Journal of Computer and Mathematical Sciences, *Vol.10(2),245-254 February 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

On Soft Nano Resolvable and Soft Nano Irresolvable Spaces

P. G. Patil[1*] and Spoorti S. Benakanawari[2]

[1,2]Department of Mathematics,
Karnatak University Dharwad-580003, Karnataka, INDIA.
pgpatil01@gmail.com[1] (Corresponding Author) and spoortisb@gmail.com[2]

(Received on: December 11, 2018)

ABSTRACT

A soft nano topological space is called soft nano resolvable space if it can be expressed as a union of two soft nano disjoint dense sets, otherwise it is termed as soft nano irresolvable. In this paper, soft nano resolvable and soft nano irresolvable spaces are introduced and relationship between soft nano dense, soft nano codense and soft nano nowhere dense set is established. New characterizations of soft nano irresolvable spaces in terms of soft nano almost open (preopen) sets are studied. Also, several weaker forms of soft nano irresolvable are introduced. Several new results of soft nano open hereditarily irresolvable spaces are obtained and precise relationships are noted between soft nano open hereditarily irresolvability, soft nano irresolvability and soft nano hereditarily irresolvability. A study on levels of soft nano irresolvability is examined. Also, a brief note on functions and soft nano irresolvability is discussed. Results pertaining to soft nano extremally disconnected spaces are obtained.

2010 AMS Subject classifi cation: 54A05, 54A10.

Keywords: Soft nano resolvable spaces, soft nano irresolvable spaces, soft nano open hereditarily irresolvable spaces.

1. INTRODUCTION

The answer for the question of when a topological space X can be written as the union of two disjoint dense subspaces was presented by Hewitt[6] and such spaces were termed as resolvable. A space is called as irresolvable if it is not resolvable. Obviously, a resolvable space is dense-in-itself. Hewitt[6] also considered whether every dense-in-itself space is resolvable and in[6] this is true for large classes of topological spaces, including metric spaces and locally compact Hausdorff spaces.

R. Bolstein[4] proves that a space is resolvable if and only if it is a finite union of sets with void interiors and calls a space almost resolvable if it is the countable union of sets each of which has void interior. The collection of spaces can be taken to be homeomorphic to expansions of the usual topology on the rational numbers to irresolvable spaces. Furthermore, Ganster[5] discussed preopen sets and resolvable spaces. Rose *et al.*,[9] studied the properties of strongly irresolvable spaces.

Molodstav[7], initiated the concept of soft set theory as a new mathematical tool for solving several real world problems in engineering, physics, computer sciences, economics, medical sciences and many other diverse fields. The notion of soft topology is introduced by Shabir and Naz[10], which is defined over an initial universe with a fixed set of parameters and studied the basic concepts such as soft open sets, soft closed sets, soft closure and soft separation axioms.

The theory of nano topology and the weaker forms of nano open sets is given by Lellis Thivagar *et al*.[11] and continuous function in nano topology is defined in terms of nano open sets and also its characterizations in terms of nano interior and nano closure is obtained. The concept of soft nano topological spaces is introduced by Benchalli *et al*.[1].

In this paper, we discuss and study in detail the properties of soft nano resolvable and soft nano irresolvable spaces. The relationship among soft nano irresolvable and soft nano open hereditarily irresolvable spaces is studied.

2. PRELIMINARIES

Definition 2.1 [1]: Let U be a non-empty finite set of objects called the universe and E be a set of parameters. Let R be a soft equivalence relation on U. The triplet (U, R, E) is said to be the soft approximation space. Let $X \subseteq U$.

(i) The soft lower approximation of X with respect to R and the set of parameters E, is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $(L_R(X), E)$. i.e. $(L_R(X), E) = \cup \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.

(ii) The soft upper approximation of X with respect to R and set of parameters E, is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $(U_R(X), E)$. i.e. $((U_R(X), E) = \cup \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The soft boundary region of X with respect to R and set of parameters E, is the set of all objects, which can be classified neither inside X nor as outside X with respect to R and is denoted by $(B_R(X), E)$. i.e. $(B_R(X), E) = (U_R(X), E) - (L_R(X), E)$.

Definition 2.2 [1]: Let U be a non-empty universal set and E be a set of parameters. Let R be a soft equivalence relation on U. Let $X \subseteq U$. Let $(\tau_R(X), U, E) = \{U, \phi, (L_R(X),E), (U_R(X),E), (B_R(X), E)\}$. Then, $(\tau_R(X),U,E)$ is a soft topology on (U,E)), called as the soft nano topology with respect to X. Elements of the soft nano topology are known as the soft nano open sets and $(\tau_R(X),U,E)$ is called soft nano topological space. The complements of soft nano open sets are called as soft nano closed sets in $(\tau_R(X), U, E)$.

