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A Transient Thermoelastic Analysis in A Semi-infinite Solid Cylinder with Laser Consecutive Pulses

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ABSTRACT

In this paper a problem of heat conduction in a semi-infinite solid cylinder when the laser consecutive irradiation pulses with a Gaussian intensity profile within the cylinder is solved. Both the energy absorption depth and the time decaying effects were considered. The heat conduction equation is assumed to be a transient state, and the integral transform technique is employed in solving the governing equations. On account of the general nature of the boundary conditions and heat absorption source considered, the solution of the problem of heat conduction yields many useful and interesting cases. The theory of thermoelasticity based on solution of Navier's equation in terms of Goodier's thermoelastic displacement potential, Michell's function and the Boussinesq's function for cylindrical co-ordinate system have been used for discussion and analysis of thermal stresses. Some numerical results for the temperature change, the displacement, and the stress distributions are shown in figures.

Keywords: semi-infinite cylinder, heat conduction, heat absorption source, potential functions, thermal stresses.

1. INTRODUCTION

Laser heat treatment finds wide application in metal industries. Laser heating alters both the mechanical and structural properties of the treated metallic parts. These alterations are related to the heating and cooling rates and the instantaneous temperature gradients between the surface and the base substrate. Consequently, the development of heating models for the accurate prediction of temperature profiles at the surface and inside the substrate is necessary. The increased usage of industrial and construction materials the interest in thermal stress problems has grown considerably due to their basic geometry, characterized by heat exchangers and cylindrical ribs of the brake disc rotor. Therefore, widespread attention has been given to the thermal stress problems in semi-infinite solid cylinder. However, in order to simplify the analysis, almost all studies were carried out under the assumption that the upper surfaces of the cylindrical structure are with sectional heat supply with a uniform heat transfer coefficient over the entire surface. For example, Noda *et al*.[1] treated the problem of non-stationary thermal elastoplastic bending of a circular plate, using the deformation increment theorem. Khobragade *et al*.[2] studied the distributed heating of thin circular plates by the finite Hankel and Fourier transforms. Kulkarni *et al*.[3] deals with the determination of transient thermal stresses in a thick annular disc considering the zero initial temperature and arbitrary heat flux on the upper and lower surfaces are at zero temperature by using integral transform technique. Kedar and Deshmukh[4] studied the temperature distribution in the thick circular plate by solving with the help of variable separation technique and the stresses are determined by suitable Michell's function and Goodier's thermoelastic displacement potential function. Sutar[5] deals the paper consisting inverse thermoelastic problem of heat conduction with the application of partially distributed heat supply and internal heat generation. In this paper unknown temperature, displacement stress functions are determined by using finite Marchi-Zgrablich transform and Fourier cosine transform. The authors[7,8] published method of integral transformation with the outer boundary of the third type of radiation conditions and internal surfaces with independent radiation constants. The author Nasser[9,10] studied the problems associated with heat sources in generalized thermoelasticity in thick plate. Many researchers do not consider the problem of thermoelasticity of any thick plate with radiation-type boundary conditions, where the source is generated in accordance with a line function of temperature, which also satisfies the time-dependent heat conduction equation. Varghese *et al*.[11] investigate the thermoelastic problems in a nonhomogeneous thick annular disc with compounded effect due to partial heating and boundary conditions of the radiation type by treating with the theory of integral transformations with independent radiation constants. It is found that all the models employed in the present study of heat conduction in a semi-infinite solid cylinder when the laser consecutive irradiation pulses with a Gaussian intensity profile within the cylinder is solved by considering Both the energy absorption depth and the time decaying effects applying the integral transform technique. On account of the general nature of the boundary conditions and considered heat absorption source, the solution of the problem of heat conduction yields many useful and interesting results.

2. FORMULATION OF THE PROBLEM

Consider a a semi-infinite circular cylinder of radius a bounded by the plane $z = 0$ and the cylinderical surface $r = a$. Laser consecutive pulse heating of cylinder is predicted for the axisymmetric heating situation. Under these more realistic prescribed conditions, the quasi-static thermal stresses are required to be determined.

