Journal of Computer and Mathematical Sciences, *Vol.10(3),410-417 March 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

$\alpha g - Normal$ Spaces and $\alpha g - Regular$ Spaces

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(Received on: February 20, 2019)

ABSTRACT

The aim of this paper is to introduce and study some normal and regular spaces using $\alpha g - open$ sets, $gp - open$ sets, $g^* - closed$ sets. We investigate some of the basic properties of these spaces. We also study the effect of some functions on these spaces.

Mathematics Subject Classification (2010) : 54A05, 54B05, 54D10.

Keywords: αg − open sets, gp − open sets, g^* − closed sets, g^* − irresolute, αg − irresolute.

1. INTRODUCTION

Levine[5,6] introduced the concept of generalized closed sets in topological spaces. S.P. Arya *et al.*[2] has discussed the notion of $gs - closed$ sets, $gsp - open$ sets. Govindappa Navalagi[3,4] has introduced the concepts of $(sp, gsp) - regular$ space, $gsp - regular$ space, $weakly\ g^*\ regular$ space and also has investigated some basic properties on allied normal spaces. The concepts of $gp - closed$ and $pg - closed$ sets were investigated by Maki *et al.*[7]. As an extension work Dontchev *et al.*[1] considered and characterized $generalized\ pre - irresolute$ and $generalized\ pre$- $continuous$ function via $gp - closed$ sets. The notion of $\alpha - open$ set was introduced and investigated by Njastad[8]. Research has been carried out by various authours by generalizing the concepts of $\alpha - open$ set, $semi - open$ sets etc.

The aim of this paper is to introduce the concepts and discuss the properties of $\alpha g - closed$ sets on regular and normal topological spaces.

Definition 1.1: A subset A of a space X is said to be
(i) $\alpha - open$ if $A \subseteq Int\left(cl\left(Int(A)\right)\right)$
(ii) $semi - open$ if $A \subseteq cl\left(Int(A)\right)$
(iii) $pre - open$ if $A \subseteq Int\left(cl(A)\right)$
(iv) $generalized - closed$ set if $cl(A) \subseteq U$ whenever $A \subseteq U$and U is open
(v) $generalized\ pre - closed$ set if $pcl(A) \subseteq U$ whenever $A \subseteq U$and U is open
(vi) $g^* - closed$ if $cl(A) \subseteq U$ whenever $A \subseteq U$and U is $g - open$

Definition 1.2: A subset A of a space X is said to be $\alpha g - closed$ if $\alpha cl(A) \subseteq U$whenever $A \subseteq U$and U is open.

Definition 1.3: A function $f: X \to Y$ is $\alpha g - continuous$ if inverse image of each open set of Y is $\alpha g - open$ in X.

Definition 1.4: A function $f: X \to Y$ is $\alpha - irresolute$ if $f^{-1}(V)$ is $\alpha - open$ in X for every $\alpha - open$ set V of Y.

Definition 1.5: A function $f: X \to Y$ is $\alpha g - irresolute$ if $f^{-1}(V)$ is $\alpha g - open$ in X for every $\alpha g - open$ set V of Y.

Definition 1.6: A topological space X is said to be $pre - gp\ closed$ if $f(F)$ is $gp\ closed$ in Y for every $pre -\ closed$ set F of X.

2. PROPERTIES OF $\alpha g - NORMAL$ SPACE

Definition 2.1: A topological space X is said to be $\alpha g - normal$ if for any pair of disjoint $\alpha g - closed$ sets A and B there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 2.1: A topological space X is $\alpha g - normal$ if and only if for any disjoint $\alpha g - closed$ sets A and B there exist open sets U and V such that $A \subseteq U$ and $B \subseteq V$ and $cl(U) \cap cl(V) = \varphi$.

Proof: Necessity: Let A and B be any disjoint $\alpha g - closed$ sets of X. There exist open sets U_0 and V of X such that $A \subseteq U_0$, $B \subseteq V$ and $U_0 \cap V = \varphi$. Hence $U_0 \cap cl(V) = \varphi$.

Since X is $\alpha g - normal$ there exist open sets G and H of X such that $A \subseteq G$ and $cl(V) \subseteq H$ and $G \cap H = \varphi$. Hence $cl(G) \cap H = \varphi$.

Let $U = U_0 \cap G$. Then U and V are open sets open sets such that $A \subseteq U$ and $B \subseteq V$ and $cl(U) \cap cl(V) = \varphi$.