Definition 2.3 [1]: If ($\tau_R(X), U, E$) is a soft nano topological space with respect to X and E, where $X \subseteq U$ and if $(A,E) \subseteq (U,E)$, then the soft nano interior of (A, E) is defined as the union of all soft nano open subsets of (A, E) and it is denoted by $NInt(A,E)$.i.e.$NInt(A,E)$ is the largest soft nano open subset of (A, E). Further, the soft nano closure of (A, E) is defined as the intersection of all soft nano closed sets containing (A, E) and it is denoted by NCl (A, E). i.e. (A, E) is the smallest soft nano closed set containing (A, E).

Definition 2.4 [2]: Let (τ_R(X), U, E) be a soft nano topological space and (K, E) be any soft set over U. Then, (K, E) is said to be

(i). Soft nano semi-open if (K, E) ⊆ NCl (NInt (K, E)). Here the complement of (K, E) is said to be soft nano semi-closed.

(ii). Soft nano pre-open if (K, E) ⊆ NInt (NCl (K, E)). Here the complement of (K, E) is said to be soft nano pre-closed.

(iii).Soft nano α-open if (K, E) ⊆ NInt(NCl(NInt(K, E))). Here the complement of (K, E) is said to be soft nano α-closed.

(iv).Soft nano β-open if (K, E) ⊆ NCl(NInt(NCl(K, E))). Here the complement of (K, E) is said to be soft nano β-closed.

Definition 2.5 [8]: Consider a soft nano topological space (τ_R(X), U, E). Let Y be a non-empty subset of U such that X ⊆ Y ⊆ U and Y/R ⊆ U/R. Then the soft nano relative topology {τ_R^*(X), Y, E} on Y is defined as
{τ_R^*(X), Y, E} = {∅, Y, (L_R^*(X), E), (U_R^*(X), E), (B_R^*(X), E)}.
Here (L_R^*(X), E) = (L_R(X), E) ∩ Y, (U_R^*(X), E) = (U_R(X), E) ∩ Y and (B_R^*(X), E) = (B_R(X), E) ∩ Y = (U_R^*(X), E) - (L_R^*(X), E), where (L_R(X), E), (U_R(X), E), (B_R(X), E) ∈ Soft nano open set of (U, E).
The elements of soft nano relative topology are known as soft nano Y -open sets and the family of all soft nano Y-open sets is denoted by SNO (Y, E). The complements of soft nano Y -open sets are known as soft nano Y -closed sets in (τ_R^*(X), Y, E).

3. SOFT NANO RESOLVABLE SPACES

Definition 3.1: A soft subset (A, E) of (τ_R(X), U, E) is called soft nano dense if SNCl(A, E) = U and it is termed soft nano codense if U - (A, E) = (D, E) is soft nano dense (equivalently, SNInt(A, E) =ϕ).

Definition 3.2: A soft subset (A, E) of U is soft nano nowhere dense if SNInt(SNCl(A, E)= ϕ and the family of all soft nano nowhere dense subsets of U is SNN(τ_R (X), U, E).

Definition 3.3: A soft nano topological space ($\tau_R(X)$, U, E) is called soft nano resolvable, if there exists a soft nano dense set (A, E) such that its complement (A, E) U is also soft nano dense in U. A soft nano topological space $(\tau_R(X), U, E)$ is called soft nano irresolvable if it is not resolvable.