2.1 Temperature distribution

The equation for heat conduction in a semi-infinite solid cylinder of radius a bounded by the plane $z = 0$ and the curved surface at $r = a$ within cylindricalcoordinates is

$$\frac{\partial ¯}{\partial t} = © \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial ¯}{\partial r} \right) + \frac{\partial^2 ¯}{\partial z^2} \right] + \Theta(r, z, t) \tag{1}$$

subjected to the initial condition

$$¯(r, z, t)\big|_{t=0} = f(r, z) \tag{2}$$

and boundary conditions

$$¯(r, z, t)\big|_{r=a} = g(z, t) \tag{3}$$

$$¯(r, z, t)\big|_{z=0} = ¯_0 \tag{4}$$

in which thermal diffusivity $© = ª / °C$, ª being the conductivity of the material, ° is the density and C is the calorific capacity, $\Theta(r, z, t, ¯)$ is the source function, $¯_0$ is a constant, $f(r, z)$ is the reference temperature, $g(z, t)$ is the sectional heat supply at $r = a,\ z > 0$ and $t > 0$, respectively.

The heat energy absorbed $\Theta = \Theta(r, z, t)$ by the plate, which is expressed[6] as

$$\Theta(r, z, t) = I_0\, ¢\, (I - R_f) \exp(-¢\, z) \exp[-(r / a_0)^2]\, f(t) \tag{5}$$

with I_0 as the laser peak power intensity, ¢ the energy absorption coefficient, R_f is the reflection coefficient, a_0 is the Gaussian parameter, and the function $f(t)$ represents the consecutive pulses as.

$$f(t) = \begin{cases} 0, & t = 0 \\ 1, & t_r \le t \le t_f \\ 0, & t = t_p \\ 0, & t_p \le t \le t_c \end{cases} \tag{6}$$

where t_r is the pulse rise time, t_f is the pulse fall time, t_p is the pulse length, t_c is the end of cooling period. $f(t)$ repeats when the second consecutive pulse begins, provided that time $t = t_f + t_c$ corresponds to the starting time of the second pulse. The same mathematical arguments can apply for the other consecutive pulses after the second pulse.

2.2 Thermal displacements and thermal stresses

The Navier's equations in the absence of body forces for the two-dimensional thermoelastic problem can be expressed as

$$\nabla^2 u_r - \frac{u_r}{r} + \frac{1}{1-2³}\frac{\partial e}{\partial r} - 2\frac{1+³}{1-2³}\sim_t \frac{\partial ¯}{\partial r} = 0,$$
$$\nabla^2 u_z - \frac{1}{1-2³}\frac{\partial e}{\partial z} - 2\frac{1+³}{1-2³}\sim_t \frac{\partial ¯}{\partial z} = 0 \quad (7)$$

where u_r and u_z are the displacement components in the radial and axial directions, $\sim_t$ being the coefficient of thermal expansion and ³ is Poisson's ratio, the dilatation *e* as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$$

and

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

the displacement and the stresses are represented as

$$u_r = \frac{\partial ¤}{\partial r} - \frac{\partial^2 M}{\partial r \partial z},$$
$$u_z = \frac{\partial ¤}{\partial z} + 2(1-³)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \quad (8)$$

$$\pm_{rr} = 2G\left\{\frac{\partial^2 ¤}{\partial r^2} - K¯ + \frac{\partial}{\partial z}\left(³\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\},$$
$$\pm_{——} = 2G\left\{\frac{1}{r}\frac{\partial ¤}{\partial r} - K¯ + \frac{\partial}{\partial z}\left(³\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right\},$$
$$\pm_{zz} = 2G\left\{\frac{\partial^2 ¤}{\partial r^2} - K¯ + \frac{\partial}{\partial z}\left((2-³)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}, \quad (9)$$
$$\pm_{rz} = 2G\left\{\frac{\partial^2 ¤}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-³)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}$$

in which G is the shear modulus and Goodier's potential ¤ must satisfy the equation

$$\nabla^2 ¤ = K¯ \quad (10)$$

with $¤ = 0$ for $t = 0$.

and the Boussinesq harmonic functions ¨ and · must satisfy the following equations

$$\nabla^2 ¨ = \nabla^2 · = 0 \quad (11)$$

Michell's function M must satisfy the equation

$$\nabla^2(\nabla^2 M) = 0 \tag{12}$$

The boundary condition on the traction free surface stress functions are

$$r = a, z = 0 : \pm_{zz} = \pm_{rz} = 0 \tag{13}$$

Eqs. (1) to (13) constitutes the mathematical formulation of the problem.

