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An Analytical Solution for the Diffraction Problem by a Pair of Submerged Cylinders in Water

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ABSTRACT

In the present work, a theoretical approach is developed to describe diffraction of water wave by a pair of coaxial submerged cylinders in water of finite depth which is based on linear water wave theory. Using the method of eigenfunction expansion and separation of variables method, we obtain an analytical expression of diffracted velocity potential for each region which is applied to derive exciting wave force acting on the cylinders.

Keywords: Finite depth, Exciting force, Virtual boundary.

INTRODUCTION

Many researchers have approached theoretically to develop the problem of diffraction of water wave by using different structure. Our present investigation is also related to the diffraction problem of water wave by two coaxial submerged cylinders under the assumptions of linearized water wave theory.

Bhatta and Rahmann (2003) and Jiang *et al*. (2010) discussed the interaction of waves with a cylinder in water of finite depth and derived the analytical expression of velocity potential for both interior and exterior regions. Hassan and Bora (2012, 2013) discussed wave forces on a pair of coaxial cylinders in water of finite depth. Macamy (1954) gave an analytical expression of diffracted velocity potential for a single vertical cylinder in water of arbitrary depth. Rahmann and Bhatta (1993) developed non-linear water wave theory in which they investigated second order force for a pair of cylinders using Graft's addition theorem. Shen *et al*. (2005) analysed the diffraction and radiation of water wave by rectangular floating structure considering a bottom still effect. Bhattacharjee and Soares (2010) investigated the diffraction problem of a floating structure near a wall with step type bottom topography. Wu *et al*. (2004, 2006) analysed the diffraction and radiation of water waves by two vertical cylinders in water of finite depth. Zheng *et al*. (2009) formulated the problem of diffraction and radiation for two vertical truncated cylinders in water of finite depth.

Since the diffraction of water wave by a pair of structures is much more complex than that of an isolated structure in water. In our present paper, we divide the complicated fluid domain into five parts and the expression of velocity potential for each defined region is being obtained by the method of eigenfunction expansion approach and separation of variables approach.

Notation

h_1 : Uniform water depth

ω : Angular frequency

g : Gravitational acceleration

ρ : Fluid Density

p : Fluid Pressure

$J_m(.)$: Bessel function of the first kind of order m

$H_m^{(1)}(.)$: Hankel function of first kind of order m

$H_m^{(2)}(.)$: Hankel function of second kind of order m

$I_m(.)$: Modified Bessel function of first kind of order m

$K_m(.)$: Modified Bessel function of second kind of order m

MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider a linear water wave propagating in ideal water of finite depth h_1 with two submerged coaxial vertical cylinders in the device. Let the radius of upper cylinder is R that occupies the region $r \le R,\ 0 \le \theta \le 2\pi,\ -e_1 \le z \le -e_2$ and the radius of lower cylinder is $R_b\left(\ge R\right)$ that occupies the region $r \le R_b,\ 0 \le \theta \le 2\pi,\ -h_2 \le z \le -h_3$ and here onwards they will be called as cylinder 1 and cylinder 2 respectively. A right-handed Cartesian coordinate system is defined on undisturbed free surface with origin at $O,$ as shown in Figure 1 in which $z-$axis is measured vertically upward and propagation of water wave is directed along $x-$axis.

Clearly, under the assumption of linear water wave theory of time –harmonic motion, the velocity potential can be written as

$$\phi(r,\theta,z,t) = \mathrm{Re}[\phi(r,\theta,z)e^{-i\omega t}],$$

where Re[.] stands for real part of complex variable, $i=\sqrt{-1},\ \phi(r,\theta,z)$ represents spatial part of velocity potential which is time-independent. Therefore, $\phi(r,\theta,z)$ satisfies the Laplace's equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial \theta^2}+\frac{\partial \phi}{\partial z^2}=0. \quad (1)$$

