} \quad (19)$$

Similarly, Love's function solution is assumed so as to satisfy the governed condition of (11) as

$$L(r,z,t) = \frac{©}{m^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\int_0^{\infty}\left\{k(m,z)\left[\int_0^t \exp[-© \Lambda_{n,m}(t-¸)][©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸ + \bar{\bar{T}}_0(n,m)\exp[-©\Lambda_{n,m}t]\right]dm\right\}[B_{nm}\sinh(«_n z)+C_{nm}\, z\cosh(«_n z)] \quad (20)$$

$$\times S_0(k_1,k_2,«_n r)\exp[\int_0^t ·(¸)d¸]/C_n\Lambda_{n,m}$$

In this manner, two displacement functions in the cylindrical coordinate system $¤(r,z,t)$ and $L(r,z,t)$ are fully formulated. Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential from Eq. (19) and Love's function from Eq. (20) in Eq. (9), one obtains the required thermal displacement. Similarly, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ¤(r,z,t) from Eq. (19) and Love's function $L(r,z,t)$ from Eq. (20) in Eq. (12), one obtains

$$
\begin{aligned}
±_{rr} = \frac{2G}{m^2 r}\Bigg\{ & \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Big[©¬\int_0^{\infty}\Big\{k'''(m,z)\Big[\int_0^{t}\exp[-©\Lambda_{n,m}(t-¸)] \\
& \times[©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸ \Big\} dm)[C_{nm}\cosh(«_n z)+B_{nm}\cosh(«_n z)\Big] \\
& +\int_0^{\infty}\Big\{k''(m,z)\Big[\int_0^{t}\exp[-©\Lambda_{n,m}(t-¸)][©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸\, dm \Big\} \\
& \times rS_0(k_1,k_2,«_n r)]]\}+(-K+3©¬(C_{nm}+B_{nm}«_n)\cosh(«_n z) \\
& +C_{nm}z\sinh(«_n z))+k«_n\int_0^{\infty}\Big\{k'(m,z)\Big[\int_0^{t}\exp[-©\Lambda_{n,m}(t-¸)] \\
& \times[©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸\, dm \Big\}(3r\,³\,S_0(k_1,k_2,«_n r) \\
& \times(C_{nm}z\cosh(«_n z)+(C_{nm}+B_{nm}«_n)\sinh(«_n z))(C_{nm}z\cosh(«_n z) \\
& +B_{nm}\sinh(«_n z))(³\,S_0'(k_1,k_2,«_n r)+r«_n(1-³)S_0''(k_1,k_2,«_n r))) \\
& +«_n\int_0^{\infty}\Big\{k(m,z)\Big[\int_0^{t}\exp[-©\Lambda_{n,m}(t-¸)][©\bar{E}(n,¸)+\bar{\bar{¡}}(n,m,¸)]d¸ \\
& +\bar{\bar{T}}_0(n,m)\exp[-©\Lambda_{n,m}t]\ dm\Big\}(©«_n r³\,S_0(k_1,k_2,«_n r)((3C_{nm} \\
& +B_{nm}«_n)\cosh(«_n z)+C_{nm}z\sinh(«_n z))-KS_0'(k_1,k_2,«_n r) \\
& +©((C_{nm}+B_{nm}«_n)\cosh(«_n z)+C_{nm}z«_n\sinh(«_n z)) \\
& \times(³\,S_0'(k_1,k_2,«_n r)+«_n r(1-³)S_0''(k_1,k_2,«_n r)))\Big\} \\
& \times\exp[\int_0^{t}·(¸)d¸]/C_n\Lambda_{n,m}
\end{aligned} \tag{21}
$$