3. SOLUTION TOTHE PROBLEM

3.1 Solution for the temperature distribution

In order to solve differential Eq. (1) subjected to the boundary conditions (2)-(4), using the theory on integral transformation, it is convenient to apply successively the Fourier Sine transforms and finite Hankel transform as

$$\frac{d\bar{\bar{\ }}}{dt} + \text{©}(\text{¶}^2 + \text{¦}_i^2)\bar{\bar{\ }} = \text{©}a\text{¦}_i J_1(a\text{¦}_i)\bar{g}(\text{¶},t) + \text{©¶}\frac{\bar{a}_0}{\text{¦}_i} J_1(a\text{¦}_i) + \bar{\bar{\Theta}}(\text{¦}_i,\text{¶},t) \tag{14}$$

where

$$\bar{\Theta}(r,\text{¶},t) = \int_0^\infty \Theta(r,z,t)\sin(\text{¶}z)dz, \tag{15}$$

$$\bar{\bar{\Theta}}(\text{¦}_i,\text{¶},t) = \int_0^a rJ_0(a\text{¦}_i)\bar{\Theta}(r,\text{¶},t)\,dr, \tag{16}$$

$$\bar{g}(\text{¶},t) = \bar{\ }(a,\text{¶},t) = \int_0^\infty g(z,t)\sin(\text{¶}z)dz \tag{17}$$

and ¦_i is a root of the transcendental equation of $J_0(a\text{¦}_i) = 0$.

The solution of Eq. (14) can be obtained as

$$\begin{aligned}\bar{\bar{\ }}(\text{¦}_i,\text{¶},t) = {} & \text{©}a\text{¦}_i J_1(a\text{¦}_i)\int_0^t \bar{g}(\text{¶},\text{²}')\exp[-\text{©}(\text{¶}^2 + \text{¦}_i^2)(t-\text{²}')]d\text{²}' + \frac{\bar{a}_0}{\text{¦}_i} J_1(a\text{¦}_i) \\ & \times\left\{\frac{\text{¶}\{1-\exp[-\text{©}(\text{¶}^2+\text{¦}_i^2)t]\}}{(\text{¶}^2+\text{¦}_i^2)}\right\} + \int_0^t \bar{\bar{\Theta}}(\text{¦}_i,\text{¶},t)\exp[-\text{©}(\text{¶}^2+\text{¦}_i^2)(t-\text{²}')]d\text{²}' \\ & + F(\text{¦}_i,\text{¶})\exp[-\text{©}(\text{¶}^2+\text{¦}_i^2)t\end{aligned} \tag{18}$$

where

$$F(\text{¦}_i,\text{¶}) = \bar{\ }(\text{¦}_i,\text{¶},0) = \int_0^\infty\left[\int_0^a rf(r,z)J_0(\text{¦}_i r)dr\right]\sin(\text{¶}z)dz \tag{19}$$

Applying the successively the inversion theorems of finite Hankel transform and Fourier Sine transforms, the following solution is arrived

$$
\begin{aligned}
^{-}(r,z,t) &= \frac{4}{a^2®}\sum_i \frac{J_0(r¦_i)}{[J_1(a¦_i)]^2} \\
&\times\Big\langle ©a¦_i J_1(a¦_i)\int_0^\infty\left\{\int_0^t \bar{g}(¶,²')\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶z)d¶ \\
&+\frac{a¶¯_0}{¦_i}J_1(a¦_i)\left\{\int_0^\infty \frac{¶\{1-\exp[-©(¶^2+¦_i^2)t]\}}{(¶^2+¦_i^2)}\right\}\sin(¶z)d¶ \\
&+\int_0^\infty\left\{\int_0^t \bar{\bar{\Theta}}(¦_i,¶,t)\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶z)d¶ \\
&+\int_0^\infty\left\{F(¦_i,¶)\exp[-©(¶^2+¦_i^2)t]\right\}\sin(¶z)d¶\Big\rangle
\end{aligned} \tag{20}
$$

The function is given in Eq. (20) represents the temperature at every instant and at all points of thesemi-infinite cylinder with sectional heating on the upper surface.

