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The Second Order Difference Equation Technique Using in Operational Amplifier Circuit

P. Mohankumar[1] and A. Ramesh[2]

[1]Professor of Mathematics,
Aaarupadaiveedu Institute of Techonology, Vinayaka Missions University,
Paiyanoor, Kancheepuram Dist., Tamilnadu, INDIA.
[2]Ph.D Scholar,
Vinayaka Missions University, Salem, Tamilnadu, INDIA.

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ABSTRACT

An Operational Amplifier circuit was designed and tested to solve any second order linear difference equation. Using the various simulator input types were testes across the input terminals of computer and Examples. Consider the generic difference equation of the form

$$a_n\Delta^2 y_n + b_n\Delta y_n + c_n y_n = x_n\,,\ \ n \ge 1$$

2010MSC: 65Q10.

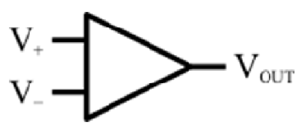
Keywords: Difference equation, Potentiometers, Mathematical Operations, Operational Amplifier

I. INTRODUCTION

A computer is a collection of the op amps, Potentiometers, resistors, capacitors, etc. which can easily be connected to simulate systems or build active filters. We make connections on the computer with patch cords; hence, we say that we patch the computer. The voltage transfer characteristics of certain circuits are the mathematical operations. The operations of interest here will be summation, multiplication. Note that these operations are sufficient for solving linear difference equations, and on building a generic amplifier block (GOPA) which is flexible to both Dimensions and topology. GOPA consists of array of scalable active and passive devices. The GOPA structure is capable of realizing more than 15 established simulation verified amplifier structures providing complete flexibility both in the choice of the structure and in dimensioning the devices constituting the structures.

II. DEFINITION OF OPERATIONAL AMPLIFIER

An operational amplifier (or an op-amp) is an integrated circuit (IC) that operates as a voltage amplifier. An op-amp has a differential input. That is, it has two inputs of opposite polarity. An op-amp has a single output and a very high gain, which means that the output signal is much higher than input signal. An op-amp is often represented in a circuit diagram with the following symbol:



These amplifiers are called "operation" amplifiers because they were initially designed as an effective device for performing arithmetic operations in an analog circuit. The op-amp has many other applications in signal processing, measurement, and instrumentation.

III. MAIN RESULTS

Our aim is to build a generic Operational Amplifier circuit to solve a generic second order difference with any input. Consider the generic difference equation of the form

$$a_n\Delta^2 y_n + b_n\Delta y_n + c_n y_n = x_n\ ,\ n\geq 1. \qquad (1)$$

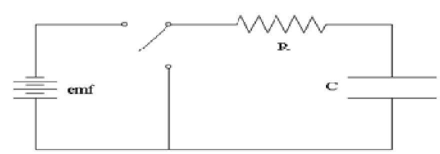
Where Δ forward difference is operatory $(\Delta y_n = y_{n+1} - y_n$ and $\Delta^2 y_n = \Delta(\Delta y_n))$ x_n is the forcing term (the input to the system represented by this difference equation) and $\{y_n\}$ is the solution (the output of the same equation). The a_n, b_n *and* c_n are some real constant[1].

Let $y_{n_1} = y_n$ *and* $y_{n_2} = \Delta y_n$. Thus we have the set of two first order difference equation of the form (assuming zero initial conditions).

$$\begin{Bmatrix} \Delta y_{n_1} = y_{n_2} \\ \Delta y_{n_2} = -\dfrac{c_n}{a_n} y_{n_2} - \dfrac{b_n}{a_n} y_n + \dfrac{1}{a_n} x_n \end{Bmatrix}. \qquad (2)$$

IV. METHODS

Consider the Operational Amplifier circuit shown in figure 1. RC (Resistor-Capacitor) Circuits In its simplest form, an R-C circuit contains a resistance, R, a capacitor, C, and an electromotive force, emf (usually a battery). A circuit diagram of an R-C circuit looks like this



The input-output relationship is given as

$$y_n = -A\frac{1}{RC}\sum_{n=1}^{\infty} x_{n_1} - B\frac{1}{RC}\sum_{n=1}^{\infty} x_{n_2} \qquad (3)$$

In the figure 1, the output $\{y_n\}$ is the input arriving at the negative terminal of operational Amplifier. The negative terminal of the $\{y_n\}$ is located at the negative terminal of operational Amplifier[2].

