

Linear Unsteady MHD Flow of Burgers' Fluid with Heat Generation/Absorption Over a Stretching Surface

P. H. Nirmala* and A. Saila Kumari

Department of Mathematics,
JNTU University, Ananthapur, A.P., 515002 INDIA.
email: *nirmalaph83@gmail.com, asailakumari@gmail.com.

(Received on: April 12, 2019)

ABSTRACT

In present article the research on the unsteady linear convection of burgers' fluid flow in presence of magnetohydrodynamic in the absence of buoyancy effect and presence of Buongiorno nano model. The erected set of linear partial differential system is changed into coupled linear ordinary differential system by take on the suitable transformations. Numerical method is used in mathematical analysis for solve the ordinary differential equations. We investigate impact of physical parameter on the magnetic field effect, velocity, temperature and concentration profiles in unsteady linear case. The present results comparative study has been done with previous published relative data. The graphical and tabulated results are specified to reflect the physical nature of the present problem. From this result we concluded that unsteady linear convection flow result is better agreement than the steady linear convection flow.

Keywords: Buongiorno model, Brownian motion, Thermophoresis effect, unsteady linear convection, Reynolds number, local Grashoff number.

INTRODUCTION

The recent investigators are focus on the Non-Newtonian materials, because these materials are important in numerous engineering, physiological and industrial procedures for plastic manufacture, food processing, instance pharmaceuticals, glass blowing, and synthetic fiber etc. One fluid model cannot be foreseen all such demands. Therefore many researchers have been recommended various constitutive relationships of such materials. Burgers liquid is part of the rate type materials. These types of materials are valuable for polymers. The following investigation associated with Burgers liquid. Hayat *et al.*¹ studied the unidirectional

and certain simple flows of a Burgers' fluid. Fetecau *et al.*² presented the steady state solutions for certain simple flow of indiscriminate Burgers fluids. Hayat *et al.*³ investigated the rotating flow of a Burgers' fluid over porous media by the influence of Hall current. Khan *et al.*⁴ analyzed the frictional Burgers' model in the accelerated flow of viscoelastic fluid. Some solutions on the longitudinal oscillations of generalized Burgers fluid are noted in cylindrical domains were computed by Fetecau *et al.*⁵ Jamil and Fetecau⁶ studied a generalized Burgers fluid exact solution in the form of rotating over cylindrical domains over rotating flows.

Recently, the investigations on heat transfer of stretched flows over a stretching sheet are a great interest topic. Because its significance in industrial and technological demands. Such demands include the occurrence of heat absorption/generation plays important role in regulating the heat transfer rate. Moreover include the following demands casting process of materials and heat giving of materials fictional in an expulsion. Further cooling of spread out sheets is projected to guarantee the top feature of the material and necessities enthusiastic regulate of temperature. Narayana and Sibanda⁷ presented the unsteady flow of nanofluid film with heat transfer past a sheet which is being stretched surface. Khan and Azeem⁸ studied the heat generation/absorption of Burgers' nanofluid in steady flow at a stretching surface. Authors have been investigated the thermophoresis on Burgers' fluid flow over a stretching surface. Several plots are drawn to discuss the considered forced convection on Burgers fluid flow over a stretching sheet has been analyzed by Khan and Khan⁹. Authors have considered a theoretical analysis to examine the transient flow and heat transfer characteristics of MHD nanofluid in various surfaces. Used Navier's slip boundary condition in nanofluid caused by nonlinear stretching sheet is investigated in the presence of magnetic field and thermal radiation has been presented by Seth and Mishra¹⁰. Moreover, the investigators computed the heat transfer on fluid flow with numerical study inclined a stretching surface has been showed in article^{11,12}.