3.2 Solution for thermal displacements and stresses

Referring to the fundamental Eq. (1) and its solution (20) for the heat conduction problem, the solution for the displacement function is represented by the Goodier's thermoelastic displacement potential governed by (10) is represented as

$$
\begin{aligned}
¤(r,z,t) &= \frac{4K}{a^2®(¶^2+¦_i^2)}\sum_i \frac{J_0(r¦_i)}{[J_1(a¦_i)]^2} \\
&\times\Big\langle ©a¦_i J_1(a¦_i)\int_0^\infty\left\{\int_0^t \bar{g}(¶,²')\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶z)d¶ \\
&+\frac{a¶¯_0}{¦_i}J_1(a¦_i)\left\{\int_0^\infty \frac{¶\{1-\exp[-©(¶^2+¦_i^2)t]\}}{(¶^2+¦_i^2)}\right\}\sin(¶z)d¶ \\
&+\int_0^\infty\left\{\int_0^t \bar{\bar{\Theta}}(¦_i,¶,t)\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶z)d¶ \\
&+\int_0^\infty\left\{F(¦_i,¶)\exp[-©(¶^2+¦_i^2)t]\right\}\sin(¶z)d¶\Big\rangle
\end{aligned} \tag{21}
$$

Similarly, Michell's solution is assumed so as to satisfy the governed condition of (12) as

$$\mathrm{M}(r,z,t)=\frac{4K}{a^2 ® (¶^2+¦_i^2)}\sum_i \frac{J_0(r¦_i)}{[J_1(a¦_i)]^2}[\mathrm{B}¦_i\, z+C¦_i \sinh(¶ z)]$$
$$\times\left\langle ©a¦_i J_1(a¦_i)\int_0^\infty\left\{\int_0^t \bar{g}(¶,²')\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶ z)d¶\right.$$
$$+\frac{a¶¯_0}{¦_i}J_1(a¦_i)\left\{\int_0^\infty\frac{¶\{1-\exp[-©(¶^2+¦_i^2)t]\}}{(¶^2+¦_i^2)}\right\}\sin(¶ z)d¶ \tag{22}$$
$$+\int_0^\infty\left\{\int_0^t \bar{\bar{\Theta}}(¦_i,¶,t)\exp[-©(¶^2+¦_i^2)(t-²')]d²'\right\}\sin(¶ z)d¶$$
$$\left.+\int_0^\infty\left\{F(¦_i,¶)\exp[-©(¶^2+¦_i^2)t]\right\}\sin(¶ z)d¶\right\rangle$$

In this manner, two displacement functions in the cylindrical coordinate system $¤(r,z,t)$ and $M(r,z,t)$ are fully formulated. Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential from Eq. (21) and Michell's function from Eq. (22) in Eq. (8), one obtains

$$u_r=\frac{1}{a^2(¶^2+¦_i^2)}\sum_i\frac{4K\,J_1(\mathrm{r}¦_i)¦_i}{[J_1(\mathrm{a}¦_i)]^2}\left\{-Q1-S1-\frac{a¯_0 ¶ J_1(\mathrm{a}¦_i)R1}{¦_i^2}-a\,k\,J_1(\mathrm{a}¦_i)\,\mathrm{P}1¦_i\right.$$
$$+\frac{1}{®}\left[A¦_i+B¦_i ¶\cosh(\mathrm{z}¶)\left(Q1+S1\frac{a\,J_1(\mathrm{a}¦_i)(¯_0 ¶ R1+k\,\mathrm{P}1¦_i^3)}{¦_i^2}\right)\right] \tag{26}$$
$$\left.+\frac{1}{®}\left[(\mathrm{A}¦_i\, z+B¦_i \sinh(\mathrm{z}¶))\left(Q1'(\mathrm{z})+\frac{a\,J_1(\mathrm{a}¦_i)(¯_0 ¶ R1'(\mathrm{z})+k\,\mathrm{P}1'(\mathrm{z})¦_i^3)}{¦_i^2}+S1'(\mathrm{z})\right)\right]\right\}$$

