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One Hundredth of Intuitionistic Fuzzy Sets

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ABSTRACT

In this paper, we propose a new type of Intuitionistic Fuzzy Sets(IFSs) namely One Hundredth of Intuitionistic Fuzzy Sets(OHIFSs) which are analogous to the intuitionistic fuzzy sets. We study the relation between OHIFSs and IFSs based on the standard operations " + " and " $\cdot$ ". Even though there exists an algebraic isomorphism between OHIFSs and IFSs, some standard results in IFSs are not true for OHIFSs. Comparision of standard results of OHIFSs and IFSs are discussed.

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Keywords: Intuitionistic fuzzy sets, Operators, Operators in IFSs and One hundredth of IFSs.

1. INTRODUCTION

In 1965, Zadeh L. A.[10] introduced fuzzy sets which was a generalisation of crisp sets. Later on Atanassov.K[1] introduced intuitionistic fuzzy set which was the development of fuzzy set in which non-membership values were considered. Atanassov. K[2] also extended the Intuitionistic Fuzzy Sets like Intuitionistic Fuzzy Sets of Second Type(IFFST), Intuitionistic L-Fuzzy Sets(ILFS) and Temporal Intuitionistic Fuzzy Sets(TIFS). In[5], Mondal. T.K and Samanta. S.K introduced Generalised Intuitionistic Fuzzy Sets(GIFS) in which the minimum of two degrees is less than or equal to half. Srinivasan.R and Palaniappan.N[8] introduced IFS of root type. Vassilev.P *et al*.,[9] introduced p^{th} type of IFS. Baloui Jamkhanesh and Nadarajah[4] introduced New Generalised Intuitionistic Fuzzy Sets($GIFS_B$). Nagalingam.R and Rajaram.S[6] introduced New Generalised Intuitionistic Fuzzy Set Type NGIFS$\left(\frac{m^*}{n}\right)$T. Now a days researchers focussed on the introduction of new intuitionistic fuzzy sets and proved new results.

The aim of this paper is to propose a new type of intuitionistic fuzzy set namely One Hundredth of Intuitionistic Fuzzy Set(OHIFS) and study relations between IFSs and OHIFSs. In section 2, some basic definitions related to the IFSs are presented.

2. PRELIMINARIES

In this section, we recall the definitions of different types of IFSs.

2.1 Definition[1]

An intuitionistic fuzzy set A defined over a non-empty set X is the elements of the form $< x, \mu_A(x), \nu_A(x) >$ where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ are the degree of the membership function and degree of non-membership function for every $x \in X$ respectively be such that $0 \le \mu_A(x) + \nu_A(x) \le 1$. Here $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitancy degree of x in A.

2.2 Set relations and operations on IFS [2]

Let IFS(X) denote the family of all intuitionistic fuzzy sets in the universe X. For every A, B $\in$ IFS(X) which are represented by $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$. Some relations and operations are defined as follows:

i. $\overline{A} = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$,

ii. $A \subset B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ $\forall x \in X$,

iii. $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ $\forall x \in X$,

iv. $A \cap B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$,

v. $A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$,

vi. $A + B = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) > | x \in X\}$,

vii. $A \cdot B = \{< x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) > | x \in X\}$,

viii. $A \ @ \ B = \{< x, \frac{\mu_A(x) + \mu_B(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\}$,

ix. $A \ \$ \ B = \{< x, \sqrt{\mu_A(x)\mu_B(x)}, \sqrt{\nu_A(x)\nu_B(x)} > | x \in X\}$,

In[2], the simplest modal operators defined over IFSs are

$\square A = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$ and

$\lozenge A = \{< x, 1 - \nu_A(x), \nu_A(x) > | x \in X\}$.

The empty IFS and the unit IFS are defined by $O^* = \{< x, 0, 1 > | x \in X\}$ and $E^* = \{< x, 1, 0 > | x \in X\}$ respectively.

2.2 Cartesian product over IFS [2]

Let X and Y be two universes and let $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and let $B = \{< y, \mu_B(y), \nu_B(y) > | x \in Y\}$ be two IFSs over X and over Y respectively.

The Cartesian product " $\times_1$ " between A and B is defined by $A \times_1 B = \{ < x, y >, \mu_A(x)\mu_B(y), \nu_A(x)\nu_B(y) > | x \in X, y \in Y\}$.