Furthermore, several researchers have been focusing to study the characteristics of Magnetohydrodynamic flow persuaded mass and heat transfer in various geometrical configurations. Many demands in chemistry, engineering and physics fields on topic of stretched flows in presence of the magnetic field. Such specific significance includes MHD generator, geothermal energy extraction, plasma studies, oil exploration and nuclear reactors. Ganga *et al.*¹³ discussed Joule heating and dissipation effects of MHD nanomaterial radiative stretched flow towards the vertical surface. For occurrence analysis of micropolar material in effectiveness of MHD in unsteady flow induced stretched surface is presented by Sandeep and Sulochana¹⁴. Farooq *et al.*¹⁵ investigated the non-linear radiation effects on viscoelastic nanofluid in presence of magnetic field. Ali *et al.*¹⁶ analyzed the heat transfer and MHD flow of stress fluid in porous medium of stretching sheet. The following authors also have done research on couple stress fluid in influence of MHD flow over a stretching surface in article Hayat *et al.*¹⁷. Khan *et al.*¹⁸ presented the numerical solution with variable properties of magnetohydrodynamic flow in stagnation point. The authors have been analyzed heat transform in unsteady magnetohydrodynamic flow over various geometrical models. The following articles are Nirmala *et al.*¹⁹ discussed the fluid flow between the two parallel plates

one is in motion and another one is in rest with unsteady magnetohydrodynamic flow. Nirmala *et al.*²⁰ analyzed the heat and mass transfer of magnetohydrodynamic non-Newtonian fluid flow over a vertical surface.

The intension of current study is the significance of unsteady magnetohydrodynamic Burgers' fluid flow of heat transfer in linear convection flow over a stretching sheet. To our information, no research has been made to analyze the unsteady and steady magnetohydrodynamic Burgers fluid flow over linear stretching sheet.

PROBLEM IMPROVEMENT

Consider the unsteady linear convection flow of Burgers' fluid in the presence of magnetic effect over a stretching surface. We analysis this problem with theoretically and depend on time. Consider the thermal and concentration effect formation into layers with heat generation/absorption. Let T_w, C_w be the temperature, concentrate at the stretching sheet surface and T_∞, C_∞ be the temperature, concentrate away from the stretching sheet.

In this analysis we have taken account of thermophoresis and Brownian motion effects. Let $u_w = \frac{ax}{1-ct}$ be the sheet velocity, where a be initial stretching rate and c be positive

constant of unsteadiness measuring. The fundamental equations governing the unsteady flow of Burgers nanofluid in the presence of magnetic field, this laws representing the continuity, momentum, energy and concentration equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + t_1 \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) + t_2 \left[\left(u^3 \frac{\partial^3 u}{\partial x^3} + v^3 \frac{\partial^3 u}{\partial y^3} \right) + u^2 \left(\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial x^2} + 2 \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial x \partial y} \right) + \right. \tag{2}$$

$$\left. 2uv \left(\frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x^2} + \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial x \partial y} \right) + 3uv \left(u \frac{\partial^3 u}{\partial x^2 \partial y} + v \frac{\partial^3 u}{\partial x \partial y^2} \right) + 3v^2 \left(\frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) \right] - \frac{\sigma \beta_0^2}{\rho} u = \nu \left[\frac{\partial^2 u}{\partial y^2} + t_3 \left(v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + u \frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} \right) \right] \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} = \tau \left[D_{Br} \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + T_D \frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where σ is the electrical conductivity, ρ is the base fluid density, u and v are the velocity

components, ν be the kinematic viscosity, $\tau = \frac{\rho C_p}{\rho C_f}$ is the ratio of base fluid and nanoparticle

heat capacity, D_T, D_B be the thermophoresis and Brownian diffusion coefficient.