$$u_r=\frac{1}{a^2(¶^2+¦_i^2)}\sum_i\frac{4K}{[J_1(\mathrm{a}¦_i)]^2}\left\{-\frac{T1}{®}+J_0(\mathrm{r}¦_i)U1-\frac{1}{®}2J_0(\mathrm{r}¦_i)\right.$$
$$\times(\mathrm{A}¦_i+B¦_i \sinh(\mathrm{z}¶)\,\mathrm{U}1-\frac{1}{®}J_0(\mathrm{r}¦_i)(\mathrm{A}¦_i\mathrm{z}+B¦_i \sinh(\mathrm{z}¶))\,\mathrm{V}1$$
$$-\frac{1}{®}(1-³)\left(\frac{-1}{\mathrm{r}¦_i}(\mathrm{A}¦_i\mathrm{z}+B¦_i \sinh(\mathrm{z}¶))(2J_1(\mathrm{r}¦_i)+\mathrm{r}(J_0(\mathrm{a}¦_i)-J_2(\mathrm{r}¦_i))¦_i)\right) \tag{27}$$
$$\times(a¯_0 ¶ J_1(\mathrm{a}¦_i)R1(\mathrm{z})+¦_i^2(\mathrm{Q}1+S1+a\,k\,J_1(\mathrm{a}¦_i)\,\mathrm{P}1(\mathrm{z})¦_i)$$
$$\left.+2T1+4J_0(\mathrm{r}¦_i)(\mathrm{A}¦_i+B¦_i ¶\cosh(\mathrm{z}¶))U1+2J_0(\mathrm{r}¦_i)(\mathrm{A}¦_i\, z+B¦_i \sinh(\mathrm{z}¶))\,\mathrm{V}1\right\}$$

in which

$$P1 = \int_0^\infty \left\{ \int_0^t \bar{g}(¶, ²') \exp[-©(¶^2 + ¦_i^2)(t - ²')] d²' \right\} \sin(¶ z) d¶$$

$$Q1 = \int_0^\infty \left\{ \int_0^t \bar{\bar{\Theta}}(¦_i, ¶, t) \exp[-©(¶^2 + ¦_i^2)(t - ²')] d²' \right\} \sin(¶ z) d¶$$

$$R1 = \left\{ \int_0^\infty \frac{¶\{1 - \exp[-©(¶^2 + ¦_i^2)t]\}}{(¶^2 + ¦_i^2)} \right\} \sin(¶ z) d¶$$

$$S1 = \int_0^\infty \left\{ F(¦_i, ¶) \exp[-©(¶^2 + ¦_i^2)t] \right\} \sin(¶ z) d¶$$

$$T1 = B_{¦_i} ¶^2 J_0(r¦_i) \sinh(z¶) \left(Q1 + S1 + \frac{a J_1(a¦_i)(¯_0 ¶ R1 + k\, P1 ¦_i^3)}{¦_i^2} \right)$$

$$U1 = \left(a k J_1(a¦_i) ¦_i\, P1'(z) + Q1'(z) + \frac{a ¯_0 ¶ J_1(a¦_i) R1'(z)}{¦_i^2} + S1'(z) \right)$$

$$V1 = \left(a k J_1(a¦_i) ¦_i\, P1''(z) + Q1''(z) + \frac{a ¯_0 ¶ J_1(a¦_i) R1''(z)}{¦_i^2} + S1''(z) \right)$$

in which, (’) represents the derivative with respect to the assigned variable.

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential $¤(r, z, t)$ from Eq. (21) and Michell’s function $M(r, z, t)$ from Eq. (22) in Eq. (9). The equations of stresses are rather lengthy. Consequently, the same has been omitted here for the sake of brevity but have been considered during graphical discussion using MATHEMATICA software. In order to solve the constants $A_{¦_i}$ and $B_{¦_i}$, using the stress-free condition (13), one obtains as $A_{¦_i} = -® / 4³$ and $B_{¦_i} = 3® / 4³ ¶$.