If we set $RC = 1$ in equation (3) we will have

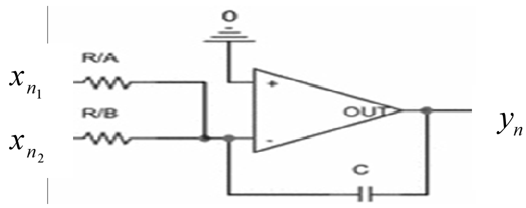


Fig.1 Operational Amplifier Circuit

$$y_n = -A\sum_{n=1}^{\infty} x_{n_1} - B\sum_{n=1}^{\infty} x_{n_2} \tag{4}$$

One step before we attempt to implement Equation (4) the solution of a generic first order linear constant coefficient differential equation.[6]. Consider the circuit given in figure 2

The input-output relationship is

$$y_n = \frac{R_f}{R} x_n \tag{5}$$

If $R_f = R$ then we have pure inversion (unity gain). The circuit containing an inverter and an integrator connected in series can solve the difference equation given in (6).

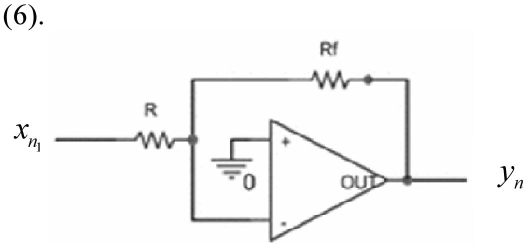


Fig.2 Inverter

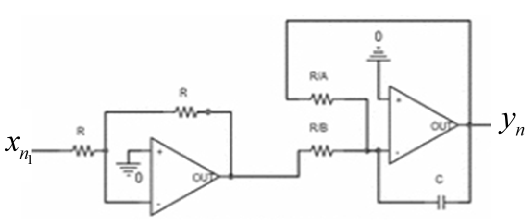


Fig.3 A Circuit to Solve Equation 6

The circuit in figure 3 to solve any first order difference equation of the form

$$\Delta y_n + A_n y_n = B_n x_n. \tag{6}$$

The $\{y_n\}$ solution of the equation (6) is $(-2)^n$ ($A_n = 1,\ B_n \geq 0$ *and* $x_n \geq 0$)

In building a circuit to solve the given difference equation (1) we will use the set of equations in (2). We can consider various types of input functions, which forcing term of the equation. We will consider the case when x_n is a unit step and pick two sets of values of a_n, b_n *and* c_n in equation (1). One set solutions is real modes and other set of solutions is complex modes.

Example: 1

If $a_n = 1, b_n = 4$ *and* $c_n = 3$ the Z-transformation of the system of equation (1) becomes

$$\Delta^2 y_n + 4\Delta y_n + 3y_n = x_n\ , \tag{7}$$

$$\frac{Y_s}{X_s} = \frac{1}{z^2 + 4z + 3} = \frac{1}{(z+3)(z+1)} \tag{8}$$

The modes are at -1 and -3 respectively. When the input is unit step of amplitude 3, the initial value of output $\{y_n\}$, initial value is zero and the final value is 1. The circuit to solve this case it shown below in figure 4 and the result is shown in figure 5.

Example: 2

If $a_n = 1, b_n = 1$ *and* $c_n = 1$ the Z-transformation of the system of equation (1) becomes

