

# A Two-dimensional Problem in Fiber-reinforced Magnetoelastostatics under a Mechanical Load

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## ABSTRACT

The present manuscript is aimed at studying the propagation of plane waves in a fiber-reinforced, anisotropic, thermoelastic half-space with magnetic effects. The formulation is applied to generalized thermoelasticity based on the Green-Lindsay (G-L) theory. A mechanical load is applied on the surface of the half-space. The analytical expressions for the displacement components, stresses and temperature field are obtained in the physical domain by using normal mode analysis. Moreover, some particular cases of interest have been deduced from the present investigation.

**Keywords:** Fiber-Reinforced; Magnetoelastostatics; Normal mode analysis; Relaxation times; Mechanical Load; Green-Lindsay theory.

## INTRODUCTION

Fiber-reinforced composites are widely used in a variety of structures due to their superiority to structural materials in applications requiring high strength and stiffness in lightweight components. The mechanical behaviour of many fiber-reinforced composite materials is of particular importance for structural design using these materials. To explain the mechanical properties of such materials, a continuum model is used. It is usual to assume transverse isotropy in case of an elastic solid reinforced by a series of parallel fibers. Fiber-reinforced materials have many applications in automotive fields and aerospace as well as in shipbuilding and mainly in modern bicycles and motorcycles, where their high strength to weight ratio is of importance. With the modern technology, the improved techniques are reducing the costs and time to manufacture, making it common in small consumer goods such as laptops, paint ball equipment, racquet frames, classical guitar strings etc.

Hashin and Rosen<sup>1</sup> introduced the elastic moduli for fiber-reinforced materials. For the last four decades, the analysis of stress and deformation of fiber-reinforced composite materials has been an important subject of solid mechanics. Pipkin<sup>2</sup> and Rogers<sup>3</sup> did pioneer work on the subject. Belfield *et al.*<sup>4</sup> gave the idea of introducing continuous self-reinforcement at every point of an elastic solid. Chattopadhyay and Choudhury<sup>5</sup> investigated the propagation, reflection and transmission of magnetoelastic shear waves in self-reinforced media. The problem of surface waves in fiber-reinforced, anisotropic elastic media was discussed by Sengupta and Nath<sup>6</sup>. Abbas<sup>7</sup> studied the plane waves in a fiber-reinforced anisotropic thermoelastic half-space by employing the theory of thermoelasticity with energy dissipation. By using Green and Naghdi theory, Othman and Atwa<sup>8</sup> discussed the propagation of plane waves in a fiber-reinforced, anisotropic thermoelastic half-space. Lotfy<sup>9</sup> studied a two-dimensional problem of an electro-magneto-thermoelastic perfectly conducting solid half-space subjected to thermal shock in the purview of two-temperature coupled, L-S and G-L theories. By using normal mode analysis, Lotfy and Hassan<sup>10</sup> analyzed the propagation of thermoelastic waves in a two-temperature thermoelastic half-space subjected to thermal shock. Bayones<sup>11</sup> proposed an analytical procedure for evaluation of the displacement and stresses in fiber-reinforced anisotropic elastic media under the effect of rotation and initial magnetic field. Othman and Said<sup>12</sup> examined the wave propagation in a fiber-reinforced magneto-thermoelastic medium in the context of the three-phase-lag and Green-Naghdi theory without energy dissipation by applying normal mode analysis. Said and Othman<sup>13</sup> investigated the influence of initial stress and gravity field on the considered field variables in a fiber-reinforced thermoelastic medium under two-temperature three-phase-lag model. Under the effect of initial stress and gravity field, Said<sup>14</sup> studied the wave propagation in a fiber-reinforced thermoelastic medium with an internal heat source that is moving with a constant speed. Said and Othman<sup>15</sup> investigated the effect of mechanical force, rotation and moving internal heat source in a two-temperature fiber-reinforced thermoelastic medium under three-phase-lag model. Kalkal *et al.*<sup>16</sup> aimed at studying the propagation of plane waves in a fiber-reinforced, anisotropic, thermoelastic half-space with diffusion. The formulation is applied to generalized thermoelasticity based on Green-Lindsay (GL) theory. A thermal shock is applied on the surface of the half-space, which is taken to be traction free. The problem is solved using normal mode analysis.