Sufficiency: Result is obvious.

Theorem 2.2: A topological space X is $\alpha g - normal$ if and only if for every αg -closed set F and for every αg -open set G containing F there exist $open$ set U such that $F \subseteq U \subseteq cl(U) \subseteq G$.

Proof: Necessity: Let F be a αg -closed set in X and G be an αg -open set in X such that $F \subseteq G$. $X - G$ is αg -closed set and $(X - G) \cap F = \varphi$.

Since X is $\alpha g - normal$ space there exist open sets U and V of X such that $U \cap V = \varphi, X - G \subseteq V$ and $F \subseteq U$, $U \subseteq X - V$.

$cl(U) \subseteq cl(X - V) = X - V \subseteq G$. Hence $F \subseteq U \subseteq cl(U) \subseteq G$.

Sufficiency: Let the given condition hold. Let A and B be disjoint $\alpha g - closed$ sets.
Let $X - B = V$ where V is $\alpha g - open$ and $A \subseteq X - B = V$. By hypothesis there exist open set U such that $A \subseteq U \subseteq cl(U) \subseteq V$. Hence $X - V \subseteq X - cl(U)$ ie., $B \subseteq X - cl(U)$. U *and* $X - cl(U)$ are disjoint open sets containing A and B respectively.

Theorem 2.3: If $f: X \to Y$ is open, $\alpha g - irresolute$ and bijective and X is $\alpha g - normal$ then Y is $\alpha g - normal$.

Proof: Let A and B be any disjoint $\alpha g - closed$ sets of Y. Since f is $\alpha g - irresolute$, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\alpha g - closed$ sets of X. Since X is $\alpha g - normal$ there exist disjoint open sets U, V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.
Since f is open and bijective $A \subseteq f(U)$, $B \subseteq f(V)$ and $f(U) \cap f(V) = \varphi$. Also $f(U)$ *and* $f(V)$ are open sets of Y. Hence Y is $\alpha g - normal$.

Lemma 2.1: A subset A of a space X is $\alpha g - open$ iff $F \subseteq \alpha Int(A)$ whenever $F \subseteq A$ and F is closed in X

Proof: Since F is closed and A is $\alpha g - open$, $F \subseteq A$,
$X - F$ is open and $X - A$ is $\alpha g - closed$, $X - A \subseteq X - F$.
$\alpha cl(X - A) \subseteq X - F$ *ie.*, $X - \alpha Int(A) \subseteq X - F$. Hence $F \subseteq \alpha Int(A)$.
Now let us assume $X - A \subseteq U$, U is open.
Let $U = X - F$, F is closed. Then $X - A \subseteq X - F$.
$F \subseteq A$ ie., $F \subseteq \alpha int(A)$. Hence $X - \alpha int(A) \subseteq X - F$ ie., $\alpha cl(X - A) \subseteq X - F = U$
Therefore $X - A$ is $\alpha g - closed$. Hence A is $\alpha g - open$.

Definition 2.2: A topological space X is said to be $(\alpha, \alpha g) - normal$ if for any pair of disjoint $\alpha - closed$ sets A and B there exist disjoint $\alpha g -$ open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.3: A function $f: X \to Y$ is called $pre - \alpha g\ closed$ if for each $\alpha - closed$ set F of X, $f(F)$ is $\alpha g\ closed$ in Y.

Theorem 2.4: A bijective function $f: X \to Y$ is $pre - \alpha g\ closed$ if and only if for each subset B of Y and $\alpha - open$ set U of X containing $f^{-1}(B)$ there exist an $\alpha g - open$ set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$

Proof: Necessity: Suppose that f is $pre - \alpha g\ closed$. Let B be any subset of Y and U any $\alpha - open$ set of X containing $f^{-1}(B)$.
Let $V = Y - f(X - U)$. Then V is $\alpha g - open$ in Y, $B \subseteq V$ and $f^{-1}(V) \subseteq U$.
Sufficiency: Let F be any $\alpha - closed$ set of X. Put $B = Y - f(F)$. Then we have $f^{-1}(B) \subseteq X - F$ and $X - F$ is $\alpha - open$ in X. There exist an $\alpha g - open$ set V of Y such that $B = Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Therefore we obtain $f(F) = Y - V$ and hence $f(F)$ is $\alpha g\ closed$ in Y.