Example 3.4: Let U = {y_1, y_2, y_3, y_4}, E = {s_1, s_2, s_3} and U/R = {{y_1}, {y_2, y_3}, {y_4}}
Let X= {y_1, y_2, y_3}.
Then soft nano topology is given by {$(\tau_R(X), U, E)$}= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)} where (U_R(X), E) = {(s_1, {y_1, y_2, y_3}), (s_2,{y_1, y_2, y_3}),(s_3, {y_1, y_2, y_3})},
(L_R(X), E) = {(s_1, {y_1, y_2, y_3}), (s_2, {y_1, y_2, y_3}), (s_3, {y_1, y_2, y_3})} and (B_R(X), E) = {(s_1, ϕ), (s_2, ϕ), (s_3, ϕ)}
Then the soft nano dense sets are U, (Z_1, E), (Z_2, E), (Z_3, E), (Z_4, E), (Z_5, E), (Z_6, E), (Z_7, E), (Z_8, E), (Z_9, E), (Z_{10}, E), (Z_{11}, E), (Z_{12}, E), (Z_{13}, E).
Then soft nano codense sets are: (Z_1, E) = {(s_1, {y_2}), (s_2, {y_2}), (s_3, {y_2})}, (Z_2, E) = {(s_1, {y_3}), (s_2, {y_3}), (s_3, {y_3})}, (Z_3, E) = {(s_1, {y_4}), (s_2, {y_4}), (s_3, {y_4})}, (Z_4, E) = {(s_1, {y_2, y_4}), (s_2, {y_2, y_4}), (s_3, {y_2, y_4})}, (Z_5, E) = {(s_1, {y_3, y_4}), (s_2, {y_3, y_4}), (s_3, {y_3, y_4})}
Then soft nano nowhere dense set is: (Z_3, E) = {(s_1, {y_4}), (s_2, {y_4}), (s_3, {y_4})}
Here (Z_1, E) = {(s_1, {y_2}), (s_2, {y_2}), (s_3, {y_2})} and its complement (Z_5, E) = {(s_1, {y_1, y_3}), (s_2, {y_1, y_3}), (s_3, {y_1, y_3})} are soft nano dense.
Hence U is soft nano resolvable.

Theorem 3.5: For a soft nano topological space $(\tau_R (X), U, E)$, the following statements are equivalent:

(i) $(\tau_R (X), U, E)$ is soft nano resolvable.

(ii) For any soft nano dense set (A, E) in U, SNInt(A, E) ≠ ϕ.

Proof: (i)⇒(ii): Let (A, E) be any soft nano dense set in U. Then we have SNCl((A, E)□) ≠U, hence SNInt(A, E) ≠ ϕ.
(ii)⇒(i): Suppose $(\tau_R (X), U, E)$ is a soft nano resolvable space. Then there exists a soft nano dense set (A, E) in U such that SNCl((A, E)□) is also soft nano dense in $(\tau_R (X), U, E)$. It follows that SNInt(A, E) = ϕ, which is a contradiction to the hypothesis. Hence, $(\tau_R (X), U, E)$ is soft nano resolvable.

Definition 3.6: In a soft nano topological space $(\tau_R(X), U, E)$, if any two soft nano dense sets in $(\tau_R(X), U, E)$ intersects then it is termed soft nano irresolvable.

Example 3.7: In Example 3.4, the soft nano dense sets (Z_4, E) and (Z_5, E) intersect. Therefore, U is soft nano irresolvable.

Definition 3.8: $(\tau_R(X), U, E)$, a soft nano topological space is called soft nano maximally resolvable if it is SNΔ(U)resolvable where SNΔ(U) = min{ ∣(G, E) ∣ : (G, E) is non empty soft nano open set} and cardinality of SNΔ(U) is called soft nano dispersion character of U.

Example 3.9: Let U= {y_1, y_2, y_3, y_4, y_5}, E = {s_1, s_2, s_3} and U / R = {{y_1, y_3}, {y_2}, {y_4, y_5}}.
Let X = {y_1, y_3, y_4, y_5} ⊆ U.

Then soft nano topology is $\{(\tau_R(X), U, E)\}$= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)} where (U_R(X), E) = {(s_1, {y_1, y_3, y_4, y_5}), (s_2, {y_1, y_3, y_4, y_5}), (s_3, {y_1, y_3, y_4, y_5})},
(L_R(X), E) = {(s_1, {y_1, y_3}), (s_2, {y_1, y_3}), (s_3, {y_1, y_3})} and
(B_R(X), E) = {(s_1, {y_4, y_5}), (s_2, {y_4, y_5}), (s_3, {y_4, y_5})}
Here ∣U∣= 5, ∣ (U_R(X), E) ∣= 4, ∣ (L_R(X), E) ∣= 2, ∣ (B_R(X), E) ∣= 2
Here SNΔ(U) = min {2, 4, 5} =2 and soft nano dispersion character is 3.

Definition 3.10: $(\tau_R(X), U, E)$, a soft nano topological space is called soft nano quasi maximal space if for every soft nano dense set (D, E) in $(\tau_R(X), U, E)$ with SNInt(D, E) ≠ ϕ, SNInt(D, E) is also soft nano dense in $(\tau_R(X), U, E)$.

Example 3.11: Let U = {y_1, y_2, y_3}, E = {s_1, s_2, s_3} and U / R = {{y_1}, {y_2}, {y_3}}.