Biot<sup>17</sup> formulated the coupled theory of thermoelasticity to deal with a defect of the uncoupled theory that mechanical changes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory in that it predicts infinite speed of propagation for heat signals contrary to physical observations. To overcome the shortcomings of the classical coupled dynamical theory of thermoelasticity, several theories of generalized thermoelasticity were developed in an attempt to amend the classical thermoelasticity. The first two generalized thermoelastic models are Lord-Shulman<sup>18</sup> (L-S) model and Green-Lindsay<sup>19</sup> (G-L) model. In L-S model, one thermal relaxation time parameter is introduced in the classical Fourier's law of heat conduction, whereas in the G-L model, two thermal relaxation times are introduced in the constitutive relations for force stress tensor and entropy equation.

In the current work, Green-Lindsay theory is applied to study the propagation of plane waves in a fiber-reinforced, anisotropic thermoelastic medium with magnetic effects. By employing normal mode analysis, exact solutions for displacement components, stresses and temperature field are obtained in the physical domain. The present study is motivated by the importance of fiber-reinforced thermoelasticity process in the field of defence marine, aerospace and medical sciences etc.

## BASIC EQUATIONS

The basic equations for a fiber-reinforced anisotropic half-space ( $x \geq 0, -\infty \leq y \leq \infty$ ), with magnetic effects under mechanical load in their context of G-L theory are as follows:

**The constitutive relation:**

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \beta_{ij} (\theta + \tau_1 \dot{\theta}) \delta_{ij}, \quad (1)$$

**The equation of motion:**

$$\rho \ddot{u}_i = \sigma_{ij,j} + F_i, \quad (2)$$

**Heat conduction equation:**

$$K_{ij} \theta_{,ij} = \rho c_E (\dot{\theta} + \tau_0 \ddot{\theta}) + T_0 \beta_{ij} \dot{u}_{i,j}, \quad (3)$$

where  $\sigma_{ij}$ 's are the components of stress,  $e_{ij}$ 's are the components of strain,  $\ddot{u}_i$  are components of displacement vector,  $\alpha$ ,  $\beta$  and  $(\mu_L - \mu_T)$  are reinforcement parameters,  $\lambda$  and  $\mu_T$  are elastic constants,  $\delta_{ij}$  is Kronecker delta,  $\theta = T - T_0$ ,  $T$  is absolute temperature,  $T_0$  is temperature of the medium in its natural state assumed to be  $|\frac{\theta}{T_0}| \ll 1$  and  $\mathbf{a} = (a_1, a_2, a_3)$ , where  $a_1^2 + a_2^2 + a_3^2 = 1$ . We choose the fiber-direction as  $\mathbf{a} = (1, 0, 0)$ .  $\rho$  is the mass density,  $\beta_{ij}$  is the thermal elastic coupling tensor,  $c_E$  is the specific heat at constant strain,  $K_{ij}$  is thermal conductivity,  $F_i$  are components of body force  $\tau_0$  and  $\tau_1$  are relaxation times. In the above equations, a comma denotes material derivative, dot indicates partial derivative with respect to time and the summation convention is used.

## FORMULATION OF THE PROBLEM

A model occupying a two-dimensional half-space, ( $x \geq 0, -\infty \leq y \leq \infty$ ), has been considered which is shown in fig. A. It is assumed that the considered model is made up of a fiber-reinforced transversely isotropic thermoelastic material and has been kept under magnetic field. As the problem is two-dimensional, therefore all the considered functions will depend on time  $t$  and co-ordinates  $x$  and  $y$ .

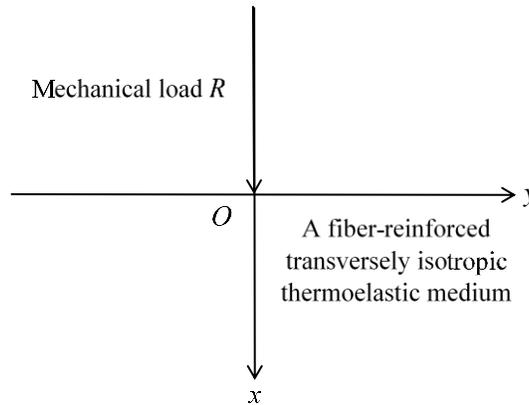


Figure A: Geometry of the problem

Taking into consideration expression (1), the non-vanishing stress components are reduced to the forms