$$\Delta^2 y_n + \Delta y_n + y_n = x_n \; ; \qquad (9)$$

$$\frac{Y_s}{X_s} = \frac{1}{z^2 + z + 1} = . \qquad (10)$$

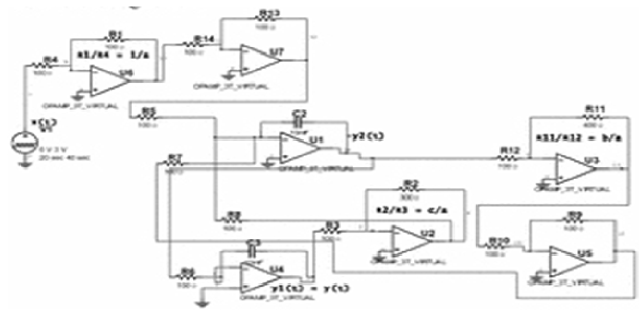


Fig.4.Circuit to solve : $\Delta^2 y_n + 4\Delta y_n + 3 y_n = x_n$

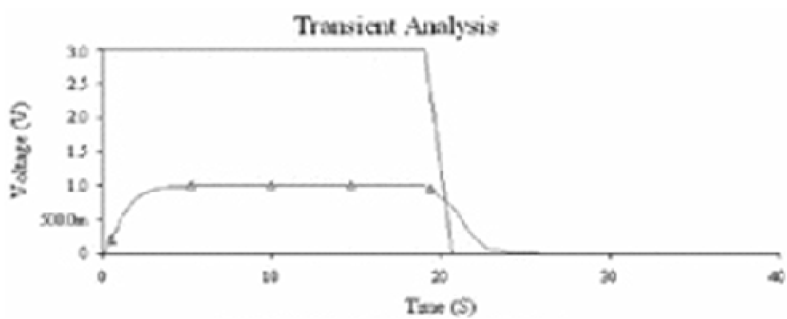


Fig.5 Output for Circuit in Fig.4

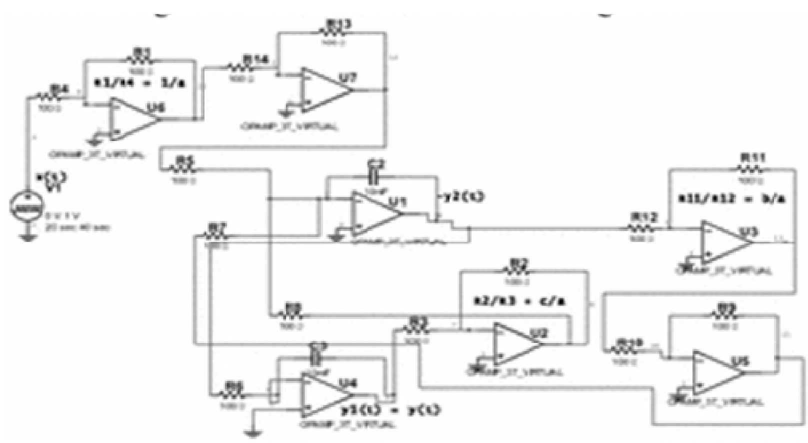


Fig.5.Circuit to solve : $\Delta^2 y_n + \Delta y_n + y_n = x_n$

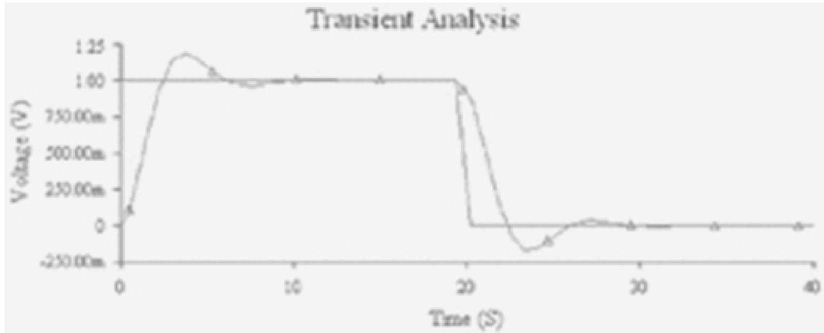


Fig.7 Output for Circuit in Fig.6

The modes are complex. When the input is unit step of amplitude 3, the initial value of output $\{y_n\}$, initial value is zero and the final value is 1 with oscillations in between. The circuit to solve this case it shown below in figure 6 and the result is shown in figure 7.

V. CONCLUSION

From the Graphs and also by comparing the results with what was derived analytically that the circuits worked as desired. The difference equation was solved and it outputs were as solution to the given input. In practically to solve any second order difference equation with any arbitrary coefficients requires a huge set of resistive values. However since the constant values of a_n, b_n *and* c_n be translate to ratio of resistor values. The issues related to amplifier saturation should be also be studied[5].

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