2.3 Operator $G_{\alpha,\beta}$ [2]

Let $\alpha, \beta \in [0,1]$. Given intuitionistic fuzzy set A, the operator $G_{\alpha,\beta}$ is defined by $G_{\alpha,\beta} = \{< x, \alpha\mu_A(x), \beta\nu_A(x) > | x \in X\}$.

2.4 Closure C and Interior I [2]

For every $A \in IFS(X)$, the Closure C and Interior I of A is defined by $C(A) = \{< x, K, L > | x \in X\}$ where $K = \underset{y \in X}{Max}\, \mu_A(y)$, $L = \underset{y \in X}{Min}\, \mu_A(y)$ and $I(A) = \{< x, k, l > | x \in X\}$ where $k = \underset{y \in X}{Min}\, \mu_A(y)$, $l = \underset{y \in X}{Max}\, \mu_A(y)$

2.5 Definition [2]

For every intuitionistic fuzzy set A, the operator R(A) is defined as $R(A) = \{< x, \mu_A(x)(1 - \nu_A(x)), \nu_A(x)(1 - \mu_A(x)) > | x \in X\}$.

2.6 Definition [2]

For every $A \in IFS(X)$, the level operators $!(A)$ and $?(A)$ are defined by $!(A) = \{< x, max(\frac{1}{2}, \mu_A(x)), min(\frac{1}{2}, \nu_A(x)) > | x \in X\}$ and $?(A) = \{< x, min(\frac{1}{2}, \mu_A(x)), max(\frac{1}{2}, \nu_A(x)) > | x \in X\}$.

2.7 Definition [7]

An intuitionistic fuzzy set of second type (IFSST) A defined over a non-empty set X is the form of elements $< x, \mu_A(x), \nu_A(x) >$ where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ are the degree of membership and degree of non-membership for every $x \in X$ respectively be such that $0 \le (\mu_A(x))^2 + (\nu_A(x))^2 \le 1$.

2.8 Definition [8]

An intuitionistic fuzzy set of root type (IFSRT) A defined over a non-empty set X is the form of elements $< x, \mu_A(x), \nu_A(x) >$ where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ are the degree of membership and degree of non-membership for every $x \in X$ respectively be such that $0 \le \frac{1}{2}\sqrt{\mu_A(x)} + \frac{1}{2}\sqrt{\nu_A(x)} \le 1$.

2.9 Definition [9]

Let p be a fixed positive real number and $\mu_A, \nu_A : X \to [0,1]$ be two arbitrary mappings. Then the set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ is said to be p-intuitionistic fuzzy set (p-IFS) if and only if $\forall\, x \in X$, $0 \le (\mu_A(x))^p + (\nu_A(x))^p \le 1$.

2.10 Definition [4]

Let X denote a non-empty set. Generalized intuitionistic fuzzy set A in X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ are the degree of membership and degree of non-membership functions of A, and $0 \le (\mu_A(x))^\delta + (\nu_A(x))^\delta \le 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, n = 1,2,…., N. The collection of all generalized IFSs is denoted by $GIFS_B(\delta, X)$.

3. ONE HUNDREDTH OF INTUITIONISTIC FUZZY SETS(OHIFSs)

The restriction of membership value and non-membership value less than or equal to 0.1 helps us to define the new concept namely, one hundredth of intuitionistic fuzzy sets(OHIFSs). The set $\{< x, 0.0a, 0.0b > | x \in X\}$ where a, b vary from 1 to 9 is equal to $\{ < x, 0.a, 0.0b > | x \in X \} \cdot \{ < x, 0.1, 0 > | x \in X \}$ and $\{ < x, 0.0a, 0.b > | x \in X \} + \{ < x, 0.1, 0 > | x \in X \}$ under the usual notation "·" and "+" give the basic concept of the improvement of results and theorems in this paper. The concept of OHIFSs is as like as the micro level of the IFSs concept.

3.1 Definition

Let X be a fixed non-empty set. One Hundredth of Intuitionistic Fuzzy Set A in X is defined as an object of the following form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ where the functions $\mu_A: X \to [0, 0.1]$ and $\nu_A: X \to [0, 0.1]$ are the degree of membership and degree of non-membership for every $x \in X$ respectively be such that $0 \leq \mu_A(x) + \nu_A(x) \leq 0.1$ for every $x \in X$.

3.2 Remark

Let OHIFS(X) denote the set of all one hundredth of intuitionistic fuzzy sets. Now we assume the following two for the calculation part of this paper

(i) In OHIFSs, the codomain of the membership and non-membership functions are restricted to [0, 0.1] but the symbols and operations in IFSs have the same meaning in OHIFSs.
(ii) All the membership and non-membership values are rounded off to two decimal places.

