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Total Edge Detour Monophonic Number of a Graph

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ABSTRACT

In this paper the concept of *total edge detour monophonic number M of a graph G* is introduced. For a connected graph $G = (V, E)$ of order at least two, a *total edge detour monophonic set M* of a graph *G* is an edge detour monophonic set such that either $M = V$ or the sub graph induced by M has no isolated vertices. The minimum cardinality of a total edge detour monophonic set of G is the *total edge detour monophonic number* of *G* and is denoted by $edm_t(G)$. We determine bounds for it and characterize graphs which realize these bounds. It is shown that if every positive integers p, a and b such that $3 \leq a \leq b \leq p-2$, then there exists a connected graph G of order p, $edm(G) = a$ and $edm_t(G) = b$. If p and k are positive integers such that $3 \leq k \leq p$, then there exists a connected graph G of order p, $edm_t(G) = k$. For positive integers a, b such that $4 \leq a \leq b$ with $b \leq 2a$ there exists a connected graph G such that $edm(G) = a$ and $edm_t(G) = b$.

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1. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q, respectively. For basic graph theoretic terminology we refer to Harary[1] and[2]. The *neighborhood* of a vertex v of G is the set $N(v)$ consisting of all vertices u which are adjacent with v. A vertex v of G is an *extreme* vertex if the sub graph induced by its neighborhood is complete. A vertex with degree one is called an *end vertex*. A vertex v of a connected graph G is called a *support vertex* of G if it is adjacent to an end vertex of G. A vertex v in a connected graph G is a *cut vertex* of G, if $G - v$ is disconnected. A *chord* of a path $u_1, u_2, u_3, \dots, u_k$ in G is an edge $u_i u_j$ with $j \geq i + 2$. A path P is called a *monophonic path* if it is a chord less path. A set M of vertices of G is a *monophonic set* of G if each vertex of G lies on $u - v$ monophonic path for some u and v in M. The minimum cardinality of a monophonic set of G is the *monophonic number* of G and is denoted by $m(G)$ [3]. A longest $x - y$ monphonic path is called an $x - y$ detour monophonic path. An edge detour monophonic set of G is a set M of vertices such that every edge of G lies on a edge detour monophonic path joining some pair of vertices in M. The minimum cardinality of an edge detour monophonic set of G is the *edge detour monophonic number* of G and is denoted by $edm(G)$ [6]. *Total edge detour monophonic set* M of a graph G is a edge detour monophonic set such that either $M = V$ or the sub graph induced by M has no isolated vertices. The minimum cardinality of a total edge detour monophonic set of G is the *total edge detour monophonic number* of G and is denoted by $edm_t(G)$.

The following theorems will be used in the sequel.

Theorem 1.1.[4] Each extreme vertex of a connected graph G belongs to every detour monophonic set of G. Moreover, if the set M of all extreme vertices of G is a detour monophonic set, then M is the unique minimum detour monophonic set of G.

Theorem 1.2.[6] Let G be a connected graph with cut – vertex v and let M be a edge detour monophonic set of G. Then every component of $G - v$ contains an element of M.

Theorem 1.3.[3] No cut vertex of a connected graph G belongs to any minimum monophonic set of G.

Theorem 1.4.[6] Each extreme vertex of a connected graph G belongs to every edge detour monophonic set of G.

Theorem 1.5.[4] If T is a tree with k end vertices, then $dm(T) = k$.

Theorem 1.6.[6] For the complete graph $K_p (p \geq 2,), edm(K_p) = p$.

Theorem 1.7.[6] Each semi-extreme vertex of a graph G belongs to every edge detour monophonic set of G. In particular, if the set M of all semi-extreme vertices of G is an edge detour monophonic set, then M is the unique minimum edge detour monophonic set of G.

Theorem 1.8[6] No cut vertex of a connected graph G belongs to any minimum edge detour monophonic set of G.

Throughout this paper G denotes a connected graph with at least two vertices.

2. TOTAL EDGE DETOUR MONOPHONIC NUMBER OF A GRAPH

Definition 2.1. An edge detour monophonic set of G is a set M of vertices such that every edge of G lies on a edge detour monophonic path joining some pair of vertices in M. The minimum cardinality of an edge detour monophonic set of G is the *edge detour monophonic number* of G and is denoted by $edm(G)$ [5]. *Total edge detour monophonic set M* of a graph G is a edge detour monophonic set such that either $M = V$ or the sub graph induced by M has no isolated vertices. The minimum cardinality of a total edge detour monophonic set of G is the *total edge detour monophonic number* of G and is denoted by $edm_t(G)$.