$$\sigma_{xx} = A_{11} \frac{\partial u}{\partial x} + A_{12} \frac{\partial v}{\partial y} - \beta_{11} (\theta + \tau_1 \dot{\theta}), \quad (7)$$

$$\sigma_{yy} = A_{12} \frac{\partial u}{\partial x} + A_{14} \frac{\partial v}{\partial y} - \beta_{22} (\theta + \tau_1 \dot{\theta}), \quad (8)$$

$$\sigma_{xy} = A_{13} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (9)$$

where  $A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta$ ,  $A_{12} = \lambda + \alpha$ ,  $A_{13} = \mu_L$  and  $A_{14} = \lambda + 2\mu_T$ . The initial magnetic field vector  $B = B(0,0, B_0)$  is applied to the considered medium and the Maxwell's equations and the Lorentz force  $F$  are given by the expressions

$$\nabla \times h = J + \epsilon_0 \frac{\partial E}{\partial t}, \quad (10)$$

$$\nabla \times E = -\mu_0 \frac{\partial h}{\partial t}, \quad (11)$$

$$\nabla \cdot h = 0, \quad (12)$$

$$E = -\mu_0 \left( \frac{\partial u}{\partial t} \times B \right), \quad (13)$$

$$F = \mu_0 J \times B, \quad (14)$$

where  $E$  is electric field,  $h$  is induced magnetic field,  $J$  is current density vector.

With the aid of equations (10) – (14), the components of Lorentz force  $F$  are as

$$F = F \left( B_0^2 \mu_0 \frac{\partial e}{\partial x} - \epsilon_0 B_0^2 \mu_0^2 \frac{\partial^2 u}{\partial t^2}, B_0^2 \mu_0 \frac{\partial e}{\partial y} - \epsilon_0 B_0^2 \mu_0^2 \frac{\partial^2 v}{\partial t^2}, 0 \right). \quad (15)$$

Plugging the stresses defined in Eqs. (7) – (9) into (2) along with the body force defined in (15), the field equation converts to

$$\rho \frac{\partial^2 u}{\partial t^2} = A_{11} \frac{\partial^2 u}{\partial x^2} + A_{21} \frac{\partial^2 v}{\partial x \partial y} + A_{13} \frac{\partial^2 u}{\partial y^2} - \beta_{11} \frac{\partial}{\partial x} (\theta + \tau_1 \dot{\theta}) + F_1, \quad (16)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = A_{13} \frac{\partial^2 v}{\partial x^2} + A_{21} \frac{\partial^2 u}{\partial x \partial y} + A_{14} \frac{\partial^2 v}{\partial y^2} - \beta_{22} \frac{\partial}{\partial y} (\theta + \tau_1 \dot{\theta}) + F_2, \quad (17)$$

where  $A_{21} = A_{12} + A_{13}$ .

By using summation convention, the heat conduction Eq. (3) takes the form:

$$K_{11} \frac{\partial^2 \theta}{\partial x^2} + K_{22} \frac{\partial^2 \theta}{\partial y^2} = \rho c_E (\dot{\theta} + \tau_0 \ddot{\theta}) + T_0 \frac{\partial}{\partial t} \left( \beta_{11} \frac{\partial u}{\partial x} + \beta_{22} \frac{\partial v}{\partial y} \right), \quad (18)$$

It is convenient to have Eqs. (7) – (9) and (16) – (18) in dimensionless forms. To this end, we introduce the following set of dimensionless quantities:

$$(x', y', u', v') = c_0 \eta_0 (x, y, u, v), \quad (t', \tau_0', \tau_1') = c_0^2 \eta_0 (t, \tau_0, \tau_1), \quad \sigma'_{ij} = \frac{1}{\rho c_0^2} (\sigma_{ij}),$$

$$\theta' = \frac{\beta_{11}}{\rho c_0^2} \theta, \quad \text{where; } \eta_0 = \frac{\rho c_E}{K_{11}}, \quad c_0^2 = \frac{A_{11}}{\rho}. \quad (19)$$