Theorem 2.5: If $f: X \to Y$ is bijective, $\alpha - irresolute$, $pre - \alpha g\ closed$ surjective and X is $\alpha - normal$ then is Y is $(\alpha, \alpha g) - normal$.

Proof: Let A and B be any disjoint $\alpha - closed$ sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\alpha - closed$ sets of X *as* f is $\alpha - irresolute$. Since X is $\alpha - normal$ there exist disjoint $\alpha - open$ sets U, V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.

Since f is $pre - \alpha g\ closed$ there exist $\alpha g - open$ sets G and H such that $A \subseteq G, B \subseteq H$, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$.
Also $G \cap H = \varphi$ because U, V are disjoint. This shows that Y is $(\alpha, \alpha g) - normal$.

3. PROPERTIES OF STRONGLY $\alpha g - NORMAL$ SPACE

Definition 3.1: A topological space X is said to be strongly $\alpha g - normal$ if for any pair of disjoint $closed$ sets A and B there exist disjoint αg −open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 3.1: The following properties are equivalent for a space X

(i) X is strongly $\alpha g - normal$

(ii) For any pair of disjoint $closed$ sets A and B there exist disjoint αg -open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

(iii) For any $closed$ set A and any $open$ set V containing A there exist αg -open set U such that $A \subseteq U \subseteq \alpha gcl(U) \subseteq V$.

Proof: (i)⇒(ii) is obvious.

(ii) ⇒(iii): Let A be any $closed$ set and V an $open$ set containing A. Since A and $X - V$ are disjoint $closed$ sets of X there exist $\alpha g - open$ sets U, W of X such that $A \subseteq U$ and $X - V \subseteq W$ and $U \cap W = \varphi$.

By lemma 2.1 we have $X - V \subseteq \alpha Int(W)$. Since $U \cap \alpha Int(W) = \varphi$ we have $\alpha gcl(U) \cap \alpha Int(W) = \varphi$ and hence $\alpha gcl(U) \subseteq X - \alpha Int(W) \subseteq V$

Therefore we obtain $A \subseteq U \subseteq \alpha cl(U) \subseteq V$

(iii) ⇒(i): Let A and B be any disjoint $closed$ sets of X. Since $X - B$ is $open$ set containing A, there exist an $\alpha g - open$ set G such that $A \subseteq G \subseteq \alpha gcl(G) \subseteq X - B$. But $A \subseteq \alpha Int(G)$. If $U = \alpha Int(G)$ and $V = X - \alpha gcl(G)$ then U, V are disjoint $\alpha g - open$ sets such that $A \subseteq U$ and $B \subseteq V$. Hence X is strongly $\alpha g - normal$.

Theorem 3.2: If $f: X \to Y$ is continuous, $pre - \alpha g\ closed$, bijective and X is strongly $\alpha g - normal$ then is Y is strongly $\alpha g - normal$.

Proof: Let A and B be any disjoint $closed$ sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $closed$ sets of X as f is continuous. Since X isstrongly $\alpha g - normal$ there exist disjoint $\alpha g - open$ sets U ,V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.

Since f is $pre - \alpha g\ closed$ there exist $\alpha g - open$ sets G and H such that $A \subseteq G, B \subseteq H$, $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$.

Also $G \cap H = \varphi$ since U, V are disjoint. This shows that Y is strongly $\alpha g - normal$.

Definition 3.2: A topological space X is said to be (g^*, gp) $-normal$ if for any pair of disjoint$g^* - closed$ sets A and B of X there exist disjoint gp −open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 3.3: A topological space X is said to be (g^*, p) $-normal$ if for any pair of disjoint$g^* - closed$ sets A and B of X there exist disjoint pre −open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 3.4: A topological space X is said to be $(g^*, \alpha g)$ $-normal$ if for any pair of disjoint$g^* - closed$ sets A and B of X there exist disjoint αg −open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Example3.1:

Let $X = \{a, b, c\}, \tau = \{\varphi, X, \{a\}, \{a, b\}\}$. Then $C(X) = \{\varphi, X, \{c\}, \{b, c\}\}$, $gC(X) = \{\varphi, X, \{c\}, \{b, c\}, \{a, c\}\}$ and $g^*C(X) = \{\varphi, X, \{c\}, \{b, c\}, \{a, c\}\}$. Also$\alpha\, C(X) = \{\varphi, X, \{b\}, \{c\}, \{b, c\}\}$ and