3.3 RESULTS

Since $\{ < x, 0.0a, 0.0b >| x \in X \} = \{ < x, 0.a, 0.0b >| x \in X \} \cdot \{< x, 0.1, 0 >| x \in X \}$ or is equal to $\{< x, 0.0a, 0.b >| x \in X\} + \{< x, 0.1, 0 >| x \in X \}$, we have the following results:
(i) Every OHIFSs can be written as the product of two IFSs (or)
(ii) Every OHIFSs can be written as the sum of two IFSs.

3.4 Example

Let $A = \{ < a, 0.09, 0.01 > , < b, 0.07, 0.01 >, < c, 0.02, 0.05 > | x \in X \}$

Then $A = \{ < a, 0.9, 0.01 > , < b, 0.7, 0.01 >, < c, 0.2, 0.05 > | x \in X \} \cdot \{ < a, 0.1, 0 > , < b, 0.1, 0 >, < c, 0.1, 0 > | x \in X \}$ and

$A = \{ < a, 0.09, 0.1 > , < b, 0.07, 0.1 >, < c, 0.02, 0.5 > | x \in X\} + \{ < a, 0, 0.1 > , < b, 0. 0.1 >, < c, 0, 0.1 > | x \in X \}$.

So we observe that $A = A_1 + E_1$ and $A = A_2 \cdot E_2$ where

A_2 ={ < a, 0.9, 0.01 > , < b, 0.7, 0.01 >, < c, 0.2, 0.05 > $| x \in X$ },
E_2 ={ < a, 0.1, 0 > , < b, 0.1, 0 >, < c, 0.1, 0 > $| x \in X$ },
A_1 = { < a, 0.09, 0.1 > , < b, 0.07, 0.1 >, < c, 0.02, 0.5 > $| x \in X$ } and
E_1 = { < a, 0, 0.1 > , < b, 0. 0.1 >, < c, 0, 0.1 > $| x \in X$ }. From this example, we verified $A = A_1 + E_1$ and $A = A_2 \cdot E_2$ and conclude that $\mu_{A_2}(x) = 10\mu_A(x)$, $\nu_{A_1}(x) = 10\nu_A(x)$, $\mu_{A_1}(x) = \mu_A(x)$ and $\nu_{A_2}(x) = \nu_A(x)$.

3.5 Theorem

Let $A \in$ OHIFS(X). Then
(i) $G_{\alpha,1}(G_{\alpha,\beta}(A)) = G_{\alpha^2,\beta}(A) = G_{\alpha,\beta}(A_2) \cdot G_{\alpha,\beta}(E_2)$
(ii) $G_{\alpha,\beta^2}(A) = G_{\alpha,\beta}(A_1) + G_{\alpha,\beta}(E_1)$

Proof

From the definition, clearly $G_{\alpha,1}(G_{\alpha,\beta}(A)) = G_{\alpha^2,\beta}(A)$ and

$$G_{\alpha,\beta}(A_2) \cdot G_{\alpha,\beta}(E_2) = \{< x, \alpha\mu_{A_2}(x), \beta\nu_{A_2}(x) > | x \in X\} \cdot \{< x, \quad 0.\alpha, 0 > | x \in X\})$$
$$= \{< x, 10\alpha\mu_A(x), \beta\nu_A(x) > | x \in X\} \cdot \{< x, 0.\alpha, 0 > | x \in X\}$$
$$= \{< x, \alpha^2\mu_A(x), \beta\nu_A(x) > | x \in X\}$$
$$= G_{\alpha^2,\beta}(A).$$

Similarly, we can prove $G_{\alpha,\beta^2}(A) = G_{\alpha,\beta}(A_1) + G_{\alpha,\beta}(E_1)$ □

3.6 Theorem

Let $A \in$ OHIFS(X). Then
(i) $\square A = \square A_2 \cdot \square E_2 = \square A_1 + \square E_1$
(ii) $\Diamond A = \Diamond A_2 \cdot \Diamond E_2 = \Diamond A_1 + \Diamond E_1$

Proof.

From the definition $\square A = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$, we have $\square A_2 = \{< x, 10\mu_A(x), 1 - 10\mu_A(x) > | x \in X\}$ and $\square E_2 = \{< x, 0.1, 1 - 0.1 > | x \in X\}$
$= \{< x, 0.1, 0.9 > | x \in X\}$.