Let X = {y_1} ⊆ U. Then soft nano topology is $\{(\tau_R(X), U, E)\}$= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)} where (U_R(X), E) = {(s_1, {y_1}), (s_2, {y_1}), (s_3, {y_1})}, (L_R(X), E) = {(s_1, {y_1}), (s_2, {y_1}), (s_3, {y_1})} and (B_R(X), E) = {(s_1, ϕ), (s_2, ϕ), (s_3, ϕ)}
The soft nano dense sets are (N_1, E), (N_2, E), (N_3, E), U. Here (N_1, E) = {(s_1, {y_1}), (s_2, {y_1}), (s_3, {y_1})}, (N_2, E) = {(s_1, {y_1, y_2}), (s_2, {y_1, y_2}), (s_3, {y_1, y_2})} and (N_3, E) = {(s_1, {y_1, y_3}), (s_2, {y_1, y_3}), (s_3, {y_1, y_3}). Therefore, U is soft nano quasi maximal.

Definition 3.12: A soft subset (S, E) of a space $(\tau_R(X), U, E)$ is soft nano faintly open if either (S, E) = ϕ or SNInt(S, E) ≠ ϕ. The collection of all soft nano faintly open sets is denoted by SNFOS.

Example 3.13: In Example 3.4, the soft nano faintly open sets are (O_1, E), (O_2, E), (O_3, E), (O_4, E), (O_5, E), (O_6, E), (O_7, E), (O_8, E), (O_9, E), (O_{10}, E), (O_{11}, E), (O_{12}, E), (O_{13}, E), U, ϕ.

Definition 3.14: A soft subset (D, E) is SNα-dense if every non empty soft subset (B, E) ∈ SNα$(\tau_R(X), U, E)$ has (B, E) ∩ (D, E) ≠ ϕ.

Example 3.15: Let U = {y_1, y_2, y_3, y_4, y_5}, E = {s_1, s_2, s_3} and U / R = {{y_1, y_2}, {y_3}, {y_4, y_5}}. Let X = {y_1, y_3} ⊆ U. Then, (U_R(X), E) = {(s_1, {y_1, y_2, y_3}), (s_2, {y_1, y_2, y_3}), (s_3, {y_1, y_2, y_3})}, (L_R(X), E) = {(s_1, {y_1, y_2}), (s_2, {y_1, y_2}), (s_3, {y_1, y_2})} and (B_R(X), E) = {(s_1, {y_3}), (s_2, {y_3}), (s_3, {y_3})}

Thus soft nano topology is $\{(\tau_R(X), U, E)\}$= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)} and $\{(\tau_R^{\alpha}(X), U, E)\}$= {U, ϕ, {($s_1$, {$y_3$}), ($s_2$, {$y_3$}), ($s_3$, {$y_3$})}, {($s_1$, {$y_1$, y_2}), (s_2, {y_1, y_2}), (s_3, {y_1, y_2})}, {(s_1, {y_1, y_2, y_3}), (s_2, {y_1, y_2, y_3}), (s_3, {y_1, y_2, y_3})}, {(s_1, {y_1, y_2, y_3, y_4}), (s_2, {y_1, y_2, y_3, y_4}), (s_3, {y_1, y_2, y_3, y_4})}, {(s_1, {y_1, y_2, y_3, y_5}), (s_2, {y_1, y_2, y_3, y_5}), (s_3, {y_1, y_2, y_3, y_5})}}

SNα-dense sets are (N_1, E), (N_2, E), (N_3, E), (N_4, E), (N_5, E), (N_6, E), (N_7, E), (N_8, E), (N_9, E), (N_{10}, E), (N_{11}, E), U.

Theorem 3.16: A soft subset (D, E) is SNα-dense if and only if (D, E) ∪ SNCl (SNInt(SNCl (D, E)) = U.

Proof: The union of all soft nano α-open subsets of U - (D, E) is (U - (D, E)) $\cap$ SNInt (SNCl(SNInt (D, E))). If (D, E) is SNα-dense, (U - (D, E)) $\cap$ SNInt (SNCl(SNInt (D, E))), so that U = (D, E) $\cup$SNCl (SNInt(SNCl (D, E))).

Definition 3.17: A soft nano topological space ($\tau_R(X)$, U, E) is soft nano open hereditarily(strongly) irresolvable if each soft nano open subspace is soft nano irresolvable.

Example 3.18: Let U = {y_1, y_2, y_3, y_4}, E = {s_1, s_2, s_3} and U / R = {{y_1}, {y_2}, {y_3, y_4}}

Let X = {y_2} $\subseteq$ U.