4. NUMERICAL RESULTS, DISCUSSION AND REMARKS

For the sake of simplicity of calculation, we introduce the following dimensionless values

$$\left.\begin{array}{l} \bar{r} = r/a,\ \bar{z} = z/a,\ ² = © t/a^{2}, \bar{} = \bar{} / \bar{}_{0}, \\ K_1 = k_1 a, K_2 = k_2 a, H_1 = h_1 a, \\ \bar{L} = \dfrac{(1-³)}{(1+³)\sim_t a^{3}\bar{}_0} L,\ \bar{¤} = \dfrac{(1-³)}{(1+³)\sim_t a^{2}\bar{}_0} ¤ \\ \bar{u}_i = \dfrac{(1-³)}{(1+³)\sim_t a\bar{}_0} u_i,\ \bar{±}_{ij} = \dfrac{1}{2G}\dfrac{(1-³)}{(1+³)\sim_t \bar{}_0} ±_{ij}\ (i, j = r, z) \end{array}\right\} \tag{28}$$

An illustrative particular case is the following, setting

$$\left.\begin{array}{l} \cdot\,(¸) = \cdot_{0}, \Phi(r,z,t) = \dfrac{¢(r-r_0)}{r_0} ¢(z)\exp(-bt), \\ T_0(r,z) = 0,\ Z(r,t) = \dfrac{Q_0}{ª}\exp(-\mu t)¢(r-r_0) \end{array}\right\} \tag{29}$$

in which $¢(r-r_0)$ is the Dirac Delta function having $a \le r_0 \le b$, Q_0 is the heat flux with constant strength, respectively. Substituting the values in Eqs. (26) and (27), we obtained the expressions for the temperature, displacement and stresses respectively for our numerical discussion. The numerical computations have been carried out for Aluminum metal with parameter *a*=2.65 cm, *b*=3.22 cm, *h*=2.00 cm, Modulus of Elasticity *E*= 6.9 × 10^6N/cm^2, Shear modulus *G*= 2.7 × 10^6N/cm^2, Poisson ratio ³ = 0.281, Thermal expansion coefficient, $\sim_t$ = 25.5× 10^6cm/cm-^{0}C, Thermal diffusivity©= 0.86 cm^2/sec, Thermal conductivity ª = 0.48 calsec^{-1}/cm ^{0}C with $q_{n,m} = 0.0763$, 0.3467, 0.8956, 1.6784, 2.6784, 3.5675, 4.9634, 6.5674, 7.8945, 8.9604, 10.4574, 13.5674, 15.7895, 18.6784, 21.8956, 24.8934, 27.6723, 30.6784, 34.7853, 38.7845are the positive & real roots of the transcendental equation. The foregoing analysis is performed by setting the radiation coefficients constants, $k_i = 0.86\ (i = 1,2)$ so as to obtain considerable mathematical simplicities. In order to examine the influence of uniform heating on the membrane, we performed the numerical calculation for time $² = 0.001, 0.05, 0.12, 0.30, 0.70..\infty$ and numerical calculations are depicted in the following figures with the help of MATHEMATICA software. Here Fig. 1(a) depicted the temperature distribution along the radial direction for different values of time and it increases as the time increases along radial direction and attains maximum at the outer end. The maximum value of temperature magnitude occurs at outer edge due to sectional heat supply with internal heat energy throughout the body. Fig. 1(b) shows the temperature distribution along time parameter for different values of radius, the curve shows maximum value of temperature magnitude occurring at the inner edge that is energized due to reference heat supply at the initial time of cylinder.