The relevant boundary conditions are:

If $y \rightarrow 0$ then $u \rightarrow U_w, v \rightarrow 0, T \rightarrow T_w, C \rightarrow C$

If $y \rightarrow \infty$ then $u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, v \rightarrow 0$ (5)

The following system of dimensionless quantities are introducing for transforming the above problem into a simpler form

$$u = -\frac{ax}{ct-1} f^1 \eta, v = \left(\frac{av}{ct-1}\right)^{\frac{1}{2}} f \eta, \eta = y \left(-\frac{a}{v ct-1}\right)^{\frac{1}{2}} \tag{6}$$

$$T = T_\infty + T_w - T_\infty \theta \eta, C = C_w - C_\infty \phi \eta$$

Where ψ is the stream function and it is represented as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

Subsequently, the above governing expressions reduce to

$$f^{iv} = 1/ \beta_2 f^3 - \beta_3 f \left[Af' + 0.5A\eta f'' + f'^2 - ff'' - \beta_1 f^2 f''' + \beta_2 6f^2 f f''' - Mf' \right] \tag{7}$$

$$\theta'' = (0.5P_r \eta A \theta' - P_r f \theta' - P_r N_b \theta' \phi' - P_r N_t \theta'^2) \tag{8}$$

$$\phi'' = \left(0.5A\eta S_c \phi' - f S_c \phi' - \frac{Nt}{Nb} S_c \theta'' \right) \tag{9}$$

$$f(0) = 0, f'(0) = 1, \phi(0) = 1, \theta(0) = 1 \tag{10}$$

$$f(\infty) \rightarrow 0, f'(\infty) \rightarrow 0, f''(\infty) \rightarrow 0, f'''(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0$$

where β_1, β_3 are Deborah number and β_2 be the fluid viscoelastic particle interaction parameter for temperature, A be the unsteadiness parameter, M be the Magnetic parameter, P_r be the Prandtl number, (N_b, N_t) be the Brownian motion and thermophoresis parameter. The exemplifications of used parameters are

$$A = \frac{c}{a}, M = \frac{\sigma \beta_0^2 (1-ct)}{a \rho}, Re_x = \frac{U_w x}{\nu}, \nu = \frac{\mu}{\rho}, P_r = \frac{\nu}{\alpha}, S_c = \frac{T_D}{D_B},$$

$$N_b = \frac{\tau D_{Br} C_w - C_\infty}{\nu}, N_t = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}, \beta_1 = \frac{t_1 a}{1-ct}, \beta_2 = \frac{a^2 t_2}{1-ct}, \beta_3 = \frac{t_3 a}{1-ct}, \tag{11}$$

$$N = \frac{\beta_C C_w - C_\infty}{\beta_T T_w - T_\infty}, Gr_x = \frac{\beta_T g T_w - T_\infty x^3}{\nu^2}$$

Where Re_x be the local Reynolds number and Gr_x be the local Grashoff number. We can get results of viscous fluid from the condition $\beta_1 = \beta_2 = \beta_3 = 0$.

The present problem physical interest parameters are the Nu_x local Nusselt number and Sh_x local Sherwood number are given by

$$Nu_x = \frac{-x}{T_w - T_0} \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}, \quad Sh_x = \frac{-x}{C_w - C_0} \left(\frac{\partial C}{\partial y} \right) \Big|_{y=0} \tag{12}$$

The dimensionless quantities are obtained

$$\begin{aligned} Nu_x / Re_x^{0.5} &= -\theta^l \quad 0 \\ Sh_x / Re_x^{0.5} &= -\phi^l \quad 0 \end{aligned} \tag{13}$$

RESULT ANALYSIS AND DISCUSSION

In present problem, we are solving the Eqs. (7) to (9) with a numerical method and analyzed the heat and mass transfer of fluid flow graphically. Mainly in this article, we compare the unsteady linear convection fluid flow with steady linear convection flow graphically.