Example 2.2. In Figure 1, $M_1 = \{v_1, v_3\}$ is a minimum edge detour monophonic set. That is $edm(G) = 2$. The set $M_2 = \{v_1, v_3, v_4\}$ is a total edge detour monophonic set. Thus $edm_t(G) = 3$.

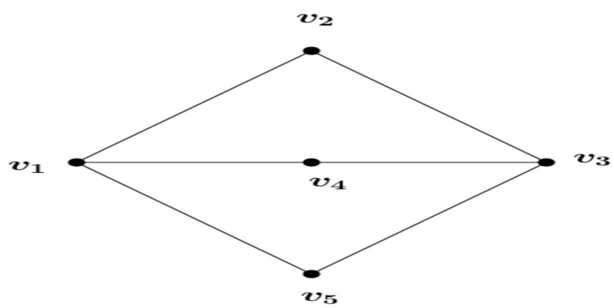

Figure 1

Theorem 2.3. Each extreme vertex and support vertex of a connected graph G belongs to every total edge detour monophonic set of G. Moreover, if the set M of all extreme vertices of G is a total edge detour monophonic set, then M is the unique minimum total edge detour monophonic set of G.

Proof : Since every total edge detour monophonic set of G is a edge detour monophonic set of G, by Theorem 1.1, each extreme vertex of G belongs to every total edge detour monophonic set of G. Since a total edge detour monophonic set of G contains no isolated vertices , it follows that each support vertex of G belongs to every total edge detour monophonic set of G. If the set M is the set of all extreme vertices and support vertices of G, then $edm_t(G) \geq |M|$. On the other hand, if M is a total edge detour monophonic set of G, then $edm_t(G) \leq |M|$. Therefore $edm_t(G) = |M|$ and M is the unique minimum total edge detour monophonic set of G.

In particular if G is a extreme graph then $V(G)$ is the unique minimum total edge detour monophonic set of G. Note that the set of all extreme vertices need not form a total edge detour monophonic set of G. Path graph with more than 5 vertices is an example.

Corollary 2.4. For the complete graph $K_p (p \geq 2)$, $edm_t(G) = p$.

Theorem 2.5. Let G be a connected graph with cut vertices and let M be a total edge detour monophonic set of G . If v is a cut - vertex of G then every component of $G - v$ contains an element of M.

Proof : Since every total edge detour monophonic set of G is a edge detour monophonic set of G, the result follows from Theorem 1.2.

Theorem 2.6. For a connected graph G of order p, $2 \leq dm(G) \leq edm(G) \leq edm_t(G) \leq p$.

Proof: Any detour monophonic set of G needs at least two vertices and so that $dm(G) \geq 2$. Since every detour monophonic set of G is also a edge detour monophonic set of G, it follows that $dm(G) \leq edm(G)$. Also Since every total edge detour monophonic set of G is also an edge detour monophonic set of G, it follows that $edm(G) \leq edm_t(G)$. Since $V(G)$ is a total edge detour monophonic set of G, it is clear that $edm_t(G) \leq p$. Hence $2 \leq dm(G) \leq edm(G) \leq edm_t(G) \leq p$.

Corollary 2.7. Let G be a connected graph, if $edm_t(G) = 2$, then $edm(G) = 2$.

For any non – trivial path of order at least 5, the edge detour monophonic number is 2 and the total edge detour monophonic number is 4. This shows that the converse of the Corollary 2.7 need not be true.

Remark 2.8. The bounds in Theorem 2.6 are sharp. For complete graph $G = K_2$, $edm_t(G) = 2$ and for the complete graph $G = K_p$, $edm_t(G) = p$. For the graph given in Figure 2, $M_1 = \{v, w, z\}$ is a minimum detour monophonic set of G. That is $dm(G) = 3$. $M_2 = \{v, w, x, z\}$ is a minimum edge detour monophonic set of G. That is $edm(G) = 4$. The set $M_3 = \{v, w, x, y, z\}$ is a minimum total edge detour monophonic set of G. Thus $edm_t(G) = 5$. Clearly order of the graph G is $p = 6$ so that $2 < dm(G) < edm(G) < edm_t(G) < p$. Hence all the parameters in Theorem 2.6 are distinct.