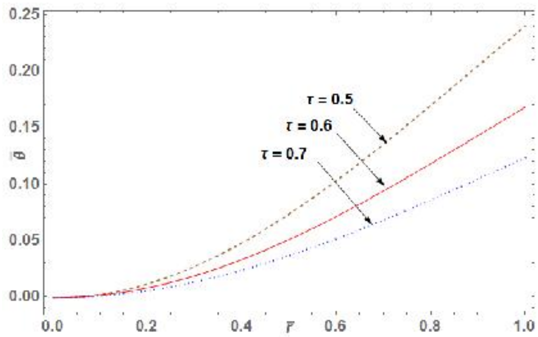

Fig. 1 (a): Temperature distribution along $\overline{r}$ -direction for different time

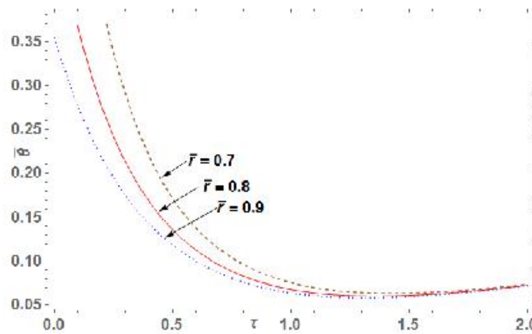

Fig. 1 (b): Temperature distribution along time parameter for different $\overline{r}$

Fig. 2(a) shows the thermal radial stress along radial direction for different values of time, in which radial stress appears minimum at the inner core while maximum the outer core. The compressive stress occurring on the inner core of the cylinder followed by the expansion occurs on the outer core due to sectional heat supply. Fig. 2(b) depicts the thermal radial stress along time parameter for different values of radius. Maximizes its magnitude towards inner as well as outer edge, it may be due to energized heat supply, whereas Minimizes at the middle edge, it may be due to compressive force.

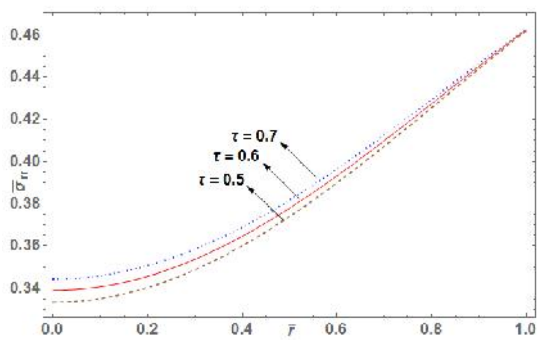

Fig. 2 (a): Thermal stress $\pm_{rr}$ along $\overline{r}$ -direction for different time

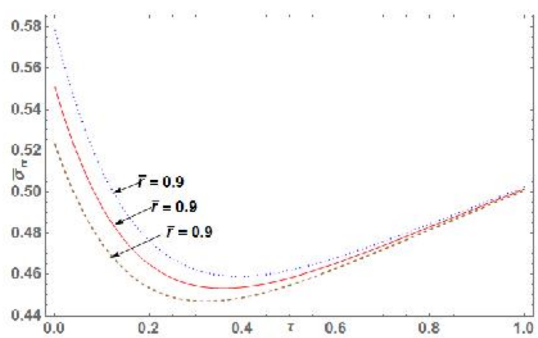

Fig. 2 (b): Thermal stress $\pm_{rr}$ along time parameter for different $\overline{r}$

The tangential stress increases gradually along time for different values of radial as shown in Fig. 2(c). The stresses attain certain minimum value and increases gradually at the outer surface. Fig. 2(d) indicates the thermal axial stress along radial direction for different values of time, and it was observed that thermal stresses attain maximum expansion at its inner core and zero at its outer core satisfying the boundary condition eq. (13). Fig. 2(e) presents curve for the axial stress along $\overline{z}$ -direction for different values of time is increases linearly towards outer edge satisfying the stress-free condition.