Making use of dimensionless parameters given in (19) Eqs. (7) – (9) and (16) – (18) can be recast into the following forms (dropping the primes):

$$\sigma_{xx} = \frac{\partial u}{\partial x} + I_1 \frac{\partial v}{\partial y} - \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \theta, \quad (20)$$

$$\sigma_{yy} = I_1 \frac{\partial u}{\partial x} + I_2 \frac{\partial v}{\partial y} - I_4 \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \theta, \quad (21)$$

$$\sigma_{xy} = I_3 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (22)$$

$$I_5 \frac{\partial^2 u}{\partial t^2} = I_8 \frac{\partial^2 u}{\partial x^2} + I_9 \frac{\partial^2 v}{\partial x \partial y} + I_3 \frac{\partial^2 u}{\partial y^2} - \frac{\partial}{\partial x} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \theta, \quad (23)$$

$$I_5 \frac{\partial^2 v}{\partial t^2} = I_3 \frac{\partial^2 v}{\partial x^2} + I_9 \frac{\partial^2 u}{\partial x \partial y} + I_{10} \frac{\partial^2 v}{\partial y^2} - I_4 \frac{\partial}{\partial y} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \theta, \quad (24)$$

$$\frac{\partial^2 \theta}{\partial x^2} + I_{11} \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial}{\partial t} \left( 1 + \tau_0 \frac{\partial}{\partial t} \right) \theta + \frac{\partial}{\partial t} \left( I_{12} \frac{\partial u}{\partial x} + I_{13} \frac{\partial v}{\partial y} \right), \quad (25)$$

Where  $(I_1, I_2, I_3, I_7) = \frac{1}{A_{11}} (A_{12}, A_{14}, A_{13}, A_{21})$ ,  $I_4 = \frac{\beta_{22}}{\beta_{11}}$ ,  $I_5 = 1 + \frac{\epsilon_0 B_0^2 \mu_0^2}{\rho}$ ,  $I_6 = \frac{B_0^2 \mu_0}{A_{11}}$ ,  
 $I_8 = (1 + I_6)$ ,  $I_9 = (I_6 + I_7)$ ,  $I_{10} = (I_2 + I_6)$ ,  $I_{11} = \frac{K_{22}}{K_{11}}$ ,  $I_{12} = \frac{T_0 \beta_{11} \beta_{11}}{A_{11} K_{11} \eta_0}$  and  $I_{13} = \frac{T_0 \beta_{11} \beta_{22}}{A_{11} K_{11} \eta_0}$ .

## SOLUTION OF THE PROBLEM

In the current section, normal mode technique is adopted which gives exact solutions without any assumed restrictions on the temperature, displacement and stress distributions. The solutions of the physical variables can be decomposed in terms of normal modes in the following form:

$$[u, v, \theta, \sigma_{ij}](x, y, t) = [u^*, v^*, \theta^*, \sigma^*_{ij}](x) e^{(\omega t + imy)}, \quad (26)$$

where  $\omega$  is the complex time constant (frequency),  $i$  is the imaginary unit,  $m$  is the wave number in the  $y$  – direction and  $u^*, v^*, \theta^*$  and  $\sigma^*_{ij}$  are the amplitudes of the corresponding functions.

By virtue of (26), Eqs. (23) – (25) transform to:

$$(I_8 D^2 - C_1) u^* + C_2 D v^* - C_3 D \theta^* = 0, \quad (27)$$

$$C_2 D u^* + (I_3 D^2 - C_4) v^* - C_5 \theta^* = 0, \quad (28)$$

$$C_6 D u^* + C_7 v^* - (D^2 - C_8) \theta^* = 0, \quad (29)$$

where  $C_1 = I_3 m^2 + I_5 \omega^2$ ,  $C_2 = im I_9$ ,  $C_3 = 1 + \tau_1 \omega$ ,  $C_4 = I_{10} m^2 + I_5 \omega^2$ ,  $C_5 = im I_4 C_3$ ,  
 $C_6 = \omega I_{12}$ ,  $C_7 = im \omega I_{13}$ ,  $C_8 = I_{11} m^2 + \omega(1 + \tau_0 \omega)$ .