$PC(X) = \{\varphi, X, \{b\}, \{c\}, \{b, c\}\}$, $\alpha g\ \ C(X) = \{\varphi, X, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Therefore X is normal but not $\alpha g - normal\ and\ \alpha - normal. But\ \ X\ is\ (g^*, p) - normal.$

Example3.2:

Let $X = \{a, b, c\}, \tau = \{\varphi, X, \{a\}, \{b, c\}\}$. Then $C(X) = \{\varphi, X, \{a\}, \{b, c\}\}\ and\ \alpha C(X) = \{\varphi, X, \{a\}, \{b, c\}\}$. Here $\alpha gC(X)$, $\alpha gO(X)\ and\ g^*C(X)$ are discrete. X is normal but not $\alpha g - normal$. Also X is strongly normal and $(g^*, \alpha g) - normal$.

Theorem 3.3: A bijective function $f: X \to Y$ is $pre - gp\ closed$ if and only if for each subset B of Y and each $U \in PO(X)$ containing $f^{-1}(B)$ there exist a $gp - open$ set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$

Proof: Necessity: Let f be $pre - gp\ closed$. $B \subseteq Y$ and $f^{-1}(B) \subseteq U$.

$X - U$ is pre-closed in X. $f(X - U)$ is $gp - closed$ in Y.

$V = Y - f(X - U)$ is $gp - open$ in Y. $f^{-1}(V) = X - f^{-1}(f(X - U)) = X - (X - U) = U$.

$B \subseteq f(U) = V$.

Sufficiency: Let F be $pre-\ closed$ in X. $f^{-1}(f(F^c)) \subseteq F^c$ and F^c is pre-open in X. By assumption there exist $gp - open$ set V of Y such that $f(F^c) \subseteq V$ and $f^{-1}(V) \subseteq F^c$. This implies $F \subseteq \left(f^{-1}(V)\right)^c$. Hence $V^c \subseteq (f(F^c))^c = f(F) \subseteq f\left(f^{-1}(V)\right)^c \subseteq V^c$. So $f(F) = V^c$ which is $gp\ closed$.

Definition 3.5: A function $f: X \to Y$ is $g^* - irresolute$ if $f^{-1}(V)$ is $g^* - open$ in X for every $g^* - open$ set V of Y.

Theorem 3.4: If $f: X \to Y$ is $g^* - irresolute,\ pre - gp\ closed$, bijective and X is $(g^*, p) - normal$ then is Y is $(g^*, p) - normal$.

Proof: Let A and B be any disjoint $g^* - closed$ sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $g^* - closed$ sets of X , since f is $g^* - irresolute$ X is $(g^*, p) - normal$. There exist disjoint pre $open$ sets U , V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.

Since f is $pre - gp\ closed$ there exist $gp - open$ sets G and H in Y such that $A \subseteq G, B \subseteq H, f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$.

Also $G \cap H = \varphi$ since U, V are disjoint. Hence Y is $(g^*, p) - normal$.

4. PROPERTIES OF αg − REGULAR SPACE

Definition 4.1: A topological space X is said to be $\alpha g - regular$ if for any pair of $\alpha g - closed$ set F of X and each point $x \in X - F$ there exist disjoint open sets U and V of X such that $x \in U$ and $F \subseteq V$.

Theorem 4.1: A topological space X is $\alpha g-regular$ if and only if for each $\alpha g-closed$ set F of X and each point $x \in X-F$ there exist open sets U and V such that $x \in U$, $F \subseteq V$ and $cl(U) \cap cl(V) = \varphi$.

Proof: Necessity: Let F be a $\alpha g-closed$ sets of X and $x \in X-F$ there exist open sets U_0 and V of X such that $x \in U_0$, $F \subseteq V$ and $U_0 \cap V = \varphi$. Hence $U_0 \cap cl(V) = \varphi$.

Since X is $\alpha g-regular$ there exist open sets G and H of X such that $x \in G,\ cl(V) \subseteq H$ and $G \cap H = \varphi$. Hence $cl(G) \cap H = \varphi$.

Let $U = U_0 \cap G$. Then U and V are open sets open sets such that $x \in U$ and $F \subseteq V$ and $cl(U) \cap cl(V) = \varphi$.

Sufficiency: Proof is obvious.