Table : Distinction of Nusselt number and friction factor in unsteady and steady linear fluid flow

M	N_t	N_b	P_r	$A = 1$ unsteady linear flow				$A = 0$ steady linear flow			
				Skin	Nur	shr	Time taken for execution (seconds)	Skin	Nur	shr	Time taken for execution (seconds)
0.1				-2.771	0.056	0.067	2.022092	-2.046	0.178	0.187	2.844523
0.5				-2.686	0.057	0.068		-1.869	0.189	0.197	
0.7				-2.643	0.058	0.069		-1.766	0.197	0.205	
	0.1			-2.686	0.057	0.055	2.429807	-1.869	0.188	0.198	2.704263
	0.2			-2.690	0.056	0.075		-1.876	0.185	0.196	
	0.3			-2.693	0.055	0.083		-1.884	0.182	0.194	
		1		-2.708	0.051	0.063	2.058621	-1.921	0.166	0.197	2.509626
		2		-2.736	0.043	0.061		-1.986	0.138	0.193	
		3		-2.763	0.036	0.061		-2.045	0.115	0.189	
			0.1	-2.535	0.123	0.068	2.202976	-1.894	0.173	0.198	2.695296
			0.2	-2.613	0.086	0.068		-1.882	0.181	0.198	
			0.3	-2.686	0.057	0.068		-1.869	0.188	0.197	

In the table we can observe the result numerically and clear representation of the comparison between unsteady and linear MHD flow of Burgers’ fluid. We analyzed the fluid flow based on time taken for execution in seconds. The table shows a clear picture of numerical accurate and time saving flow convection that is the unsteady flow is taking less time taken for execution compare to steady flow, the heat and concentration transform effects are more in steady flow compare to unsteady flow.

Our main aim is to investigate the characteristic of different parameters on the velocity, concentration and temperature distributions of unsteady and steady linear Burgers’ fluid in an occurrence of magnetic field. Graphical analyses the heat transfer, mass transfer and the flow physical characteristics have been conceded out to know the present problem.

Figs. 1 to 3 reveals the variation of the velocity, temperature and concentration distributions in response to a change in the thermophoresis parameter N_t .

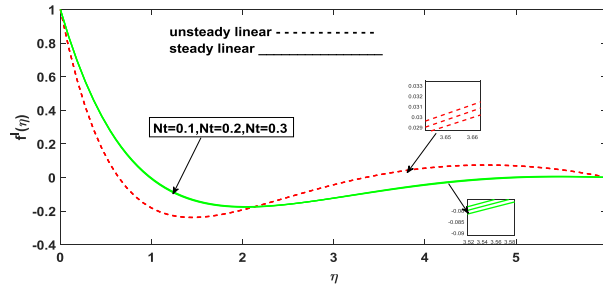


Fig. 1 The sway of N_t on velocity profiles $f' \eta$ when the following values are fixed

$$Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$$

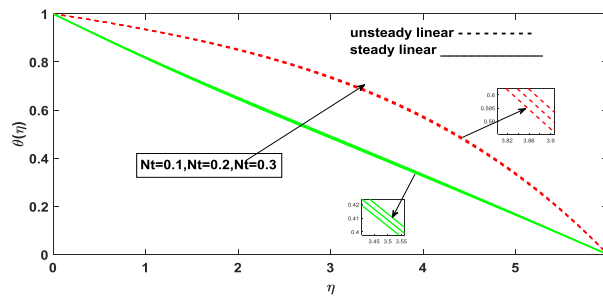


Fig.2 the sway of N_t on temperature profiles $\theta \eta$ when the fixed values are

$$Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$$

Generally, the characteristics of thermophoresis in a mechanism are it pull small particles far-off from the broiling surface to a held in reserve one. The concentration distribution enhances and the behavior of velocity and temperature is opposite for higher values of thermophoresis parameter. We identified the smaller quantity error values and behavior of fluid flow at boundary layer is a better agreement in unsteady linear flow than the steady linear flow.