The total velocity potential $\phi(r,\theta,z)$ can be written as $\phi=\phi_d+\phi_i$, where ϕ_d is the diffracted velocity potential due to diffraction and ϕ_i is the incident velocity potential. The fluid domain can be divided into five subdomains viz. I, II, III, IV and V, as indicated in Figure 1 and hence the diffracted velocity potential for each region can be expressed as $\phi_d^{\,I},\phi_d^{\,II},\phi_d^{\,III},\phi_d^{\,IV}$ and $\phi_d^{\,V}$ respectively. The expression of incident velocity potential given by MaCamy and Fuchs (1954) with unit amplitude is given by

$$\phi_i=-\frac{ig}{\omega}\frac{\cosh[k(z+h_1)]}{\cosh(kh_1)}\sum_{m=0}^{\infty}\varepsilon_m J_m(kr)\cos m\theta, \quad (2)$$

where the wave number k can be determined from the dispersion relation $\omega^2=gk\tanh(kh_1)$ and ε_m is given by

$$\varepsilon_m=\begin{cases}2i^m, & m>0\\ 1, & m=0\end{cases}$$

THE GOVERNING EQUATION AND BOUNDARY CONDITIONS

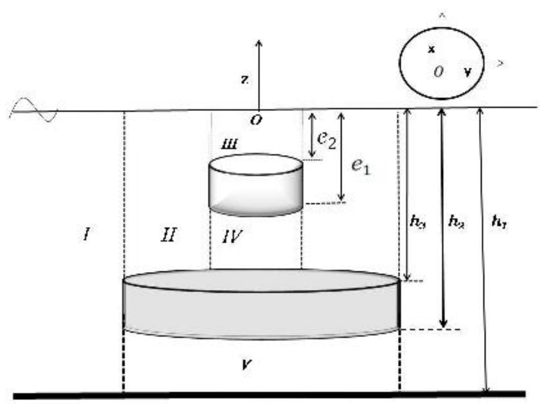

Figure 1. Physical geometry of the problem

The diffracted velocity potential ϕ_d satisfies the following governing equation and boundary conditions:

$$\nabla^2\phi_d=0 \quad \left(0<r<\infty,\ 0\le\theta\le 2\pi;\ -h_1<z<0 \text{ or } -h_3<z<0 \text{ or } -e_2<z<0\right) \quad (3)$$

$$\frac{\partial \phi_d}{\partial z}-\frac{\omega^2}{g}\phi_d=0 \quad (z=0) \quad (4)$$

$$\frac{\partial ¤_d}{\partial z} = 0 \quad (z = -h_1) \tag{5}$$

$$\frac{\partial ¤_d}{\partial z} = 0 \quad (z = -h_3,\, r < R_b;\, z = -e_2,\, r < R) \tag{6}$$

$$\frac{\partial (¤_d + ¤_i)}{\partial r} = 0 \quad (-h_2 < z < -h_3,\, r = R_b;\, -e_1 < z < -e_2,\, r = R) \tag{7}$$

and radiation condition is given by

$$\lim_{r\to\infty} \sqrt{kr}\left(\frac{\partial ¤_d}{\partial r} - ik¤_d\right) = 0 \tag{8}$$

MATCHING CONDITIONS

To preserve the continuity of flow, we consider the following matching conditions along the virtual and physical boundaries between the regions. Therefore, along the boundary $r = R_b$, i.e. the regions between I and II and between I and V, the conditions are given by

$$¤_d^I = \begin{cases} ¤_d^{II} & (-h_3 \le z \le 0) \\ ¤_d^V & (-h_1 \le z \le -h_2) \end{cases} \tag{9}$$

$$\frac{\partial ¤_d^{II}}{\partial r} = \begin{cases} \dfrac{\partial ¤_d^{II}}{\partial r} & (-h_3 \le z \le 0) \\ -\dfrac{\partial ¤_i}{\partial r} & (-h_2 \le z \le -h_3) \\ \dfrac{\partial ¤_d^V}{\partial r} & (-h_1 \le z \le -h_2) \end{cases} \tag{10}$$

