Journal of Computer and Mathematical Sciences, *Vol.9(12),1866-1874 December 2018* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

On Estimating Reliability of Systems in Stress-strength Set Up for Transmuted Inverted Exponential Distribution

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(Received on: November 29, 2018)

ABSTRACT

The problem of estimation of reliability of systems in stress-strength set up is an important area of research in Statistics, particularly, in Statistical Inference on reliability. The present paper deals with the estimation of reliability of stress-strength model when the strength and the stress variables follow transmuted inverted exponential distribution. The maximum likelihood estimator(MLE) and moment estimators for the reliability of stress-strength model are derived. Also, asymptotic confidence interval for reliability based on MLE is constructed. The moment estimators and MLEs are compared using mean squared error(MSE) criteria. An extensive simulation study is conducted to estimate MSEs of the estimators derived here. The real data analysis is considered.

Keywords: Stress-strength model, Reliability, Maximum likelihood estimator, moment estimator.

1. INTRODUCTION

Exponential distribution possesses constant failure rate. This makes exponential distribution unsuitable for real life problems with bathtub failure rates (Singh *et al*. (2013) for details). Hence, there is a need to generalize the exponential distribution in order to make model suitable for such real life problems. Keller and Kamath (1982) introduced one parameter inverse exponential distribution which has inverted bathtub failure rate. Further Lin *et al*. (1989) studied this as a lifetime model.

Let X denotes a random variable, which has Inverse Exponential(IE) distribution with a scale parameter α. The cumulative density function(cdf) and the probability density function(pdf)are respectively given by

$F(x) = exp\left(-\frac{\alpha}{x}\right),\ x > 0, \alpha > 0$

$f(x) = \frac{\alpha}{x^2} exp\left(-\frac{\alpha}{x}\right), x > 0, \alpha > 0.$

Shaw and Buckley (2007) have introduced an interesting method of adding a new parameter to an existing distribution using quadratic rank transmutation map(QRTM) that would offer more distributional flexibility. The generated distribution is also known as transmuted distribution thatincludes the parent distribution as a special case and gives more flexibility to model various types of data.

There has been a growing interest in transmuted distributions because of its flexibility. Aryal and Tsokos (2009) defined the transmuted generalized extreme value distribution and also studied the some basic mathematical characteristics of transmuted Gumbel distribution. Aryal and Tsokos (2011) defined the transmuted Weibull distribution. On the similar lines, various transmuted distributions such as transmuted Rayleigh, transmuted exponentiated modified Weibull, transmuted inverse Rayleigh and transmuted pareto are respectively studied by Merovci (2013), Ashour and Eltehiwy (2013), Ahmad A. *et al*. (2014), Merovci and Puka (2014). Recently, the transmuted inverse exponential distribution is studied by Oguntunde and Adejumo (2015) and they have shown the flexibility of the transmuted inverse exponential distribution (Oguntunde *et al*. (2017)).

Suppose we have an arbitrary parent cumulative distribution function $G(x)$, a random variable X is said to have a transmuted distribution if its cdf is given by

$$F(x) = (1+\lambda)G(x) - \lambda[G\ (x)\ ]^2,\ |\lambda| \le 1 \tag{1}$$

and corresponding probability density function is

$$f(x) = g(x)[1+\lambda - 2\lambda G(x)],\ |\lambda| \le 1 \tag{2}$$

where g(x) and f(x) are the associated pdf of G(x) and F(x) respectively.

It is to be noted that if$\lambda = 0$; equation (1) and (2) reduce to cdf and pdf of parent distribution respectively.

Hence, a random variable X is said to have a transmuted inverse exponential (TIE) distribution with parameters α and λ if the cumulative distribution function is given by

$$F(x) = \exp\left(-\frac{\alpha}{x}\right)\left[1+\lambda-\lambda\exp\left(-\frac{\alpha}{x}\right)\right], x>0, \alpha>0 \text{ and } |\lambda| \le 1. \tag{3}$$

The corresponding pdf is given by

$$f(x) = \frac{\alpha}{x^2}\exp\left(-\frac{\alpha}{x}\right)\left[1+\lambda-2\lambda\exp\left(-\frac{\alpha}{x}\right)\right], x>0, \alpha>0 \text{ and } |\lambda| \le 1. \tag{4}$$

Here, α and λ are the scale and transmuted parameters respectively.For $\lambda = 0$, equation (4) reduces to one parameter IE distribution. Some properties of TIE distribution are

i. The r^th^moment of a random variable X, μ_r is given by

$\mu_r = \alpha^r \Gamma(1-r)\left[(1+\lambda) - \lambda 2^r\right]$, only exists when r<1.