Then the soft nano topology {($\tau_R(X)$, U, E)}= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)}, where (U_R(X), E) = {(s_1, {y_2}), (s_2, {y_2}), (s_3, {y_2})}, (L_R(X), E) = {(s_1, {y_2}), (s_2, {y_2}), (s_3, {y_2})} and (B_R(X), E) = {(s_1, ϕ), (s_2, ϕ), (s_3, ϕ)}.Here each soft nano open subspace of ($\tau_R(X)$, U, E) is soft nano irresolvable.

Theorem 3.19: Every soft nano open hereditarily irresolvable space is soft nano irresolvable.
Proof: Let (B, E) ⊆ U be soft nano open set. Since U is soft nano open hereditarily irresolvable where X ⊆ (B, E) and X⊆ U. Also, there exists a pair of soft nano dense sets intersects, it is true for each soft nano open sets. Hence, U is soft nano irresolvable.
Remark 3.20: Converse of the Theorem 3.19 is not true as seen by the below example.

Example 3.21: Let U = {y_1, y_2, y_3}, E = {s_1, s_2, s_3} and U / R = {{y_1}, {y_2}, {y_3, y_4}}

Let X = {y_1} $\subseteq$ U. Then the soft nano topology is {($\tau_R(X)$, U, E)}= {U, ϕ, (L_R(X), E), (U_R(X), E), (B_R(X), E)} where (U_R(X), E) = {(s_1, {y_1}), (s_2, {y_1}), (s_3, {y_1})}, (L_R(X), E) = {(s_1, {y_1}), (s_2, {y_1}), (s_3, {y_1})}and (B_R(X), E) = {(s_1, ϕ), (s_2, ϕ), (s_3, ϕ).
Here the soft nano dispersion character is 1 and hence, it is soft nano irresolvable and since soft nano open subspace is not soft nano irresolvable. Therefore, the space is not soft nano open hereditarily irresolvable.

Theorem 3.22: Soft nano open hereditarily irresolvability (strongly irresolvability) of the space ($\tau_R(X)$, U, E) is equivalent to each of the following.
(i) Every soft nano open subspace is soft nano irresolvable.
(ii) Every soft nano dense subset has a soft nano dense interior.
(iii) Every soft nano codense set is soft nano nowhere dense.
(iv) Every soft nano subset is the union of a soft nano open set and a soft nano nowhere dense set.

Proof:**(i)** $\Rightarrow$ **(ii):** If (D, E) is soft nano dense and $(A,E) = U - SNCl(SNInt(D,E)) \neq \phi$, then $(A,E) - (D,E) \neq \phi$ for otherwise, $(A,E) \subseteq SNInt(D,E) - SNInt(D,E) = \phi$.
Also, $(A,E) \cap (D,E) \neq \phi$.
Since, (D, E) is soft nano dense in (A, E) and (A, E) $\subseteq$ SNCl(A, E) = SNCl((A, E) $\bigcap$ (D, E)). But $SNInt_{(A, E)}$ ((A, E) $\bigcap$ (D, E)) = SNInt((A, E) $\bigcap$ (D, E)) = ϕ .

Since SNInt((A, E) $\bigcap$ (D, E)) $\subseteq$ (A, E) $\bigcap$ SNInt(D, E) = ϕ. Thus, (A, E) - (D, E) is also soft nano dense in (A, E) and (A, E) is soft nano resolvable. This contradiction shows that U - SNCl(SNInt(D, E)) = ϕ. Hence, SNInt(D, E) is soft nano dense.

(ii) $\Rightarrow$ (iii): If (B, E) is a soft nano codense then U- (B, E) is soft nano dense and also SNInt(U-(B, E)) is soft nano dense and SNCl(Int(U-(B, E)))=U-SNInt(SNCl(B, E))=U which implies SNCl(SNInt(B, E)) = ϕ. Hence, (B, E) is a soft nano nowhere dense.

(iii)$\Rightarrow$(iv): $(\mathrm{B,E}) \subseteq \mathrm{U}$, implies $\mathrm{SNInt(B,E)} = \phi$ or $\mathrm{SNInt(B,E)} \neq \phi$.