So $\square A_2 \cdot \square E_2 = \{< x, \mu_A(x), 1 - 10\mu_A(x) + 0.9 - 0.9(1 - 10\mu_A(x)) > | x \in X\}$
$= \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$
$= \square A$ and is also equal to $\square A_1 + \square E_1$

Similarly, we can prove $\Diamond A = \Diamond A_2 \cdot \Diamond E_2 = \Diamond A_1 + \Diamond E_1$ □

3.7 Theorem

Let A, B $\in$ OHIFS(X). Then
(i) $(A_2 \times_1 B_2) \cdot (G_{\frac{1}{10},1}(E_2 \times_1 E^*)) = A \times_1 B$
(ii) $(A_1 \times_1 B_1) + (G_{1,\frac{1}{10}}(E_1 \times_1 O^*)) = A \times_1 B$

Proof

From the definition $A \times_1 B = \{ < x, y >, \mu_A(x)\mu_B(y), \nu_A(x)\nu_B(y) > | x, y \in X \}$, we have $A_2 \times_1 B_2 = \{ < x, y >, 10\mu_A(x)10\mu_B(y), \nu_A(x)\nu_B(y) > | x, y \in X \}$
$= \{ < x, y >, 100\mu_A(x)\mu_B(y), \nu_A(x)\nu_B(y) > | x, y \in X \}$ and

$G_{\frac{1}{10},1}(E_2 \times_1 E^*) = G_{\frac{1}{10},1}\{<x,y>, 0.1, 0>|\, x,y \in X\}$

$= \{<x, 0.01, 0>|\, x, y \in X\}$. Therefore,

$(A_2 \times_1 B_2) \cdot (G_{\frac{1}{10},1}((E_2 \times_1 E^*)) = \{<x, y>, \mu_A(x)\mu_B(y), \nu_A(x)\nu_B(y) >|\, x, y \in X\}$

$= A \times_1 B.$

Similarly, we can prove $(A_1 \times_1 B_1) + (G_{\frac{1}{10},1}(E_1 \times_1 O^*)) = A \times_1 B$ □

3.8 Theorem.

Let A ∈ OHIFS(X). Then **(i)** $R(A_1) + R(E_1) \subset R(A)$

(ii) $R(A_2) \cdot R(E_2) \subset R(A)$

Proof

From the definition $R(A) = \{<x, \mu_A(x)(1-\nu_A(x)), \nu_A(x)(1-\mu_A(x))>|\, x \in X\}$ (1)

we have $R(A_1) = \{<x, \mu_A(x)(1-10\nu_A(x)), 10\nu_A(x)(1-\mu_A(x))>|\, x \in X\}$,

$R(A_2) = \{<x, 10\mu_A(x)(1-\nu_A(x)), \nu_A(x)(1-10\mu_A(x))>|\, x \in X\}$,

$R(E_1) = \{<x, 0, \frac{1}{10})>|\, x \in X\}$ and $R(E_2) = \{<x, \frac{1}{10}, 0)>|\, x \in X\}$.Therefore

$R(A_2) \cdot R(E_2) = \{<x, \mu_A(x)(1-10\,\nu_A(x)), \frac{1}{10}10\nu_A(x)(1-\mu_A(x))>|\, x \in X\}$

$= \{<x, \mu_A(x)(1-10\,\nu_A(x)), \nu_A(x)(1-\mu_A(x))>|\, x \in X\}$ (2)

From **(1)** and **(2)**, we have $R(A_2) \cdot R(E_2) \subset R(A)$.

Similarly, we can prove **(i)**. □

3.9 Theorem

Let A ∈ OHIFS(X). Then

(i) $C(A) = C(A_2) \cdot C(E_2) = C(A_1) + C(E_1)$

(ii) $I(A) = I(A_2) \cdot I(E_2) = I(A_1) + I(E_1)$

(iii) $(!\,(A_2)) \cdot (!\,(E_2)) \subset !\,(A)$

(iv) $?(A_2) \subset (?\,(A_2)) + (?\,(E_2))$

Proof. From the definition, we have

$C(A_2) = \{<x, \max_x \mu_{A_2}(x), \min \nu_{A_2}(x)>|\, x \in X\}$

$= \{<x, \max_x(10\mu_A(x)), \min \nu_A(x)>|\, x \in X\}$

$= \{<x, 10\max_x \mu_A(x), \min \nu_A(x)>|\, x \in X\}$ and

$C(E_2) = \{<x, \frac{1}{10}, 0>|\, x \in X\}$.