Eliminating  $u^*, v^*$  and  $\theta^*$  from Eqs. (27) – (29), we obtain the following sixth-order differential equation:

$$[D^6 - PD^4 + QD^2 - R][u^*(x), v^*(x), \theta^*(x)] = 0, \tag{30}$$

Where  $P = \frac{Y_2}{Y_1}, Q = \frac{Y_3}{Y_1}, R = \frac{Y_4}{Y_1}, Y_1 = X_1I_3 + X_3C_2, Y_2 = X_1X_6 + I_3X_2 + I_5X_3 + X_4C_2, Y_3 = X_1X_7 + X_2X_6 + X_4X_5, Y_4 = X_2X_7, X_1 = C_5I_8 - C_2C_3, X_2 = C_5C_1, X_3 = C_3I_3, X_4 = C_5C_2 + C_4C_3, X_5 = C_2C_8 + C_5C_6, X_6 = C_8I_3 + C_4, X_7 = C_4C_8 - C_5C_7.$

Now, Eq. (30) can be factorized as

$$[(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)(D^2 - \lambda_3^2)][u^*(x), v^*(x), \theta^*(x)] = 0, \tag{31}$$

where  $\lambda_n^2 (n = 1, 2, 3)$  are the roots with positive real part, of the characteristic equation  $\lambda^6 - P\lambda^4 + Q\lambda^2 - R = 0.$

The solution of Eq. (30), which is bounded as  $x \rightarrow \infty$ , is given by

$$(u^*, v^*, \theta^*)(x) = \sum_{n=1}^3 (H_n, H'_n, H''_n)(m, \omega) e^{-\lambda_n x}, \tag{32}$$

where  $H_n, H'_n$  and  $H''_n$  are some parameters depending on  $m$  and  $\omega$  Using solutions (32) in Eqs. (27) – (29) we get the following relations:

$$(v^*, \theta^*)(x) = \sum_{n=1}^3 (N_{1n}, N_{2n})H_n(m, \omega) e^{-\lambda_n x}, \tag{33}$$

where  $N_{1n} = \frac{X_1\lambda_n^2 - X_2}{-\lambda_n(X_3\lambda_n^2 - X_4)}$  and  $N_{2n} = \frac{-\lambda_n C_2 + (I_3\lambda_n^2 - C_4)N_{1n}}{C_5}.$

Introducing the dimensionless parameters defined in (19) and invoking normal mode analysis defined in (26), we obtain the expressions for stresses as

$$(\sigma^*_{xx}, \sigma^*_{xy})(x) = \sum_{n=1}^3 (N_{3n}, N_{4n})H_n(m, \omega) e^{-\lambda_n x}, \tag{34}$$

where  $N_{3n} = -\lambda_n + imI_1N_{1n} - (1 + \tau_1\omega)N_{2n}, N_{4n} = -I_3(-\lambda_nN_{1n} + im).$

### BOUNDARY CONDITIONS

The model occupying a two-dimensional half-space, ( $x \geq 0, -\infty \leq y \leq \infty$ ), has been made up of a fiber-reinforced transversely isotropic thermoelastic material under magnetic field. The surface of the half-space i.e. the bounding plane  $x = 0$  is subjected to a normal mechanical load  $R(R, 0, 0)$  as shown in fig. A, where  $R$  is the intensity of normal load acting in the positive  $x$  direction. Mathematically, these boundary conditions can be written as:

$$\theta(0, y, t) = 0, \tag{35}$$

$$\sigma_{xx}(0, y, t) + \bar{\sigma}_{xx}(0, y, t) = -R, \tag{36}$$

$$\sigma_{xy}(0, y, t) + \bar{\sigma}_{xy}(0, y, t) = 0, \tag{37}$$

where  $\bar{\sigma}_{xj}$  is Maxwell stress defined as  $\bar{\sigma}_{xj} = \mu_0 [B_x h_j + B_j h_x - h_k B_k \delta_{xj}].$