If $\mathrm{SNInt(B,E)} = \phi$, $(\mathrm{B,E}) \in \mathrm{SNC}(\tau_{\mathrm{R}}(\mathrm{X}), \mathrm{U}, \mathrm{E}) = \mathrm{SNN}(\tau_{\mathrm{R}}(X), \mathrm{U}, \mathrm{E})$ implies $(\mathrm{B,E}) = \phi \cup (\mathrm{B,E})$.
Here (B, E) is the union of a soft nano open set and a soft nano nowhere dense set.
If $\mathrm{SNInt}((\mathrm{B,E}) - \mathrm{SNInt(B,E)}) = \phi \Rightarrow (\mathrm{B,E}) - \mathrm{SNInt(B,E)} \in \mathrm{SNC}(\tau_{\mathrm{R}}, \mathrm{U}, \mathrm{E}) = \mathrm{SNN}(\tau_{\mathrm{R}}, \mathrm{U}, \mathrm{E})$,
$(\mathrm{B,E}) = \mathrm{SNInt(B,E)} \cup ((\mathrm{B,E}) - \mathrm{SNInt(B,E)})$ is union of soft nano open and soft nano nowhere dense set.

(iv)$\Rightarrow$ (i): If (A, E) is non empty soft nano open and soft nano resolvable, then (A, E) = $(\mathrm{A, E})_1 \bigcup (\mathrm{A, E})_2$ with each $(\mathrm{A, E})_1$ soft nano dense in (A, E). We have every soft nano condense set is soft nano nowhere dense. So, $(\mathrm{B, E})_1$ = (G, E) $\bigcup$ (N, E) where (G, E) is soft nano open and (N, E) is soft nano nowhere dense.

Here $(\mathrm{G,E}) = \phi$ which implies $(\mathrm{B,E}) \subseteq \mathrm{SNInt(SNCl(B,E)} = \phi$. So, (G,E)$\neq \phi$.

Implies $\mathrm{SNInt}_{(\mathrm{A,E})}((\mathrm{A,E})_1 = \mathrm{SNInt(A,E)}_1 \neq \phi$.

Implies $(\mathrm{A,E})_1 \cap (\mathrm{A,E})_2 \neq \phi$. This contradiction shows that (A, E) is soft nano irresolvable.

4. LEVELS OF SOFT NANO IRRESOLVABILITY

Theorem 4.1: For any soft nano topological space ($\tau_R(X)$, U, E), soft nano maximally irresolvability implies soft nano hereditarily irresolvability which implies soft nano open hereditarily(strongly) irresolvability which implies soft nano irresolvability.

Proof: If U is a soft nano maximally irresolvable and (A, E) is non empty soft nano resolvable subspace with (A, E) = (A_1, E) $\bigcup$ (A_2, E) and each A_i is soft nano dense in (A, E).Then U - (A_2, E) = (U - (A, E)) $\bigcup$ (A_1, E) is soft nano dense in U and hence soft nano open.
So, (A_2, E) is soft nano closed and (A, E) $\subseteq$ SNCl(A_2, E) = (A_2, E) $\Rightarrow$ (A_1, E) = ϕ $\Rightarrow$ (A, E) = ϕ. Since (A, E) $\subseteq$ SNCl(A_1, E). This contradiction shows that U is soft nano hereditarily irresolvable. If U is soft nano hereditarily irresolvable, all subspaces including soft nano open ones are soft nano irresolvable so that U is soft nano open hereditarily(strongly) irresolvable. Certainly soft nano open hereditarily(strongly) irresolvable spaces are soft nano irresolvable.

5. CHARACTERIZATIONS

Definition 5.1: A soft subset (D, E) is soft nano almost open dense if every non empty soft subset (A, E) $\subseteq$ SNAO($\tau_R(X)$, U, E) has non empty intersection with (D, E). The smallest soft nano topology containing SNAO($\tau_R(X)$, U, E) is ($\tau_R^*(X)$, A, E) = { SNO($\tau_R(X)$, U, E)}.Then (D, E) is τ_R^*(A) -dense, if every non empty soft subset (A, E) $\in$ ($\tau_R^*(X)$, A, E) has (A, E) $\cap$ (D, E) $\neq \phi$.

Theorem 5.2: A soft subset (D, E) is SNAO-dense if and only if (D, E) $\cup$ SNCl(SNInt(D, E)) = U.

Proof: The union of all almost soft nano open subsets of U - (D, E) is U - (D, E) $\cap$ SNInt(SNCl(U - (D, E)). If (D, E) is SNAO-dense, then U - (D, E) $\cap$ SNInt(SNCl(U - (D, E)) = ϕ so that U = (D, E) $\cup$ SNCl(SNInt(D, E)).

Theorem 5.3: The following are equivalent for a soft nano topological space ($\tau_R(X)$, U, E).
(i) U is soft nano open hereditarily irresolvable (strongly irresolvable).
(ii) SNAO($\tau_R(X)$, U, E) $\subseteq$ SNSO($\tau_R(X)$, U, E).
(iii) SNAO($\tau_R(X)$, U, E) $\subseteq$ SNFO($\tau_R(X)$, U, E).
(iv) Every soft nano dense set is SNAO- dense.
(v) Every soft nano dense set is τ_R^*(A) – dense.