The remaining stresses can be obtained Eqs. (19)-(20) in Eq. (12). The equations of stresses are rather lengthy. Consequently, the same has been omitted here for the sake of brevity but have been considered during graphical discussion using MATHEMATICA software. In order to solve the constants B_{nm} and C_{nm}, using the stress-free condition (13), one obtains

$$B_{nm} = \frac{-K(1+2©³)S_0'(k_1,k_2,«_n a)}{2©³«_n D_{nm}} \tag{22}$$

$$C_{nm} = \frac{K}{2©³}\left[1+\frac{1+2©³\,S_0'(k_1,k_2,«_n a)}{D_{nm}}\right] \tag{23}$$

in which

$$D_{nm} = «_n © a(-1+³)S_0(k_1,k_2,«_n a) + ©(-2+³)(S_0'(k_1,k_2,«_n a) + a«_n S_0''(k_1,k_2,«_n a)) - 1$$

4. NUMERICAL RESULTS, DISCUSSION AND REMARKS

For the sake of simplicity of calculation, we introduce the following dimensionless values

$$\left.\begin{aligned}
&\bar{r} = r/a,\ \bar{z} = z/a,\ ² = ©\,t/a^2, \bar{\ } = \bar{\ }/\bar{\ }_0,\\
&K_1 = k_1 a, K_2 = k_2 a, H_1 = h_1 a,\\
&\bar{L} = \frac{(1-³)}{(1+³)~_t\, a^3\bar{\ }_0} L,\ \bar{¤} = \frac{(1-³)}{(1+³)~_t\, a^2\bar{\ }_0}¤\\
&\bar{u}_i = \frac{(1-³)}{(1+³)~_t\, a\bar{\ }_0} u_i,\ \bar{±}_{ij} = \frac{1}{2G}\frac{(1-³)}{(1+³)~_t\bar{\ }_0}±_{ij}\ (i,j = r,z)
\end{aligned}\right\} \tag{24}$$

An illustrative particular case is the following, setting

$$\left.\begin{aligned}
&·(¸) = ·_0, \Phi(r,z,t) = \frac{¢(r-r_0)}{r_0}¢(z)\exp(-bt),\\
&T_0(r,z) = 0,\ Z(r,t) = \frac{Q_0}{ª}\exp(-µt)¢(r-r_0)
\end{aligned}\right\} \tag{25}$$

in which $¢(r-r_0)$ is the Dirac Delta function having $a \le r_0 \le b$, Q_0 is the heat flux with constant strength, respectively. Substituting the values in Eqs. (24) and (25), we obtained the expressions for the temperature, displacement and stresses respectively for our numerical discussion. The numerical computations have been carried out for Aluminum metal with parameter a=2.65 cm, b=3.22 cm, h=2.00 cm, Modulus of Elasticity E= 6.9 × 10^6 N/cm^2, Shear modulus G = 2.7 × 10^6N/cm^2, Poisson ratio ³ = 0.281, Thermal expansion coefficient, $~_t$ = 25.5× 10^6cm/cm-^{0}C, Thermal diffusivity ©= 0.86 cm^2/sec, Thermal conductivity ª = 0.48 cal sec^{-1}/cm ^{0}C with $q_{n,m}$ = 0.0763, 0.3467, 0.8956, 1.6784, 2.6784, 3.5675, 4.9634, 6.5674, 7.8945, 8.9604, 10.4574, 13.5674, 15.7895, 18.6784, 21.8956, 24.8934, 27.6723, 30.6784, 34.7853, 38.7845 are the positive & real roots of the transcendental equation. The foregoing analysis is performed by setting the radiation coefficients constants, $k_i = 0.86$ $(i = 1, 2)$ so as to obtain considerable mathematical simplicities. In order to examine the influence of uniform heating on the membrane, we performed the numerical calculation for time $^{2} = 0.001, 0.05, 0.12, 0.30, 0.70..\infty$ and numerical calculations are depicted in the following figures with the help of MATHEMATICA software.