The boundary conditions (35) – (37), with the help of expressions (33) and (34), yield a non-homogeneous system of three equations, which can be written in matrix form as

$$\begin{bmatrix} N_{21} & N_{22} & N_{23} \\ N_{51} & N_{52} & N_{53} \\ N_{41} & N_{42} & N_{43} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -R^* \\ 0 \end{bmatrix} \tag{38}$$

where  $N_{5n} = N_{3n} + B_0^2 \mu_0 (-\lambda_n + imN_{1n}), n = (1, 2, 3).$

Finally, the expressions of  $H_n$  ( $n = 1, 2, 3$ ) obtained by solving system (38), when substituted in (32) – (34), provide us the required expressions of displacement components, temperature and stresses as

$$u^*(x) = \frac{1}{\Delta} [\Delta_1 e^{-\lambda_1 x} + \Delta_2 e^{-\lambda_2 x} + \Delta_3 e^{-\lambda_3 x}], \tag{39}$$

$$v^*(x) = \frac{1}{\Delta} [\Delta_1 N_{11} e^{-\lambda_1 x} + \Delta_2 N_{12} e^{-\lambda_2 x} + \Delta_3 N_{13} e^{-\lambda_3 x}], \tag{40}$$

$$\theta^*(x) = \frac{1}{\Delta} [\Delta_1 N_{21} e^{-\lambda_1 x} + \Delta_2 N_{22} e^{-\lambda_2 x} + \Delta_3 N_{23} e^{-\lambda_3 x}], \tag{41}$$

$$\sigma^*_{xx}(x) = \frac{1}{\Delta} [\Delta_1 N_{31} e^{-\lambda_1 x} + \Delta_2 N_{32} e^{-\lambda_2 x} + \Delta_3 N_{33} e^{-\lambda_3 x}], \tag{42}$$

$$\sigma^*_{xy}(x) = \frac{1}{\Delta} [\Delta_1 N_{41} e^{-\lambda_1 x} + \Delta_2 N_{42} e^{-\lambda_2 x} + \Delta_3 N_{43} e^{-\lambda_3 x}], \tag{43}$$

where  $\Delta = N_{31} d_1 - N_{32} d_2 + N_{33} d_3$ ,  $\Delta_1 = -R^* d_1$ ,  $\Delta_2 = R^* d_2$ ,  $\Delta_3 = -R^* d_3$ ,  
 $d_1 = N_{22} N_{43} - N_{23} N_{42}$ ,  $d_2 = N_{21} N_{43} - N_{23} N_{41}$ ,  $d_3 = N_{21} N_{42} - N_{22} N_{41}$ .

**DISCUSSION ON FIELD EQUATIONS TO ELABORATE PARTICULAR CASES**

**1. Without fiber-reinforcement:**

To discuss the problem of wave propagation in a isotropic thermoelastic medium in the presence of magnetic field under G–L theory, it is sufficient to set the values of  $\alpha, \beta$  and  $\mu_L - \mu_T$  as  $\alpha = \beta = 0$  and  $\mu_L = \mu_T$ . With these modifications, the corresponding expressions for displacements, stresses and temperature field can be obtained from expressions (39) – (43).

**2. Neglecting the magnetic effect:**

To discuss the problem of wave propagation in a transversely isotropic thermoelastic medium in the absence of magnetic field under G–L theory, it is sufficient to set the value  $B_0$  as  $B_0 = 0$ , for a mechanical load applied on the isothermal boundary  $x = 0$ .

**3. Neglecting the mechanical load:**

To discuss the problem of wave propagation in a thermoelastic medium in the presence of magnetic field with traction free isothermal boundary  $x = 0$ , under G–L theory, it is sufficient to set the value of  $R$  as  $R = 0$ .

**CONCLUSIONS**

The main purpose of present study is to introduce a new mathematical model for a fiber-reinforced transversely isotropic thermoelastic medium in the presence of magnetic field. In this study, the method of normal mode analysis is adopted which proves to be a quite productive approach to handle such type of problems. This technique gives exact solutions without any pre-assumed restrictions on the actual physical field quantities existing in the governing field equations of the considered problem. The theoretical and numerical outcomes unveil that all the considered parameters have significant effects on the field variables. The

current research has important practical usefulness. Since valuable organic and inorganic deposits beneath the earth's surface are difficult to detect by drilling randomly, normal mode technique is one of the best technique to solve these type of problems. We obtain the following conclusions based on the above analysis:

- The normal displacement, normal stress, tangential stress and temperature field are highly influenced by reinforcement parameters, while negligible effects are observed on the temperature field. In the presence of reinforcement, magnitude of normal stress increases.
- Significant impact of magnetic field is observed on all the physical fields. It has both increasing and decreasing effects on their magnitudes. This is due to the influences of cross-effects arising from the coupling of the field variables.
- All the physical quantities satisfy the boundary conditions.

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