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Some Properties and Some Example of Series

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ABSTRACT

The aim of this is to chek convergence and divergence of series. In this papers, we will take theorem and some examples of series and chek out convergence and divergence of series.

Keywords: s_n,<s_n>, $\sum v_n$, ,convergence,divergence.

INTRODUCTION

This theorem is not beneficial for those series for which $v_n \to 0$,but very beneficial for those series where v_n does not tends to zero. In this paper, we discuss the some properties of series and some examples.

Notation

We will use s_n for parital sum of series and <s_n> for sequence of partial sum of series.

Some definition

1. A sequence in a set B is a special function whose domain is the set of natural number s, and whose range is contained in the set B .
2. If <v_n> is a sequence of real numbers, then expression $v_1+v_2+v_3+$………………..$v_{n-1}+v_n+$…………………
   (i.e. the sum of terms of sequence ,which are infinite in number) is called an infinite series.
3. The nth partial sum of infinite series is sum of first n terms .
   $s_1,s_2,s_3,$…………………..,are first, second, third….. partial sum of series.
   <s_n> is called sequence of partial sum of infinite series.
4. A sequence <v_n> is said to converge to a real no. l if for given €>0 ,however small it may be, there exist a positive integer m(depending upon €) such that $|v_n-l|<€$ for all n>=m
5. A series is convergent if sequence of partial sum <s_n> is convergent.

6. These results are hold for positive terms series.

Theorem (1): IF series $\sum_{n=1} v_n$ is converges,then that $\lim_{n\to\infty} v_n = 0$.
Is converse true?
Proof :
Let s_n denotes the partial sum of series $\sum v_n$ and $<s_n>$ denotes the sequence of partial sum.
$\sum v_n$ is convergent (given)
As we know that $\sum v_n$ is convergent if $<s_n>$ is convergent.
Therefore,

$< s_n >$ is convergent.

It implies that $\lim_{n\to\infty} s_n = s$ and $\lim_{n\to\infty} s_{n-1} = s$
Now,
$s_n = v_1 + v_2 + v_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_{n-1} + v_n$
$s_{n-1} = v_1 + v_2 + v_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_{n-1}$
Therefore,

$S_n - s_{n-1} = v_n$

Therefore,

$$\lim_{n\to\infty} v_n = \lim_{n\to\infty} s_n - \lim_{n\to\infty} s_{n-1}$$
$$= s - s$$
$$= 0$$

Hence $\sum v_n$ is convergent implies $\lim_{n\to\infty} v_n = 0$
The converse of above theorem is not always true.
i.e. $\lim_{n\to\infty} v_n = 0$ even if series $\sum v_n$ is convergent

Example 1
Consider the series $\sum v_n = \sum_{n=1}^{\infty} \frac{1}{n}$

Here $v_n = \frac{1}{n}$

$\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{1}{n} = 0$

Therefore, $\lim_{n\to\infty} v_n = 0$

Suppose that series $\sum v_n$ is also convergent

Therefore , by Cauchy General Principle of convergence ,
For given €>0, there exist a positive integer m such that $|s_n - s_m| <$ € for all $n>m$
Therefore,

$|v_{m+1} + v_{m+2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_n| <$ € for all $n>m$

Taking $n=2m$, we get

$|v_{m+1} + v_{m+2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_{2m}| <$ € for all $n>m$

$|1/m+1 + 1/m+2 + 1/m+3 + \ldots\ldots\ldots\ldots\ldots + 1/2m| <$ € (1)

$|1/m+1 + 1/m+2 + 1/m+3 + \ldots\ldots\ldots\ldots\ldots + 1/2m| > 1/2m + 1/2m + 1/2m + \ldots\ldots\ldots\ldots\ldots + 1/2m$
$= m/2m$
$= 1/2$

Taking €=1/2, we get

|1/m+1+1/m+2+1/m+3+……………+1/2m|>€ (2)

From (1) and(2) we get a contradiction

Hence series $\sum v_n$ is not convergent even if $\lim_{n\to\infty} v_n=0$

Example 2

$\sum v_n= \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$

is not convergent even if $\lim_{n\to\infty} v_n=0$

Solution $\sum v_n=\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$

Here $v_n= \frac{1}{n^{\frac{3}{4}}}$

$\lim_{n\to\infty} v_n= \lim_{n\to\infty} \frac{1}{n^{\frac{3}{4}}} =0$

But series $\sum v_n= \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$ is divergent by p-series test

Note 1 if $\lim_{n\to\infty} v_n\neq 0$ than series $\sum v_n$ is not convergent

Example 1

$\sum_{n=1}^{\infty} \frac{n^3+2}{n^3+5}$

Here $v_n=\frac{n^3+2}{n^3+5}$

$\lim_{n\to\infty} v_n= \lim_{n\to\infty} \frac{n^3+2}{n^3+5} =1\neq 0$

Therefore , $\sum_{n=1}^{\infty} \frac{n^3+2}{n^3+5}$ is divergent

Example 2

Here $v_n=\frac{4n^2}{4n^2+1}$

$\lim_{n\to\infty} v_n= \lim_{n\to\infty} \frac{4n^2}{4n^2+1} =4\neq 0$

Therefore $\sum_{n=1}^{\infty} \frac{4n^2}{4n^2+1}$ is divegent

Example 3

$\sum_{n=1}^{\infty} \frac{n}{2n+2}$

Here $v_n= \frac{n}{2n+2}$

$\lim_{n\to\infty} v_n= \lim_{n\to\infty} \frac{n}{2n+2}$

$=1/2\neq 0$

Therefore $\sum_{n=1}^{\infty} \frac{n}{2n+2}$ is divegent

CONCLUSION

1. $\sum V_n = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$ is divergent although $\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{1}{n^{\frac{3}{4}}} = 0$

2. if $\lim_{n\to\infty} v_n \neq 0$ then . $\sum V_n$ is divergent .

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