Theorem 4.2: Let X be a topological space. Then the following statements are equivalent

(i) X is $\alpha g-regular$

(ii) For each point $x \in X$ and for each αg -open neighbourhood W of x there exist an open set of $V\ of\ X$ such that $x \epsilon V\ and\ cl(V) \subseteq W$.

(iii) For each point $x \in X$ and for each αg –closed set F not containing x there exist an open set V of X such that $cl(V) \cap F = \varphi$.

Proof:(i) ⇒(ii) Let W be αg -open neighbourhood of x. Then there exist a αg –open set G such that $x \in G \subseteq W$. Since $X-G$ is αg –closed and $x \notin X-G$. There exist open sets U and V such that $X-G \subseteq U, x \in V$ and $U \cap V = \varphi$.

Hence $V \subseteq X-U$. Now $cl(V) \subseteq cl(X-U) = X-U$ and $X-G \subseteq U$ implies $X-U \subseteq G \subseteq W$. Hence $cl(V) \subseteq W$

(ii) ⇒(i) Let F be any $\alpha g-closed$ set of X and let $x \notin F$. Then $x \in X-F$ and $X-F$ is αg –open neighbourhood of x . Hence there exist an open set V of X such that $x \in V$ and $cl(V) \subseteq X-F\ \ ie., F \subseteq X-cl(V)$. Then $X-cl(V)$ is open containing F and $V \cap (\ X-cl(V)) = \varphi$. Hence X is $\alpha g-regular$

(ii) ⇒(iii) Let $x \in X$ and F be $\alpha g-closed$ such that $x \notin F$. Then $X-F$ is αg –open neighbourhood of x and hence there exist an open set V of X such that $x \in V$ and $cl(V) \subseteq X-F$ and therefore $cl(V) \cap F = \varphi$.

(iii) ⇒(ii) Let $x \in X$ and W be αg –open neighbourhood of x. Then there exist $\alpha g-$ open set G such that $x \in G \subseteq W$. Since $X-G$ is αg –closed and $x \notin X-G$ there exist an open set V of x such that $cl(V) \cap (X-G) = \varphi$. Therefore $cl(V) \subseteq G \subset W$.

Theorem 4.3: A topological space X is $\alpha g-regular$ if and only if given any $\alpha g-open$ set U of X such that $x \in U$ and there is open set V such that $x \in V \subseteq cl(V) \subseteq U$

Proof: Let U be an αg -open set where $x \in U$. $X-U$ is αg −closed such that $x \notin X-U$. Since X is $\alpha g-regular$ there exist open sets V and W such that $V \cap W = \varphi,\ X-U \subseteq W, x \in V$.

Since $V \cap W = \varphi$ we have $cl(V) \subseteq cl(X-W) = X-W$.

As $X-U \subseteq W$ we have $X-W \subseteq U$. Hence we have $x \in V \subseteq cl(V) \subseteq X-W \subset U$

Conversely let F be $\alpha g - closed$ set in X and $x \notin F$. $X - F$ is αg -open such that $x \in X - F$. Hence there exist open set U such that $x \in U \subseteq cl(U) \subseteq X - F$. Let $V = X - cl(U)$. V is an $open$ set containing $F, x \in U$ and $U \cap V = \varphi$. Hence X is $\alpha g - regular$

Definition 4.2: A topological space X is said to be $(\alpha g, gp) - regular$ if for each $\alpha g - closed$ set F of X and each point $x \in X - F$ there exist disjoint gp -open sets U and V of X such that $x \in U$ and $F \subseteq V$.

Theorem 4.4: Let X be a topological space. Then the following statements are equivalent

(i) X is $(\alpha g, gp) - regular$

(ii) For each point $x \in X$ and for each αg -open neighbourhood W of x there exist $gp -$ open set $V \; of \; X$ such that $gpcl(V) \subseteq W$.

(iii) For each point $x \in X$ and for each αg –closed set F not containing x there exist $gp -$ open set V of X such that $gpcl(V) \cap F = \varphi$.

Proof: Similar to theorem 4.2

Theorem 4.5: A topological space X is $(\alpha g, gp) - regular$ if and only if given any $x \in U$ and $\alpha g -$ open set U of X there is gp –open set V such that $x \in V \subseteq gpcl(V) \subset U$

Proof: Let U be $\alpha g -$ open set where $x \in U$. $X - U$ is αg −closed such that $x \notin X - U$. Since X is $(\alpha g, gp) - regular$ there exist $gp -$ open sets V and W such that $V \cap W = \varphi,\ X - U \subseteq W, x \in V$.