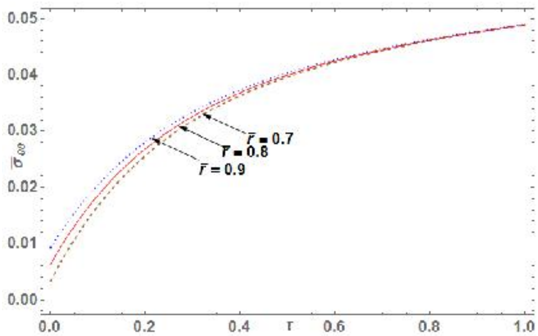

Fig. 2 (c): Thermal stress $\pm$— along time parameter for different $\overline{r}$

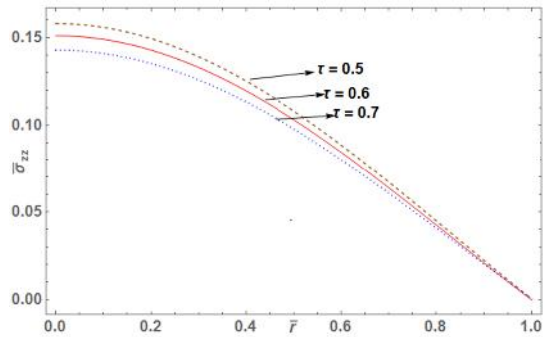

Fig. 2 (d): Thermal stress $\pm_{ZZ}$ along $\overline{r}$ -direction for different time

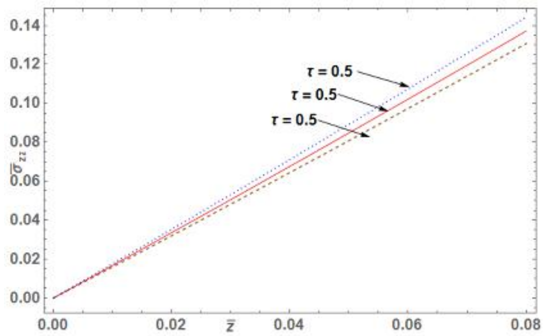

Fig. 2 (e): Thermal stress $\pm_{ZZ}$ along $\overline{Z}$ -direction for different time

Figs. 2(f) and 2(g) indicates the thermal shear stress for different values of time and radius along radial direction and time parameter respectively. In Fig. 2(f) curve shows maximum magnitude at inner core while outer core attains zero satisfying the stress-free condition, whereas in Fig. 2(g) curve attains minimum at inner and as the time increases it shows the maximum expansion.

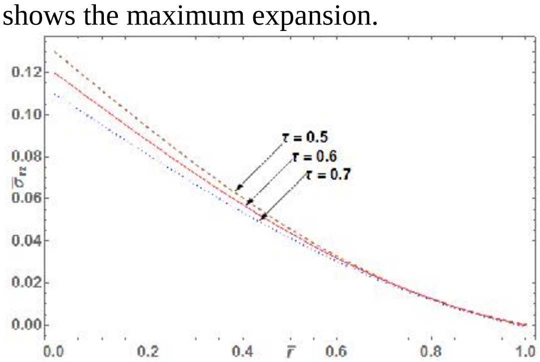

Fig. 2 (f): Thermal stress $\pm_{rZ}$ along $\overline{r}$ -direction for different time

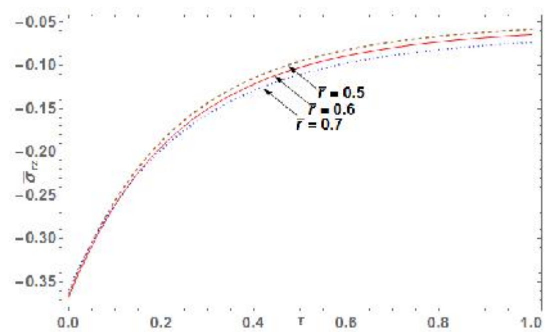

Fig. 2 (g): Thermal stress $\pm_{rZ}$ along time parameter for different $\overline{r}$

5. CONCLUSION

In this study, we have treated heat conduction problem of a semi-infinite solid cylinder when the laser consecutive irradiation pulses with a Gaussian intensity profile within the cylinder. We successfully established and obtained the temperature distribution, displacements and stress functions with additional sectional heating available at the edge $\overline{z} = 0$ of the cylinder. The solution of Navier's equation in terms of Goodier's thermoelastic displacement potential, Michell's function and the Boussinesq's function for cylindrical co-ordinate system have been used for the analysis of thermal stresses. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behaviour with radiation.

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