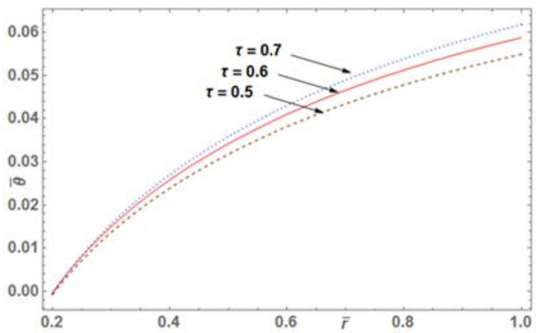

Fig. 1 (a): Temperature distribution along $\bar{r}$ -direction for different time

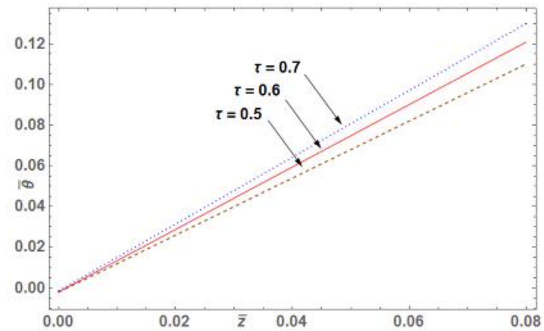

Fig. 1 (b): Temperature distribution along $\bar{z}$ -direction for different time

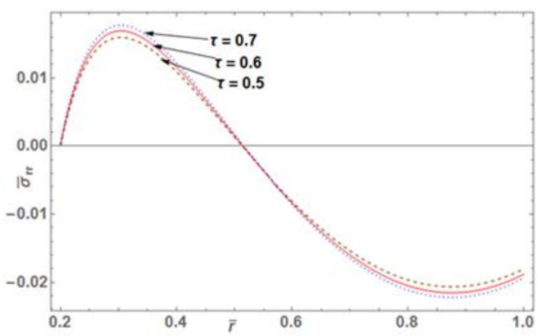

Fig. 2 (a): Thermal stress $\pm_{rr}$ along $\bar{r}$ -direction for different time

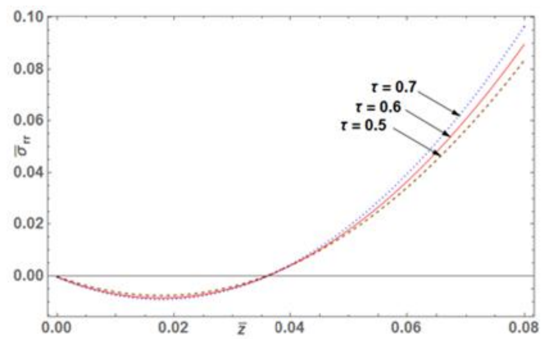

Fig. 2 (b): Thermal stress $\pm_{rr}$ along $\bar{z}$ -direction for different time

Fig. 1(a) illustrates the temperature distribution along the radial direction for different values of time and it increases as the time proceeds along radial direction and reaches maximum at the outer end. The maximum value of temperature magnitude occurs at outer edge due to additional heat supply with available internal heat energy throughout the body. Fig. 1(b) indicates the temperature distribution along *axial* direction of the plate for different values of time, but the curve shows a straight line due to the shallowness of cylinder and maximum value of temperature magnitude occurring at the edge that is energized due to sectional heat supply. Fig. 2(a) shows the thermal radial stress along radial direction for different values of time, in which radial stress appears nearly sinusoidal nature. The expansion occurs on the inner edge due to sectional heat supply followed by the compressive stress occurring on the outer core of the cylinder. Fig. 2(b) depicts the thermal radial stress along axial direction for different values of time. Minimizes its magnitude at the inner edge, it may be due to compressive force, whereas maximizes towards outer edge may be due to energized heat supply.

With thermal load the tangential stress increase gradually along the radial direction as shown in Fig. 2(c). The stresses attain certain maxima and decrease gradually at the outer surface. Fig. 2(d) shows the thermal axial stress along radial direction for different values of time and it was observed that thermal stresses attain minimum expansion at its inner core and maximum at its outer core. thermal axial stress along radial direction for different values of time. Fig. 2(e) presents the axial stress along $\overline{z}$ -direction for different values of time is also having an increasing trend. Figs. 2(g) and 2(h) indicates the thermal shear stress for different values of time along radial and axial direction respectively.