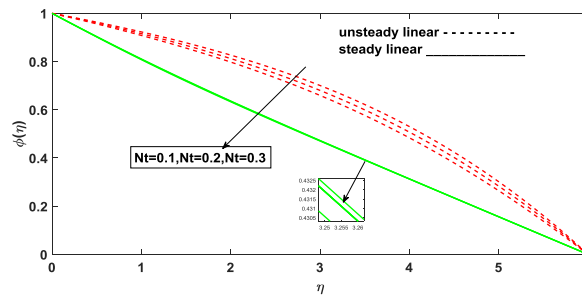


Fig.3 the sway of N_t on concentration profiles $\phi \eta$ when the fixed values are

$$Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$$

Figures 4, 5 and 6 show the Influence of the Brownian motion parameter on velocity, temperature, and concentration. Temperature, Thermal boundary layer, and concentration are enhanced for the upper value of the Brownian motion parameter. The exact opposite trend exists on velocity.

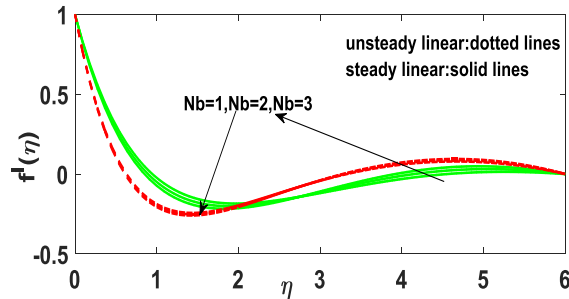


Fig.4 the sway of N_b on velocity profiles $f' \eta$ when the fixed values are $Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

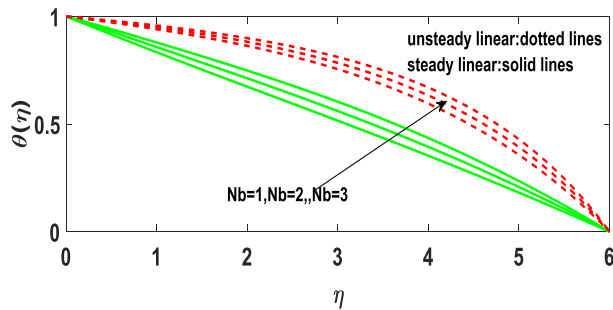


Fig.5 the sway of N_b on velocity profiles $\theta \eta$ when the fixed values are $Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

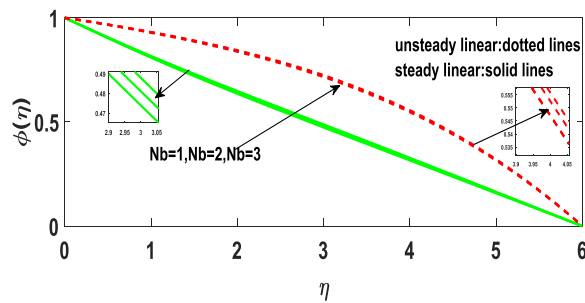


Fig.6 the sway of N_b on velocity profiles $\phi \eta$ when the fixed values are $Pr = 0.3, M = 0.5, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

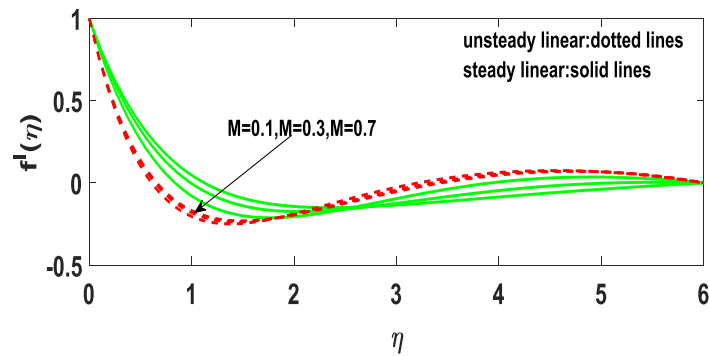


Fig.7 the sway of M on velocity profiles $f' \eta$ when the fixed values are $Pr = 0.3, N_t = 0.1, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