ii. The moment generating function of a TIE distribution as follows;

$$M_X(t) = (1+ \text{ª}) \sum_{k=0}^{\infty} \frac{(\sim t)^k}{k!} \Gamma(1+k) - \text{ª} \sum_{k=0}^{\infty} \frac{(\sim t)^k}{k!} 2^k \Gamma(1-k).$$

In this paper, the problem of estimating the stress-strength reliability (R) is studied when X,the strength and Y, the stress are independent TIE random variables. In the stress-strength setup, $R = P(X > Y)$ is the reliability of the system where Y is the stress and X is the strength of the system to overcome the possible stress. It is to be noted that the estimation of R is of greater interest in the literature. However, in the literature the problem of $P(X > Y)$ is addressed by many researchers when X and Y have different distributions. Estimation of $P(X > Y)$ with non-identical component strength is studied by Paul, R. and Borhan Uddin, M.D. (1997). In the similar manner, Rezaei, S *et al*. (2010), Rao, G.S. *et al*. (2013), Singh *et al*. (2014) and Singh *et al*. (2015) have studied the estimation of $P(X > Y)$ for generalized Pareto distribution,inverse Rayleigh distribution, Generalized Lindley distribution and Inverted exponential distribution respectively.

The present paper is organized as below. In section 2, maximum likelihood estimator (MLE) and estimator based on method of moments of R are studied. Also asymptotic distribution and confidence interval for R based on MLE are given in section 2.
Section 3 is devoted to analysis of real data. In section 4, simulation studies are conducted for comparison of different estimators of R. Section 5 gives conclusions.

2. ESTIMATION OF STRESS-STRENGTH RELIABILITY

Suppose that X and Y are independently distributed random variables with $TIE(\alpha, \lambda_1)$and $TIE(\beta, \lambda_2)$ respectively.Then the stress-strength reliability is

$$R = P(X > Y)$$

$$R = \int_0^{\infty} \left\{ \int_0^{x} f(y; \text{ª}_2, \ ) dy \right\} f(x; \text{ª}_1, \sim) dx$$

$$R = \frac{v^2[(1-\text{ª}_1)(2+\lambda_2)] + v[\lambda_1 \text{ª}_2 - \lambda_1 + 2\text{ª}_2 + 5] + 2}{(v+1)(v+2)(v+3)}, \text{ where } v = \frac{\ }{\sim}. \quad (5)$$

Now, the parametersα, λ_1, β and λ_2 are estimated as R is function of $\alpha, \lambda_1, \beta$and λ_2. These parameters are estimated by using method of maximum likelihood and method of moments.

2.1. Maximum likelihood estimation

Let $\underline{X} = (X_1, X_2, ..., X_n)$ and $\underline{Y} = (Y_1, Y_2, ..., Y_m)$ be independent random samples from $TIE(\alpha, \lambda_1)$ and $TIE(\beta, \lambda_2)$ respectively. Then the likelihood function given$(\underline{x}, \underline{y})$ isgiven by

$$L(\sim, \text{ª}_1, \ , \text{ª}_2 \mid \underline{x}; \underline{y}) = \prod_{i=1}^{n} f(x_i, \sim, \text{ª}_1) \times \prod_{j=1}^{m} f(y_j, \ , \text{ª}_2)$$

Then, the log-likelihood function is given by

$$\ell = n\log\alpha - \sum_{i=1}^{n}\log x_i^2 - \alpha\sum_{i=1}^{n}\left(\frac{1}{x_i}\right) + \sum_{i=1}^{n}\log\left[1+\lambda_1 - 2\lambda_1 e^{-\frac{\alpha}{x}}\right] + m\log\beta - \sum_{j=1}^{m}\log y_j^2 - \beta\sum_{j=1}^{m}\left(\frac{1}{y_j}\right) +$$
$$\sum_{j=1}^{m}\log\left[1+\lambda_2 - 2\lambda_2 e^{-\frac{\beta}{y}}\right]. \tag{6}$$