Along the boundary $r = R$, i.e. the regions between II and III and between II and IV, the conditions are given by

$$¤_d^{II} = \begin{cases} ¤_d^{III} & (-e_2 \le z \le 0) \\ ¤_d^{IV} & (-h_3 \le z \le -e_1) \end{cases} \tag{11}$$

$$\frac{\partial ¤_d^{II}}{\partial r} = \begin{cases} \dfrac{\partial ¤_d^{III}}{\partial r} & (-e_2 \le z \le 0) \\ -\dfrac{\partial ¤_i}{\partial r} & (-e_1 \le z \le -e_2) \\ \dfrac{\partial ¤_d^{IV}}{\partial r} & (-h_3 \le z \le -e_1) \end{cases} \tag{12}$$

SOLUTION OF THE PROBLEM

As shown in Figure 1, the fluid region is divided into five regions denoted by I, II, III, IV and V and we apply separation of variables method in each region to obtain the expression of diffracted velocity potential. The expression of the diffracted velocity potential for each region described by Wu *et al.*(2004) is given by

$$¤_d^I = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m,n} \frac{R_m(ª_n r)}{R_m(ª_n R_b)} \cos[ª_n(z+h_1)]\cos m¯, \quad (13)$$

$$¤_d^{II} = -¤_i + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[B_{m,n}\frac{S_m(\sim_n r)}{S_m(\sim_n R)} + C_{m,n}\frac{T_m(\sim_n r)}{T_{m(}(\sim_n R)}\right]\cos[\sim_n(z+h_3)]\cos m¯, \quad (14)$$

$$¤_d^{III} = -¤_i + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} D_{m,n}U_m(¥_n r)\cos[¥_n(z+e_2)]\cos m¯, \quad (15)$$

$$¤_d^{IV} = -¤_i + \sum_{m=0}^{\infty}\left[E_{m,0}r^m + \sum_{n=1}^{\infty} E_{m,n}\frac{I_m(\ \ _n r)}{I_m(\ \ _n R)}\cos[\ \ _n(z+h_3)]\right]\cos m¯, \quad (16)$$

$$¤_d^{V} = -¤_i + \sum_{m=0}^{\infty}\left[F_{m,0}r^m + \sum_{n=1}^{\infty} F_{m,n}\frac{I_m(¢_n r)}{I_m(¢_n R_b)}\cos[¢_n(z+h_1)]\right]\cos m¯, \quad (17)$$

where $A_{m,n}$, $B_{m,n}$, $C_{m,n}$, $D_{m,n}$, $E_{m,n}$ and $F_{m,n}$ are the unknown constants and $ª_n, \sim_n, ¥_n, \ \ _n$ and $¢_n$ can be determined from the dispersion relation:

$$\begin{cases} ª_n = -ik & µ^2 = gk\tanh(kh_1),\ n=0 \\ µ^2 = -gª_n\tan(ª_n h_1) & n = 1, 2, ... \end{cases} \quad (18)$$

$$\begin{cases} \sim_n = -ik_1 & µ^2 = gk_1\tanh(k_1h_3),\ n=0 \\ µ^2 = -g\sim_n\tan(\sim_n h_3) & n = 1, 2, ... \end{cases} \quad (19)$$

$$\begin{cases} ¥_n = -ik_2 & µ^2 = gk_2\tanh(ke_2),\ n=0 \\ µ^2 = -g¥_n\tan(¥_n e_2) & n = 1, 2, ... \end{cases} \quad (20)$$

$$\ \ _n = \frac{n®}{h_3 - e_1} \qquad n = 0, 1, 2, ... \quad (21)$$

and

$$¢_n = \frac{n®}{h_1 - h_2} \qquad n = 0, 1, 2, ... \quad (22)$$

where k, k_1 and k_2 are the wave numbers in regions I, II and III respectively.