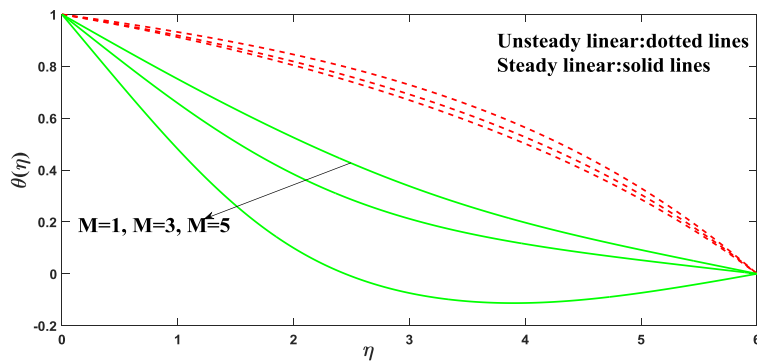


Fig.8 the sway of M on velocity profiles $\theta \eta$ when the fixed values are $Pr = 0.3, N_t = 0.1, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

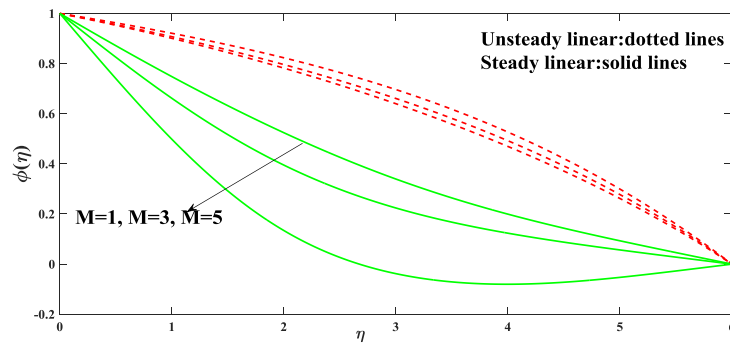


Fig.9 the sway of M on velocity profiles $\phi \eta$ when the fixed values are $Pr = 0.3, N_t = 0.1, \beta_1 = \beta_2 = \beta_3 = 0.2, t_1 = t_2 = 0.2, N_b = 0.3, S_c = 0.3$

Figs. 7-9 show that the temperature and concentration rising while increasing values of magnetic effect and decreases the velocity. Clearly, we can observe Lorentz force exists in the magnetic field in an unsteady vice versa steady flow. These graphs clearly give the result of this problem that is flow has an accurate force and less time execution in unsteady linear flow than the steady linear flow.

CONCLUSION

In an evaluation of this in the current analysis, we compared the MHD flow of Burgers' fluid in unsteady and steady linear convection. The measurement results for temperature, concentration, and no dimensional velocity profiles are offered through graphs in the following cases ($A = 0$ and $A = 1$)

1. The skin friction, mass transfer, and heat transfer are found an extremely negligible error in linear unsteady convection than the linear steady convection.
2. The temperature, concentration, and velocity are very clear with the flow in linear unsteady convection than in the linear steady convection. Through this, we define that the cooling process of linear unsteady convection is extremely beneficial.
3. The time in use for the execution in the presence of non-linear unsteady convection is very less compared to non-linear steady convection fluid flow. The thermophoresis parameter to separate the different particles in the field flow friction we can highlight this in unsteady flow to control the flow.