The likelihood equations to estimate α, λ_1, β and λ_2 are

$$\frac{d\ell}{d\alpha} = \frac{n}{\alpha} - \frac{1}{\alpha}\sum_{i=1}^{n}\left(\frac{\alpha}{x_i}\right) + 2\lambda_1\sum_{i=1}^{n}\left[\frac{\exp\left(-\frac{\alpha}{x_i}\right)\left(\frac{1}{x_i}\right)}{1+\lambda_1-2\lambda_1\exp\left(-\frac{\alpha}{x_i}\right)}\right] \tag{7}$$

$$\frac{d\ell}{d\lambda_1} = \sum_{i=1}^{n}\left[\frac{1-2\exp\left(-\frac{\alpha}{x_i}\right)}{1+\lambda_1-2\lambda_1\exp\left(-\frac{\alpha}{x_i}\right)}\right] \tag{8}$$

$$\frac{d\ell}{d\beta} = \frac{m}{\beta} - \frac{1}{\beta}\sum_{j=1}^{m}\left(\frac{\beta}{y_j}\right) + 2\lambda_2\sum_{j=1}^{m}\left[\frac{\exp\left(-\frac{\beta}{y_j}\right)\left(\frac{1}{y_j}\right)}{1+\lambda_2-2\lambda_2\exp\left(-\frac{\beta}{y_j}\right)}\right] \tag{9}$$

and

$$\frac{d\ell}{d\lambda_2} = \sum_{j=1}^{m}\left[\frac{1-2\exp\left(-\frac{\beta}{y_j}\right)}{1+\lambda_2-2\lambda_2\exp\left(-\frac{\beta}{y_j}\right)}\right]. \tag{11}$$

Solving the nonlinear system of equations $\frac{d\ell}{d\alpha}=0$, $\frac{d\ell}{d\lambda_1}=0$, $\frac{d\ell}{d\beta}=0$ and $\frac{d\ell}{d\lambda_2}=0$ we get maximum likelihood estimates of α, λ_1, β and λ_2.

The asymptotic distribution of the maximum likelihood estimators of $\hat{\underline{\theta}} = \left(\hat{\alpha}, \hat{\lambda}_1, \hat{\beta}, \hat{\lambda}_2\right)'$ is multivariate normal $N_P\left(\hat{\underline{\theta}}, I^{-1}\left(\hat{\underline{\theta}}\right)\right)$, where $I^{-1}\left(\hat{\theta}\right)$ is the observed information matrix evaluated at $\hat{\underline{\theta}}$. That is, $\sqrt{n}\left(\hat{\underline{\theta}}-\underline{\theta}\right)\to N_p\left(0, I^{-1}\left(\hat{\underline{\theta}}\right)\right)$, where $I\left(\hat{\underline{\theta}}\right)$ is the variance-covariance matrix of the unknown parameters $\underline{\theta}=\left(\alpha, \lambda_1, \beta, \lambda_2\right)'$

Approximate $100(1-\alpha)\%$ confidence intervals for α, λ_1, β and λ_2 are, respectively, given by $\left(\hat{\alpha}\pm Z_{\frac{\alpha}{2}}\sqrt{I_{11}^{-1}}\right)$, $\left(\hat{\lambda}_1\pm Z_{\frac{\alpha}{2}}\sqrt{I_{22}^{-1}}\right)$, $\left(\hat{\beta}\pm Z_{\frac{\alpha}{2}}\sqrt{I_{33}^{-1}}\right)$ and $\left(\hat{\lambda}_2\pm Z_{\frac{\alpha}{2}}\sqrt{I_{44}^{-1}}\right)$ where, $Z_{\frac{\alpha}{2}}$ is the upper $\frac{\alpha}{2}^{th}$ percentile of a standard normal distribution. Similarly, the asymptotic distribution of stress-strength reliability as $n, m\to\infty$ is given by

$$\frac{\hat{R}-R}{\sqrt{AV(\hat{R})}}\to N(0,1),$$

where,
$$AV(\hat{R})=\left[\frac{\partial R}{\partial\alpha}\right]^2 I_{11}^{-1}+2\left[\frac{\partial R}{\partial\alpha}\right]\left[\frac{\partial R}{\partial\lambda_1}\right]I_{12}^{-1}+2\left[\frac{\partial R}{\partial\beta}\right]\left[\frac{\partial R}{\partial\lambda_1}\right]I_{13}^{-1}+2\left[\frac{\partial R}{\partial\beta}\right]\left[\frac{\partial R}{\partial\lambda_2}\right]I_{24}^{-1}+$$
$$\left[\frac{\partial R}{\partial\lambda_1}\right]^2 I_{22}^{-1}+\left[\frac{\partial R}{\partial\beta}\right]^2 I_{33}^{-1}+\left[\frac{\partial R}{\partial\lambda_2}\right]^2 I_{44}^{-1}. \tag{12}$$

To construct the confidence interval of R we use the asymptotic distribution of $\hat{R}$. Thus, $100(1-\alpha)\%$ asymptotic confidence interval for the reliability R is obtained as

$$\hat{R}\pm Z_{\frac{\alpha}{2}}\sqrt{AV(R)} \tag{13}$$

2.2. Method of Moment Estimation (MME)

The method of moments estimation can be applied only when the moments exists for the underlined TIE distribution. However, the moments exists if, we consider transformed variables such as $Z_1=\frac{1}{X}$ and $Z_2=\frac{1}{Y}$.