Therefore, $C(A_2) \cdot C(E_2) = \{<x, \max_x \mu_A(x), \min \nu_A(x)>|\, x \in X\}$

$= C(A)$

Similarly, we can prove $C(A_1) + C(E_1) = C(A)$.

(ii) can be proved easily as similar to that of (i).

We know $!\,(A) = \{<x, \max_x(\frac{1}{2}, \mu_A(x)), \min(\frac{1}{2}, \nu_A(x))>|\, x \in X\}$ (3)

$= \{<x, \frac{1}{2}, \nu_A(x)>|\, x \in X\}$,

$!\,(A_2) = \{<x, \max_x(\frac{1}{2}, 10\mu_A(x)), \min(\frac{1}{2}, \nu_A(x))>|\, x \in X\}$ and

$!(E_2) = \{< x, \max(\frac{1}{2}, 0.1), \min(\frac{1}{2}, 0) > | x \in X\} = \{< x, \frac{1}{2}, 0) > | x \in X\}$.

Therefore, $(!(A_2)) \cdot (!(E_2)) = \{< x, \frac{1}{2}\max(\frac{1}{2}, 10\mu_A(x)), \min(\frac{1}{2}, \nu_A(x)) > | x \in X\}$

$$= \{< x, \max(\frac{1}{4}, 5\mu_A(x)), \nu_A(x)) > | x \in X\} \quad (4)$$

From (3) and (4), we have $(!(A_2)) \cdot (!(E_2)) \subset !(A)$.

Proof of (iv) is similar to that (iii) □

3.10 Theorem

Let A, B ∈ OHIFS(X). Then

(i) $G_{\alpha,\beta^2}(A@B) = G_{\alpha,\beta}(A_1@B_1) + G_{\alpha,\beta}(E_1)$

(ii) $G_{\alpha,\beta^2}(A@B) = G_{\alpha,\beta}(A_2@B_2) \cdot G_{\alpha,\beta}(E_2)$ where $\alpha, \beta \in [0, 1]$

Proof.

Under the usual notation, we know $A = A_2 \cdot E_2$ and $B = B_2 \cdot E_2$.

Since $\mu_{A_2}(x) = 10\mu_A(x)$ and $\nu_{A_2}(x) = \nu_A(x)$,

we have $G_{\alpha,\beta}(A_2@B_2) = G_{\alpha,\beta}(\{< x, \frac{\mu_{A_2}(x) + \mu_{B_2}(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\})$

$$= G_{\alpha,\beta}(\{< x, \frac{10\mu_A(x) + 10\mu_B(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\})$$

$$= G_{\alpha,\beta}(\{< x, 5(\mu_A(x) + \mu_B(x)), \frac{\nu_A(x) + \nu_B(x)}{2} > | x \in X\})$$

$$= \{< x, 5\alpha(\mu_A(x) + \mu_B(x)), \beta(\frac{\nu_A(x) + \nu_B(x)}{2}) > | x \in X\} \quad \text{and}$$

$G_{\alpha,\beta}(E_2) = \{< x, 0.\alpha, 0) > | x \in X\}$ which implies

$G_{\alpha,\beta}(A_2@B_2) \cdot G_{\alpha,\beta}(E_2) = \{< x, \alpha^2(\frac{\mu_A(x) + \mu_B(x)}{2}), \beta(\frac{\nu_A(x) + \nu_B(x)}{2}) > | x \in X\}$

$= G_{\alpha,\beta^2}(A@B)$. This proves (i).

Similarly, we can prove (ii). □

3.11 Theorem

Let A, B ∈ OHIFS(X). Then

(i) $(A \cup B) + C = (A + B) \cup (A + C)$

(ii) $A \cap B = (A_2 \cap B_2) \cdot E_2 = (A_1 \cap B_1) + E_1$

(iii) $A \cup B = (A_2 \cup B_2) \cdot E_2 = (A_1 \cup B_1) + E_1$

Proof. From the definition, we can prove (i) easily. We know

$A \cap B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

$A_2 \cap B_2 = \{< x, \min(\mu_{A_2}(x), \mu_{B_2}(x)), \max(\nu_{A_2}(x), \nu_{B_2}(x)) > | x \in X\}$

$= \{< x, \min(10\mu_A(x), 10\mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$ and