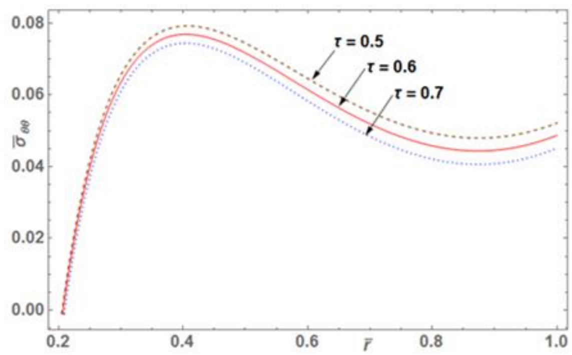

Fig. 2 (c): Thermal stress ±—— along $\overline{r}$ -direction for different time

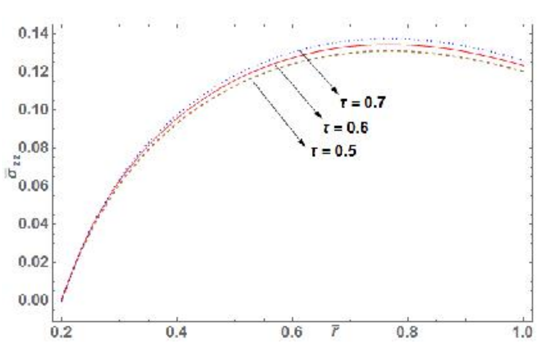

Fig. 2 (d): Thermal stress $\pm_{ZZ}$ along $\overline{r}$ -direction for different time

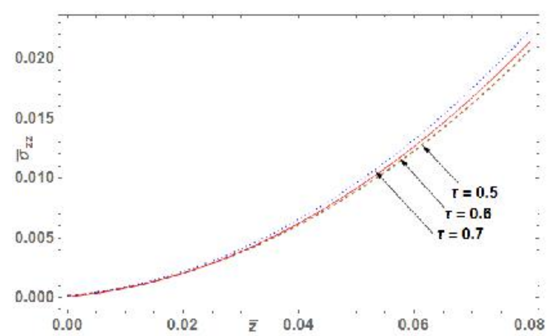

Fig. 2 (e): Thermal stress $\pm_{ZZ}$ along $\overline{Z}$ -direction for different time

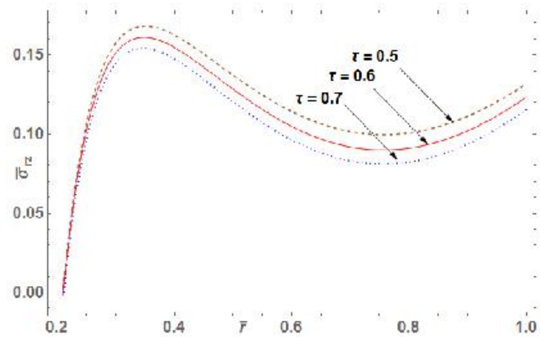

Fig. 2 (f): Thermal stress $\pm_{rz}$ along $\overline{r}$ -direction for different time

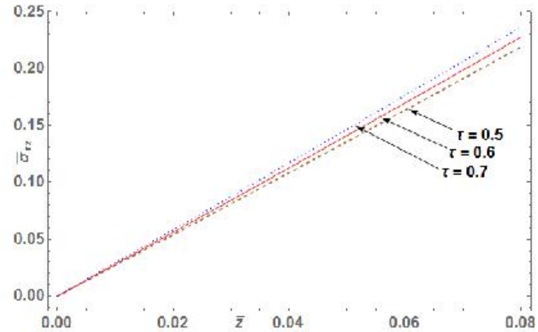

Fig. 2 (g): Thermal stress $\pm_{rz}$ along $\overline{z}$ -direction for different time

5. CONCLUSION

In this study, we have treated thermoelastic problem of a semi-infinite hollow cylinder in which sources are generated according to the linear function of the temperature. We successfully established and obtained the temperature distribution, displacements and stress functions with additional sectional heating available at the edge $\overline{z} = 0$ of the cylinder. Then, in order to examine the validity of boundary value problem, we analyze, as a particular case with mathematical model for $\cdot\,(\text{¸}\,) = \cdot\,_0$ and numerical calculations were carried out. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behaviour with radiation.

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