The pdf of Z_1 and Z_2 respectively, are given by

$$f(z_1)=\alpha\exp\left(-\alpha z_1\right)\left[1+\lambda_1-2\lambda_1\exp(-\alpha z_1)\right], z_1>0, \alpha>0, -1\le\lambda_1\le 1. \tag{14}$$

$$f(z_2)=\beta\exp(-\beta z_2)\left[1+\lambda_2-2\lambda_2\exp(-\beta z_2)\right], z_2>0, \beta>0, -1<\lambda_2<1. \tag{15}$$

For the above distributions, moments exists and the first two moments of Z_1 and Z_2 are given by

$$E(Z_1)=\frac{2+\lambda_1}{2\alpha}, E\left(Z_1^2\right)=\frac{4+3\lambda_1}{2\alpha^2}, E(Z_2)=\frac{2+\lambda_2}{2\beta} \text{ and } E(Z_2^2)=\frac{4+3\lambda_2}{2\beta^2} \tag{16}$$

Then, the estimators of $\sim, \text{ª}_1,$ and ª_2 are obtained using moment equations. The moment estimators of $\sim, \text{ª}_1,$ and ª_2 are given by

$$\tilde{\sim} = \frac{3\bar{Z}_1 + \sqrt{9\bar{Z}_1^{\,2} - 4\frac{\sum_{i=1}^{n} Z_{1i}^2}{n}}}{2\frac{\sum_{i=1}^{n} Z_{1i}^2}{n}}, \quad \tilde{\text{ª}}_1 = 2\left(\tilde{\sim}\bar{Z}_1 - 1\right), \quad \tilde{\sim} = \frac{3\bar{Z}_2 + \sqrt{9\bar{Z}_2^{\,2} - 4\frac{\sum_{j=1}^{m} Z_{2j}^2}{m}}}{2\frac{\sum_{j=1}^{m} Z_{2j}^2}{m}} \text{ and}$$

$$\tilde{\text{ª}}_2 = 2\left(\tilde{\sim}\bar{Z}_2 - 1\right) \qquad (17)$$

2.3 Likelihood ratio test criterion

To check if the fit using the Transmuted G model is statistically superior to a fit using the G model, we test $H_0 : \text{ª} = 0$ against $H_1 : \text{ª} \neq 0$. Then, the likelihood ratio statistic is as follows;

$w = 2\left\{\ell\left(\hat{\text{ª}}, \hat{¶}\right) - \ell\left(0, \breve{¶}\right)\right\}$, where $\hat{\text{ª}}, \hat{¶}$ and $\breve{¶}$ are the unrestricted and restricted estimates obtained from the maximization of $\ell(\bar{\ })$ under H_1 and H_0. Reject H0, if $w > \text{¡}^2_{(k)}(1-\sim)$. Here, for the two data sets we test $H_0 : \text{ª}_1 = 0$ against $H_1 : \text{ª}_1 \neq 0$ and $H_0 : \text{ª}_2 = 0$ against $H_1 : \text{ª}_2 \neq 0$. The test statistic w_1 and w_2 are given below.

$w_1 = 2\left\{\ell\left(\hat{\text{ª}}_1, \hat{¶}\right) - \ell\left(0, \breve{¶}\right)\right\}$ and $w_2 = 2\left\{\ell\left(\hat{\text{ª}}_2, \hat{¶}\right) - \ell\left(0, \breve{¶}\right)\right\}$.

3. DATA ANALYSIS

Here, we consider the data set, which has initially used by Effron (1988). Further Pooja *et al*. (2014) and Singh et al (2015) are also used. The data represent the patients of two groups suffering from head and neck cancer disease. The data set of first group(X) represent the survival times of 51 head and neck cancer patients treated with radiotherapy whereas the other group(Y) of data set represent the survival times of 45 head and neck cancer patients treated with combined radiotherapy and chemotherapy. The data sets are as follows:

Data (X): 6.*53, 7, 10.42, 14.48, 16.10, 22.70, 34, 41.55, 42, 45.28, 49.40, 53.62, 63, 64, 83, 84, 91, 108, 112, 129, 133,* 133, *139, 140, 140, 146, 149, 154, 157, 160, 160, 165, 173, 176, 218, 225, 241, 248, 273,* 277,*297, 405, 417, 420, 440, 523, 583, 594, 1101,1146, 1417.*