The radial functions $R_m(.), S_m(.), T_m(.)$ and $U_m(.)$ are given by

$$R_m(ª_n r) = H_m^{(1)}(kr) = H_m^{(1)}(iª_0 r), \quad n = 0 \tag{23}$$

$$R_m(ª_n r) = K_m(ª_n r), \quad n = 1, 2, \ldots \tag{24}$$

$$S_m(\sim_n r) = H_m^{(1)}(k_1 r), \quad n = 0 \tag{25}$$

$$S_m(\sim_n r) = K_m(\sim_n r), \quad n = 1, 2, \ldots \tag{26}$$

$$T_m(\sim_n r) = H_m^{(2)}(k_1 r), \quad n = 0 \tag{27}$$

$$T_m(\sim_n r) = I_m(\sim_n r), \quad n = 1, 2, \ldots \tag{28}$$

$$U_m(¥_n r) = J_m(k_2 r), \quad n = 0 \tag{29}$$

$$U_m(¥_n r) = I_m(¥_n r), \quad n = 1, 2, \ldots \tag{30}$$

WAVE FORCES

The relation between the dynamic fluid pressure and the velocity potential can be obtained from Bernoulli's equation which is given by

$$p(r, ^-, z, t) = -° \frac{\partial ¤(r, ^-, z, t)}{\partial t}. \tag{31}$$

Now we determine the horizontal wave exciting force acting on both cylinders due to diffraction which is generated by a combine action of an incident velocity and diffracted velocity potential. Let F_i be the horizontal exciting force due to incident velocity potential $¤_i$ and F_d be the horizontal exciting force due to diffracted velocity potential $¤_d$. Then total horizontal exciting force is given by

$$F_h = F_i + F_d = i°µ\left(\int_W ¤_i n_x ds + \int_W ¤_d n_x ds\right) \tag{32}$$

where $\vec{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$ is the unit normal vector to the surface of cylinder, W is the wetted surface of the cylinder and ds is the small surface element. Let F_{h1} be the total horizontal force acting on the cylinder 1, then we can write

$$F_{h1} = F_{i1} + F_{d1}, \tag{33}$$

where F_{i1} is the horizontal exciting force on cylinder 1 due to incident velocity potential and F_{h1} is the horizontal exciting force due to diffracted velocity potential $¤_d^{II}$ acting on cylinder 1. Using equations (2), (14), (32) and (33), we get the analytical expression of total horizontal wave exciting force on cylinder 1 as

$$F_{h1} = -i°µR\sum_{n=0}^{\infty}\left(B_{1,n} + C_{1,n}\right)\frac{\sin[\sim_n(h_3 - e_2)] - \sin[\sim_n(h_3 - e_1)]}{\sim_n}. \tag{34}$$

Similarly, if F_{h2} is the total horizontal wave exciting force acting on the cylinder 2, then we have

$$F_{h2} = F_{i2} + F_{d2}, \tag{35}$$

where F_{d2} is the horizontal exciting force due to diffracted velocity potential $¤_d^I$ acting on cylinder 2. Now from equations (2), (13), (32) and (35), the total horizontal wave exciting force acting on cylinder 2 is given by

$$\begin{aligned} F_{h2} = {} & \frac{-2i®°gR_b J_1(kR_b)\{\sinh[k(h_1-h_3)]-\sinh[k(h_1-h_2)]\}}{k\cosh(kh_1)} - \\ & i®°\mu R_b \sum_{n=0}^{\infty} A_{1,n} \frac{\sin[ª_n(h_1-h_3)]-\sin[ª_n(h_1-h_2)]}{ª_n}. \end{aligned} \tag{36}$$