REFERENCES

1. T. Hayat , C. Fetecau , S. Asghar, Some simple flows of a Burgers' fluid. *International Journal of Engineering Science* 44 1423–1431 (2006).
2. C. Fetecau, T. Hayat, Corina Fetecau, Steady-state solutions for some simple flows of generalized Burgers fluids, *International Journal of Non-Linear Mechanics* 41, 880 – 887 (2006).
3. T. Hayat, SB. Khan, M. Khan, Influence of Hall current on the rotating flow of a Burgers' fluid through a porous space, *J Porous Med.* 11, 277-287 (2008).
4. M. Khan, S.H. Ali, H. Qi, On accelerated flows of a viscoelastic fluid with the fractional Burgers' model, *Nonlinear Analysis: Real World Applications* 10, 2286–2296 (2009).
5. C. Fetecau, T. Hayat, M. Khan, A note on longitudinal oscillations of a generalized Burgers fluid in cylindrical domains, *J Non-Newtonian Fluid Mech* 165, 350-361 (2010).
6. M. Jamil, C. Fetecau, Some exact solutions for rotating flows of a generalized Burgers' fluid in cylindrical domains. *Journal of Non-Newtonian Fluid Mechanics* 165, 1700–1712 (2010).
7. M. Narayana, P. Sibanda, Laminar flow of a nanoliquid film over an unsteady stretching sheet. *International Journal of Heat and Mass Transfer* 55, 7552–7560 (2012).
8. M. Khan, W. Azeem, Steady flow of Burgers' nanofluid over a stretching surface with heat generation / absorption. *J Braz. Soc. Mech. Sci. Eng.* (2014).

9. M. Khan, W.A. Khan, Forced convection analysis for generalized Burgers nanofluid flow over a stretching sheet. *AIP Advances* 5, 107138 (2015).
10. G.S. Seth, M.K. Mishra, Analysis of transient flow of MHD nanofluid past a non-linear stretching sheet considering Navier's slip boundary condition. *Advanced Powder Technology* 28, 375–384 (2017).
11. T. Thumma, O.A. Bég, A. Kadir, Numerical study of heat source / sink effects on dissipative magnetic nano fluid flow from a non-linear inclined stretching / shrinking sheet. 232, 159–173 (2017).
12. T. Hayat, M. I. Khan, M. Waqas, A. Alsaedi and M. Farooq, Numerical simulation for melting heat transfer and radiation effects in stagnation point flow of carbon—water nanofluid. *Computer Methods Appl. Mech. Eng.* 315, 1011-1024 (2017).
13. B. Ganga, S. M. Y. Ansari, N. V. Ganesh and A. K. A. Hakeem, MHD radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects. *J. Nigerian Math. Soc.* 34, 181-194 (2015).
14. N. Sandeep and C. Sulochana, Dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source/sink. *Eng. Sci. Tech. An Int. J.* 18 738-745 (2015).
15. M. Farooq, M. I. Khan, M. Waqas, T. Hayat, A. Alsaedi and M. I. Khan, MHD stagnation point flow of viscoelastic nanofluid with non-linear radiation effects. *J. Mol. Liq.* 221, 1097-1103 (2016).
16. N.Ali, S. U. Khan, M. Sajid and Z. Abbas, MHD flow and heat transfer of couple stress fluid over an oscillatory stretching sheet with heat source/sink in porous medium. *Alex Eng. J.* 55, 915-924 (2016).
17. T. Hayat, A. Aziz, T. Muhammad and B. Ahmad, Influence of magnetic field in three dimensional flow of couple stress nanofluid over a nonlinearly stretching surface with convective condition. *Plos One* 10 e0145332 (2016).
18. M. I. Khan, M. Z. Kiyani, M. Y. Malik, T. Yasmeen, M. W. A. Khan and T. Abbas, Numerical investigation of magnetohydrodynamic stagnation point flow with variable properties, *Alex. Eng. J.* 55, 2367-2373 (2016).
19. P.H. Nirmala, A.S. Kumari, C.S.K. Raju, Unsteady MHD Couette flow between two Parallel Plates with Uniform Suction. *Research J. Science and Tech.* 9(3) 476–483 (2017).
20. P.H. Nirmala, A.S. Kumari, C.S.K. Raju, An Integral Vonkarman Treatment of Magnetohydrodynamic Natural Convection on Heat and Mass Transfer Along a Radiating Vertical Surface in a Saturated Porous. *Journal of nanofluids* 7, 1–9 (2018).