Data (Y): 12.*20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81, 43, 84, 92,* 94, *110, 112, 119, 127, 130, 133, 140,146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725,* 817, *1776.*

To check goodness of fit of TIE distribution to the above data Kolmogrov Smirnov test, log-likelihood criterion, Akaike information criterion (AIC), Akaike information criterion corrected (AICc) and Bayesian information criterion (BIC) are considered. It can be seen that the TIE model is a better fit to the data than the distributions generalized inverse exponential distribution (GIED), inverse Rayleigh distribution (IRD) and inverse exponential distribution, which were considered in the literature. The maximum likelihood estimate of the parameters $(\alpha, \lambda_1, \beta, \lambda_2)$ and stress strength parameter R are obtained as (40.6839, -0.6811, 49.5925, -0.7165) and $\hat{R} = 0.4530$ respectively.We have applied the likelihood ratio test criterion, which is discussed in section 2.3 to the above data sets. The values of w_1 and w_2 are given below.

$w_1 = 98.508$ ($\chi^2_{(1)}(0.95) = 0.024$) and $w_2 = 0.6156$ ($\chi^2_{(1)}(0.95) = 0.024$) . Hence, we reject H_0 in both cases and conclude that the underlined TIE distribution is statistically superior than IED.

The information regarding the fitness of the model to the data sets is given in table 1 and table 2.

Table 1:
Data set – I

Model	Estimate	AIC	BIC	AIC$_C$	-logL	KS
GIED	(0.7771, 49.2410)	713.1815	777.3024	773.4316	384.5908	0.2453
IRD	741.3652	840.1341	842.066	840.2158	419.0671	0.6039
IED	59.1258	773.3742	775.4346	773.4558	385.6811	0.2875
TIED	(40.6839, -0.6811)	676.8654	676.2805	677.1154	336.4323	0.203

Table 2:
Data set – II

Model	Estimate	AIC	BIC	AIC$_C$	-logL	KS
GIED	(1.1799, 83.8998)	572.4309	576.0443	572.715	284.215	0.1901
IRD	2547.4170	962.4993	964.4993	962.8082	480.3576	0.3783
IED	75.3793	571.0622	572.8689	571.155	284.5311	0.1823
TIED	(46.9356, -0.7588)	572.4446	571.7510	572.7303	284.2233	0.0786

And the confidence intervals for $(\alpha, \lambda_1, \beta, \lambda_2)$ are given intable 3.

Table 3:

	Parameters			
	α	**λ_1**	**β**	**λ_2**
Confidence intervals	(36.5728, 44.7949)	(-0.9752, -0.3861)	(41.1224, 52.7487)	(-1, -0.3729)

4. SIMULATION STUDY

A simulation study is conducted to evaluate the performance of estimators of system reliability using mean squared error criterion. The mean square errors (MSEs) of MLE and moment estimators of system reliability are computed by generating 100000 samples of size $n, m = 10, 20, 30, 40$. The samples are generated from TIE distribution with parameters

$(\sim, a_1, \ , a_2)$. The following table 4 gives the MSEs of MLE and moment estimator of reliability.

Table 4: MLEs, moment estimators and MSEs for estimators of R.

$\sim = 3$, $a_1 = 0.5$, $a_2 = 0.8$ and $= 3.2$					
Sample size	R	$\hat{R}$	MSE($\hat{R}$)	$\tilde{R}$	MSE($\tilde{R}$)
$n=10, m=10$	0.5301	0.4937	0.0046	0.5064	0.0210
$n=20, m=20$		0.4958	0.0028	0.5014	0.0108
$n=30, m=30$		0.4978	0.0021	0.5021	0.0064
$n=40, m=40$		0.4923	0.0023	0.5053	0.0051
$\sim = 4$, $a_1 = 0.5$, $a_2 = -0.5$ and $= 1$					
$n=10, m=10$	0.6778	0.4865	0.0405	0.5079	0.0499
$n=20, m=20$		0.5216	0.0276	0.4885	0.0464
$n=30, m=30$		0.5114	0.0289	0.5161	0.0319
$n=40, m=40$		0.4453	0.0547	0.4924	0.0378

5. CONCLUSION

In this paper, we have considered stress-strength reliability of the component when the strength variable X and stress variable Y are independent TIE distributions. The estimators of reliability are compared using mean square error criterion. The MLE has smaller MSE than moment estimator. It can be observed that MSEs of both the estimators increase as sample size increase. Hence, MLEs are recommended to estimate reliability. However, moment estimator has closed form expression.

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