Since $V \cap W = \varphi$ we have $gpcl(V) \subseteq gpcl(X - W) = X - W$.

As $X - U \subseteq W$ we have $X - W \subseteq U$. Hence we have $x \in V \subseteq gpcl(V) \subseteq X - W \subseteq U$

Conversely let F be $\alpha g - closed$ set in X and $x \in X - F$. $X - F$ is αg −open such that $x \in X - F$. Hence there exist $gp -$ open set U such that $x \in U \subseteq gpcl(U) \subseteq X - F$. Let $V = X - gpcl(U)$. V is $gp - open$ set containing F and $U \cap V = \varphi$. Hence X is $(\alpha g, gp) - regular$

Definition 4.3: A topological space X is said to be $(\alpha g, \alpha) - regular$ if for each $\alpha g - closed$ set F of X and each point $x \in X - F$ there exist disjoint α -open sets U and V of X such that $x \in U$ and $F \subseteq V$.

Lemma 4.1: A subset A of a space X is $gp - open$ iff $F \subseteq pInt(A)$ whenever $F \subseteq A$ and F is closed in X

Proof: Similar to lemma 2.1

Theorem 4.6: Let X be a topological space. Then the following statements are equivalent

(i) X is $(\alpha g, \alpha) - regular$

(ii) For each point $x \in X$ and for each αg - neighbourhood W of x there exist α - open set V of X such that $x \in V$ and $\alpha cl(V) \subseteq W$.

(iii) For each point $x \in X$ and for each αg –closed set F not containing x there exist $\alpha -$ open set V of X such that $\alpha cl(V) \cap F = \varphi$.

Proof: Similar to theorem 4.2

Definition 4.4: A topological space X is said to be strongly $\alpha g - regular$ if for each closed set $F \; of \; X$ and each point $x \in X - F$ there exist disjoint αg -open sets U and V of X such that $x \in U$ and $F \subseteq V$.

Theorem 4.7: Let X be a topological space. Then the following statements are equivalent

(i) X is $strongly\ \alpha g - regular$

(ii) For each point $x \in X$ and for each open neighbourhood W of x there exist αg - open set V of X such that $\alpha gcl(V) \subseteq W$.

(iii) For each point $x \in X$ and for each closed set F not containing x there exist $\alpha g -$ open set V of X such that $\alpha gcl(V) \cap F = \varphi$.

Proof: (i) ⇒(ii) Let W be open neighbourhood of x. Then there exist a open set G such that $x \in G \subseteq W$. Since $X - G$ is closed and $x \notin X - G$ there exist αg -open sets U and V such that $X - G \subseteq U, x \in V$ and $U \cap V = \varphi$.

Hence $V \subseteq X - U$. Now $\alpha gcl(V) \subseteq \alpha gcl(X - U) = X - U$ and $X - G \subseteq U$ implies $X - U \subseteq G \subseteq W$. Hence $\alpha gcl(V) \subseteq W$

(ii) ⇒(i) Let F be any $closed$ set of X and let $x \notin F$. Then $x \in X - F$ and $X - F$ is open neighbourhood of x . Hence there exist αg open set V of X such that $x \in V$ and $\alpha g\ cl(V) \subseteq X - F\ ie., F \subseteq X - \alpha gcl(V)$. Then $X - \alpha gcl(V)$ is αg -open containing F and $V \cap (X - \alpha gcl(V)) = \varphi$. Hence X is strongly $\alpha g - regular$

(ii) ⇒(iii) Let $x \in X$ and F be $closed$ such that $x \notin F$. Then $X - F$ is open neighbourhood of x and hence there exist an $\alpha g -$open set V of X such that $\alpha gcl(V) \subseteq X - F$ and therefore $\alpha gcl(V) \cap F = \varphi$.

(iii) ⇒(ii) Let $x \in X$ and W be open neighbourhood of x. Then there exist open set G such that $x \in G \subseteq W$. Since $X - G$ is closed and $x \notin X - G,$ there exist $\alpha g -$ open set V of X such that $\alpha gcl(V) \cap (X - G) = \varphi$. Therefore$\alpha g\ cl(V) \subseteq G \subseteq W$.

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