METHOD TO FIND THE UNKNOWN CONSTANTS

To find the unknown constants appearing in above expression of potentials, first we use equations (9)- (12) followed by multiplication of both sides by a set of eigenfunction. Hence we use the property of orthogonality of eigenfunction, we get the following equations:

$$\int_{-h_3}^{0} ¤_d^I(R_b,¯,z).\cos[\sim_l(z+h_3)]dz = \int_{-h_3}^{0} ¤_d^{II}(R_b,¯,z).\cos[\sim_l(z+h_3)]dz \tag{37}$$

$$\int_{-h_1}^{-h_2} ¤_d^I(R_b,¯,z).\cos[¢_l(z+h_1)]dz = \int_{-h_1}^{-h_2} ¤_d^{III}(R_b,¯,z).\cos[¢_l(z+h_1)]dz \tag{38}$$

$$\begin{aligned} \int_{-h_1}^{0} \frac{\partial ¤_d^I(R_b,¯,z)}{\partial r}.\cos[ª_l(z+h_1)]dz = {} & \int_{-h_3}^{0} \frac{\partial ¤_d^{II}(R_b,¯,z)}{\partial r}.\cos[ª_l(z+h_1)]dz + \\ & \int_{-h_2}^{-h_3} \frac{\partial ¤_i(R_b,¯,z)}{\partial r}.\cos[ª_l(z+h_1)]dz + \int_{-h_1}^{-h_2} \frac{\partial ¤_d^V(R_b,¯,z)}{\partial r}.\cos[ª_l(z+h_1)]dz \end{aligned} \tag{39}$$

$$\int_{-e_3}^{0} ¤_d^{II}(R,¯,z).\cos[¥_l(z+e_2)]dz = \int_{-e_2}^{0} ¤_d^{III}(R,¯,z).\cos[¥_l(z+e_2)]dz \tag{40}$$

$$\int_{-h_3}^{-e_1} ¤_d^{II}(R,¯,z).\cos[\ _l(z+h_3)]dz = \int_{-h_3}^{-e_1} ¤_d^{IV}(R,¯,z).\cos[\ _l(z+h_3)]dz \tag{41}$$

$$\begin{aligned} \int_{-h_3}^{0} \frac{\partial ¤_d^{II}(R,¯,z)}{\partial r}.\cos[\sim_l(z+h_3)]dz = {} & \int_{-e_2}^{0} \frac{\partial ¤_d^{III}(R,¯,z)}{\partial r}.\cos[\sim_l(z+h_3)]dz + \\ & \int_{-e_1}^{-e_2} \frac{\partial ¤_i(R,¯,z)}{\partial r}.\cos[\sim_l(z+h_3)]dz + \int_{-h_3}^{-e_1} \frac{\partial ¤_d^{IV}(R,¯,z)}{\partial r}.\cos[\sim_l(z+h_3)]dz \end{aligned} \tag{42}$$

Again, let us define the following functions

$$M(x_n, y_n, a_1, a_2, z_1, z_2) = \int_{z_1}^{z_2} \cos[x_n(z+a_1)].\cos[y_n(z+a_2)]\,dz, \tag{43}$$

$$N(x_n, a_1, z_1, z_2) = \int_{z_1}^{z_2} \cos^2[x_n(z+a_1)]\,dz. \tag{44}$$

Applying equations (43) and (44) to equations $(37)-(42)$, we get

$$\sum_{n=0}^{\infty} A_{m,n}\, M(ª_n, \sim_l, h_1, h_3, -h_3, 0) = \frac{ig}{µ}\frac{£_m J_m(kR_b)}{\cosh(kh_1)} M(ª_0, \sim_l, h_1, h_3, -h_3, 0) + \left[B_{m,l}P_{ml} + C_{m,l}Q_{ml}\right] N(\sim_l, h_3, -h_3, 0) \tag{45}$$

$$\sum_{n=0}^{\infty} A_{m,n}\, M(ª_n, ¢_l, h_1, h_1, -h_1, -h_2) = \frac{ig}{µ}\frac{£_m J_m(kR_b)}{\cosh(kh_1)} M(ª_0, ¢_l, h_1, h_1, -h_1, -h_2) + F_{m,l}R^b_{ml} N(¢_l, h_1, -h_1, -h_2) \tag{46}$$