Proof: (i) $\Rightarrow$ (ii): If ($\tau_R(X)$, U, E) is soft nano open hereditarily irresolvable (strongly irresolvable) and (P, E) = (Q, E) $\cap$ (D, E) $\in$ SNAO($\tau_R(X)$, U, E) for some (Q, E) $\in$ ($\tau_R(X)$, U, E) and soft nano dense (D, E). Then SNInt(D, E) is soft nano dense and SNCl(U $\cap$ SNInt(D, E)) = SNCl(Q, E) so that (Q, E) $\cap$ (D, E) $\subseteq$ (Q, E) $\subseteq$ SNCl(SNInt(Q, E) $\cap$ SNInt(D, E)) $\subseteq$ SNCl(SNInt(Q, E) $\cap$ (D, E)). Thus, (Q, E) $\in$ SNSO($\tau_R(X)$, U, E).

(ii) $\Rightarrow$ (iii): It is obvious since SNSO($\tau_R(X)$, U, E) $\subseteq$ SNFO($\tau_R(X)$, U, E).

(iii) $\Rightarrow$ (iv): Since every non empty soft nano faintly open set has non empty soft nano interior which must then intersect every soft nano dense set.

(iv) $\Rightarrow$ (i): If (D, E) is soft nano dense in ($\tau_R(X)$, U, E) then (D, E) is SNAO-dense and by Theorem 5.2, U = (D, E) $\cup$ SNCl(SNInt(D, E)).
If SNInt(D, E) is not soft nano dense, (Q, E) = U $\subseteq$ (SNInt(D, E)) - (SNInt(D, E)) = ϕ.
This contradiction shows that (SNInt(D, E) is soft nano dense and hence, ($\tau_R(X)$, U, E) is soft nano open hereditarily irresolvable.

(ii) $\Rightarrow$ (v): Since we have that every soft nano dense set is SNAO-dense and therefore every soft nano dense set is SNAO-dense and hence every soft nano dense set is τ_R^*(A) – dense.

(v) $\Rightarrow$ (iv): Since SNAO($\tau_R(X)$, U, E) $\subseteq$ ($\tau_R^*(X)$, A, E).

Definition 5.4: A soft nano topological space ($\tau_R(X)$, U, E) is soft nano extremally disconnected if SNCl(O, E) is soft nano open for every soft nano open set (O, E).

Theorem 5.5: A soft nano topological space ($\tau_R(X)$, U, E) is soft nano extremally disconnected if and only if SNSO($\tau_R(X)$, U, E) $\subseteq$ SNAO (τ_R (X), U, E).

Proof: If SNSO($\tau_R(X)$, U, E) $\subseteq$ SNAO (τ_R (X), U, E) and (B, E) $\in$ ($\tau_R(X)$, U, E), then SNCl(B, E) $\in$ SNSO($\tau_R(X)$, U, E) implies SNCl(B, E) $\subseteq$ SNInt(SNCl(SNCl(B, E) = SNInt(SNCl(B, E) implies SNCl(B, E) $\in$ {($\tau_R(X)$, U, E)} and hence ($\tau_R(X)$, U, E) is soft nano extremally disconnected.

Conversely, if ($\tau_R(X)$, U, E) is soft nano extremally disconnected and (A, E) $\in$ SNSO($\tau_R(X)$, U, E), then (A, E) $\subseteq$ SNCl(SNInt(A, E) $\in$ ($\tau_R(X)$, U, E) which implies (A, E) $\subseteq$ SNInt(SNCl(SNInt(A, E))) $\subseteq$ SNInt(SNCl(A, E)) and (A, E) $\in$ SNAO($\tau_R(X)$, U, E)

6. FUNCTIONS ON SOFT NANO IRRESOLVABILITY

Definition 6.1: A bijective function f: ($\tau_R(X)$, U, E) $\rightarrow$(τ_{R1}(Y), V, E) is soft nano faint homeomorphism if both f and f^{-1} preserve soft nano faintly open sets.

Theorem 6.2: If f: ($\tau_R(X)$, U, E) $\rightarrow$(τ_{R1}(Y), V, E) is a bijective, f is a soft nano faint homeomorphism if and only if both f and f^{-1} preserve soft nano dense sets.

Corollary 6.3: A composition of two soft nano faint homeomorphisms is a soft nano faint homeomorphism.