$E_2 = \{< x, 0.1, 0 > | x \in X\}$. Therefore

$(A_2 \cap B_2) \cdot E_2 = \{< x, 0.1 \min(10\mu_A(x), 10\mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

$= \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

$= A \cap B$. Similarly, we can prove $(A_1 \cap B_1) + E_1 = A \cap B$

This proves **(ii)** and **(iii)** can be proved easily. □

3.12 Theorem
Let A, B ∈ OHIFS(X). Then
(i) $(A_1 @ A_1) + (E_1 @ E_1) = A$
(ii) $(A_2 \cap A_2) \cdot (E_2 \cap E_2) = A$
(iii) $(A_1 \cup A_1) + (E_1 \cup E_1) = A$
If we replace @ by $ and # then the result (i) is true.

Proof.
Since $A_1 @ A_1 = A_1$ and $E_1 @ E_1 = E$, we have $(A_1 @ A_1) + (E_1 @ E_1)$ $= A_1 + E_1 = A$. Similarly, (ii) and (iii) can be proved. □

3.13 Theorem
Let A, B ∈ OHIFS(X). Then
(i) $A_2 \times_1 B_2 = G_{100,1}(A \times_1 B)$
(ii) $A + B \subset (A + B) \cdot E_2$
(iii) $\square(A_2 \cdot B_2) \cdot \square E_2 \subset \square(A \cdot B)$
(iv) $\Diamond(A_1 + B_1) + \Diamond E_1 \subset \Diamond(A + B)$

Proof
From the definition,

$$A_2 \times_1 B_2 = \{ < x, y >, 10\mu_A(x)10\mu_B(y), \nu_A(x)\nu_B(y) > \mid x, y \in X \}$$
$$= \{ < x, y >, 100\mu_A(x)\mu_B(y), \nu_A(x)\nu_B(y) > \mid x, y \in X \}$$
$$= G_{100,1}(A \times_1 B)$$

(ii) can be proved easily.
By applying the definition,

$$\square(A \cdot B) = \{ < x, \mu_A(x)\mu_B(x), 1-\mu_A(x)\mu_B(x) > \mid x \in X\} \quad (5)$$

$\square(A_2 \cdot B_2) = \{ < x, 100\mu_A(x)\mu_B(x), 1-100\mu_A(x)\mu_B(x) > \mid x \in X\}$ and
$\square E_2 = \{< x, \frac{1}{10}, 1 - \frac{1}{10} > \mid x \in X\} = \{< x, \frac{1}{10}, \frac{9}{10} > \mid x \in X\}$, we have

$$\square(A_2 \cdot B_2) \cdot \square E_2 = \{< x, 10\mu_A(x)\mu_B(x), \frac{9}{10} + (1 - 100\mu_A(x)\mu_B(x)) - \frac{9}{10}(1-100\mu_A(x)\mu_B(x)) > \mid x \in X \}$$
$$= \{ < x, 10\mu_A(x)\mu_B(x), 1-10\mu_A(x)\mu_B(x) > \mid x \in X \} \quad (6)$$

From the above equations (5) and (6), we have $\square(A_2 \cdot B_2) \cdot \square E_2 \subset \square(A \cdot B)$
Since $\Diamond(A + B) = \{ < x, 1 - \nu_A(x)\nu_B(x), \nu_A(x)\nu_B(x) > \mid x \in X \}$ (7)
$\Diamond(A_1 + B_1) = \{< x, 1 - 100\,\nu_A(x)\nu_B(x), 100\,\nu_A(x)\nu_B(x) > \mid x \in X \}$ and
$\Diamond E_1 = \{< x, \frac{9}{10}, \frac{1}{10} > \mid x \in X\}$, we have

$$\Diamond(A_1 + B_1) + \Diamond E_1 = \{ < x, 1 - 10\nu_A(x)\nu_B(x), 10\,\nu_A(x)\nu_B(x) > \mid x \in X \} \quad (8)$$

From (7) and (8), we have $\Diamond(A_1 + B_1) + \Diamond E_1 \subset \Diamond(A + B)$ □

4 CONCLUSION

In this paper, we have introduced new type of intuitionistic fuzzy sets namely OHIFSs. Some theorems related to set relation and set operators in OHIFSs are proved. By using the usual operator " + " and " · ", some results in IFSs are discussed in OHIFSs also and we observe that most of the results in IFSs are valid in OHIFSs but some results in IFSs are not true in OHIFSs.

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