$$A_{m,l}N(ª_l, h_1, -h_1, 0) = \frac{ig}{µ}\frac{£_m kJ'_m(kR_b)}{\cosh(kh_1)} N(ª_0, h_1, -h_1, 0) + \sum_{n=0}^{\infty}\left[B_{m,n}V_{mn} + C_{m,n}W_{mn}\right] M(\sim_n, ª_l, h_3, h_1, -h_3, 0) + \sum_{n=0}^{\infty} F_{m,n}X_{mn}M(¢_n, ª_n, h_1, h_1, -h_1, -h_2) \tag{47}$$

$$\sum_{n=0}^{\infty}(B_{m,n} + C_{m,n}) M(\sim_n, ¥_l, h_3, e_2, -e_2, 0) = D_{m,l}U(¥_l R) N(¥_l, e_2, -e_2, 0) \tag{48}$$

$$\sum_{n=0}^{\infty}(B_{m,n} + C_{m,n}) M(\sim_n, \quad_l, h_3, h_3, -h_3, -e_1) = E_{m,l}R_{ml}N(\quad_l, h_3, -h_3, -e_1) \tag{49}$$

$$\left[B_{m,l}G_{ml} + C_{m,l}H_{ml}\right] N(\sim_l, h_3, -h_3, 0) = \sum_{n=0}^{\infty} D_{m,n}¥_n U'(¥_n R) M(¥_n, \sim_l, e_2, h_3, -e_2, 0) + \sum_{n=0}^{\infty} E_{m,n}O_{mn}M(\quad_n, \sim_l, h_3, h_3, -h_3, -e_1) \tag{50}$$

where

$$P_{ml} = \frac{S_m(\sim_l R_b)}{S_m(\sim_l R)},\ Q_{ml} = \frac{T_m(\sim_l R_b)}{T_m(\sim_l R)}.$$

$$R^b_{ml} = \begin{cases} R_b^m & l = 0 \\ 1 & l = 1,2,\ldots \end{cases}$$

$$V_{mn} = \frac{\sim_n S'_m(\sim_n R_b)}{S_m(\sim_n R)},\ W_{mn} = \frac{\sim_n T'_m(\sim_n R_b)}{T_m(\sim_n R)},\ X_{mn} = \begin{cases} mR_b^{m-1} & n = 0 \\ \dfrac{¢_n I'_m(¢_n R_b)}{I_m(¢_n R_b)} & n = 1,2,\ldots \end{cases}$$

$$R_{ml} = \begin{cases} R^m & l = 0 \\ 1 & l = 1,2,\ldots \end{cases}$$

$$G_{ml} = \frac{\sim_l S'_m(\sim_l R)}{S_m(\sim_l R)},\; H_{ml} = \frac{\sim_l T'_m(\sim_l R)}{T_m(\sim_l R)},\; O_{mn} = \begin{cases} mR^{m-1} & n = 0 \\ \dfrac{{}_n I'_m(\;{}_n R)}{I_m(\;{}_n R)} & n = 1,2,\ldots \end{cases}$$

Since each expression of velocity potential is an infinite series, therefore to compute the value of unknown constants it is necessary to truncate each series suitably. Then the unknown constants $A_{m,n}$, $B_{m,n}$, $C_{m,n}$, $D_{m,n}$, $E_{m,n}$ and $F_{m,n}$ are obtained by solving the system of equations $(45)-(50)$.

CONCLUSION

By theoretical approach that based on the method of eigenfunction expansion and separation of variables, we obtained an analytical expression for the diffracted velocity potential of the problem of diffraction of water wave by two coaxial submerged cylinders in water of finite depth. Set of horizontal wave exciting forces acting on the cylinders have also derived theoretically from the expressions of potentials. Then to find the unknown coefficients appearing in the expression, we have applied the appropriate matching conditions along the physical and virtual boundaries between the regions.

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