Theorem 6.4: Every soft nano faint homeomorphism directly and inversely preserves soft nano nowhere dense sets.

Proof: Let f: ($\tau_R(X)$, U, E) $\rightarrow$(τ_{R1}(Y), V, E) be a soft nano faint homeomorphism.

Let (N, E) be a soft nano nowhere dense, where (N, E) is a soft subset of U. Let (O, E) be any non empty soft nano open subset of V. Then, we have SNInt(f^{-1}(O, E)) = (M, E) $\neq \phi$ and so there exists a non empty soft nano open subset $(\mathrm{M,E})' \subseteq (\mathrm{M,E})$ such that (M, E)□ $\bigcap$ (N, E) = ϕ.

It follows that $\phi \neq (\mathrm{N, E})' = \mathrm{SNInt}(\mathrm{f}(\mathrm{O,E})') \subseteq (\mathrm{N, E})$ and $(\mathrm{N, E})' \cap \mathrm{f}(\mathrm{M,E}) = \phi$ showing that f(E) is soft nano nowhere dense.

So, f and, by symmetry of argument, f^{-1} perserve soft nano nowhere dense sets.

Theorem 6.5: If f: ($\tau_R(X)$, U, E) $\rightarrow$(τ_{R1}(Y), V, E) is a bijection, f is a soft nano faint homeomorphism, then direct and inverse images under f of non empty soft nano almost open sets contain non empty soft nano almost open sets.

Proof: Suppose that $(\mathrm{A, E}) \subseteq \mathrm{U}$ is soft nano almost open and non empty. Then, (A, E) = U $\cap$ (D, E) for some non empty soft nano open set $(\mathrm{D, E}) \subseteq \mathrm{U}$. Then, f(O, E) = f(O, E) $\cap$ f(D, E). Since f is a soft nano faint homeomorphism, f(D, E) is soft nano dense in (τ_{R1}(Y), V, E) and $(\mathrm{V, E}) = \mathrm{SNInt}(\mathrm{f}(\mathrm{O,E})) \neq \phi$. Thus, the soft nano almost open set (V, E) $\cap$ f(D, E) is non empty soft nano subset of f(B, E). Similar is the argument for inverse images.

7. CONCLUSION

This paper presents the aspects of soft nano resolvable and soft nano irresolvable spaces in soft nano topology. We defined soft nano irresolvable spaces in terms of soft nano almost open (preopen) sets. Also, the characterizations of soft nano irresolvable and soft nano open hereditarily irresolvable spaces are obtained. Further, weaker forms of soft nano irresolvable are studied. Soft nano topology plays a prominent role in solving real world problems.

REFERENCES

1. S.S. Benchalli, P.G. Patil, N.S. Kabbur and J.Pradeepkumar, On soft nano topological spaces, (Communicated).
2. S.S. Benchalli, P.G. Patil, N.S.Kabbur and J.Pradeepkumar, Weaker forms of soft nano open sets, *J. Comput. & Math. Sci*., Vol.8 (11), 589-599 (2017).
3. S.S.Benchalli, P.G.Patil, N.S.Kabbur and J.Pradeepkumar, On δ-operation in soft nano topological spaces, *J. Comput. & Math. Sci*., Vol.9 (8), 1001-1016 (2018).
4. R.Bolstein, Sets of points of discontinuity, *Proc. Amer. Math. Society*, Vol. 38(1), 193-197 (1973).
5. M. Ganster, Preopen sets and resolvable spaces, *Kyungpook Math. J*., Vol.27 (2), 135- 143 (1987).
6. E. Hewitt, A problem of set theoretic topology, *Duke Math. J*., Vol. 10, 309-333 (1943).
7. D. Molodtsov, Soft set theory-first results, *Comput. Math. Appl*, Vol.37, 19-31 (1999).
8. P. G. Patil and Nagashree N. Bhat, On some weaker forms of generalization of soft nano closed sets in soft nano topological spaces (Communicated).
9. D. Rose, K. Sizemore and B.Thurston, Strongly irresolvable spaces, Hindawi Publishing Corporation, Vol., 1-12 (2006).
10. M. Shabir and M. Naz, On soft topological spaces, *Comput. Math. Appl*, Vol.61, 1786-1799 (2011).
11. M. L. Thivagar and C. Richard, On nano forms of weakly open sets, *Int. J. of Math. & Stat. Invention*, Vol.1 (1), 31-37 (2013).
12. M. L. Thivagar and J. Kavitha, On nano resolvable spaces, *Missouri J. Math. Sci*., Vol. 29(1), 80-91 (2017).