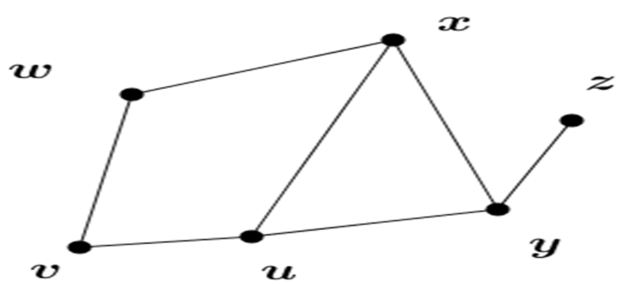

Figure 2

Theorem 2.9. Let G be a connected graph of order $p \geq 2$. Then, $G = K_2$ if and only if $edm_t(G) = 2$.

Proof : If $G = K_2$, then $edm_t(G) = 2$. Conversely, let $edm_t(G) = 2$. Let $M = \{u, v\}$ be a minimum total edge detour monophonic set of G. Then uv is an edge. It is clear that a vertex different from u and v cannot lie on a $u - v$ total edge detour monophonic path of G and so $G = K_2$.

Corollary 2.10. For any nontrivial tree T of order p, $edm_t(G) = p$.

Proof : It follows from Theorems 1.5 and 2.3.

Result 2.1. For the Wheel graph W_n with n vertices, $edm_t(W_n) = n - 1, n \geq 6$.

Result 2.12. For the Star graph $G = K_{1,n-1}$, $edm_t(G) = n$.

Result 2.13: For the complete bipartite graph $K_{m,n}$, $edm_t(G) = \begin{cases} 3, if\ 2 = m \leq n \\ 4, \ if\ 3 \leq \ m \leq n \end{cases}$.

3. REALIZATION RESULTS

Theorem 3.1. If every positive integers p, a and b such that $3 \leq a \leq b \leq p-2$, then there exists a connected graph G of order p, $edm(G) = a$ and $edm_t(G) = b$.

Proof : We prove this result by considering two cases.

Case 1: Let $3 \leq a = b \leq p-2$. Let K_{a-2} be the complete graph with the vertex set $x_1, x_2, x_3 \dots, x_{a-2}$ and C_4 be the cycle of order 4 with vertices x, y, z, w, x. Let H be the graph obtained from K_{a-2} and C_4 by joining each $x_i (1 \leq i \leq a-2)$ to the vertices z and y in cycle C_4. Let G be the graph obtained from H by adding $p-a-2$ new non adjacent vertices $v_1, v_2, v_3 \dots, v_{p-a-2}$ to the graph H and join each $v_i (1 \leq i \leq p-a-2)$ to the vertices x and z in cycle C_4. The graph G is shown in Figure 3.

Let $M = \{x_1, x_2, x_3 \dots, x_{a-2}\}$ be the set of all extreme vertices of G by Theorem 1.1 every edge detour monophonic set contains M. It is clear that M is not an edge detour monophonic set of G. $M_1 = M \cup \{x, y\}$ is an edge detour monophonic set and also M_1 is the total edge detour monophonic set. Therefore clearly $edm(G) = a = edm_t(G)$.

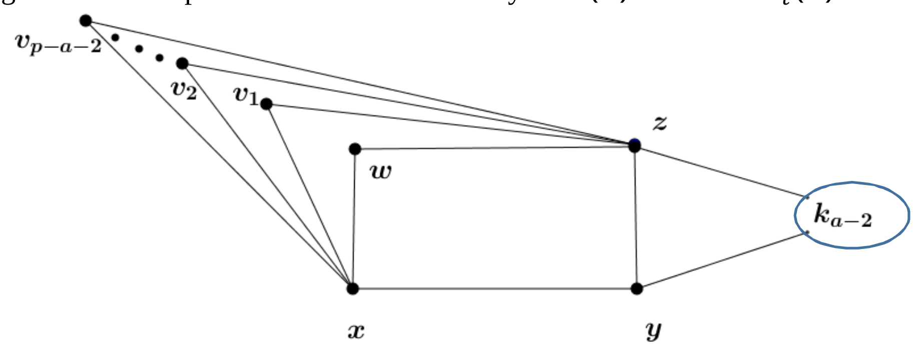

Figure 3

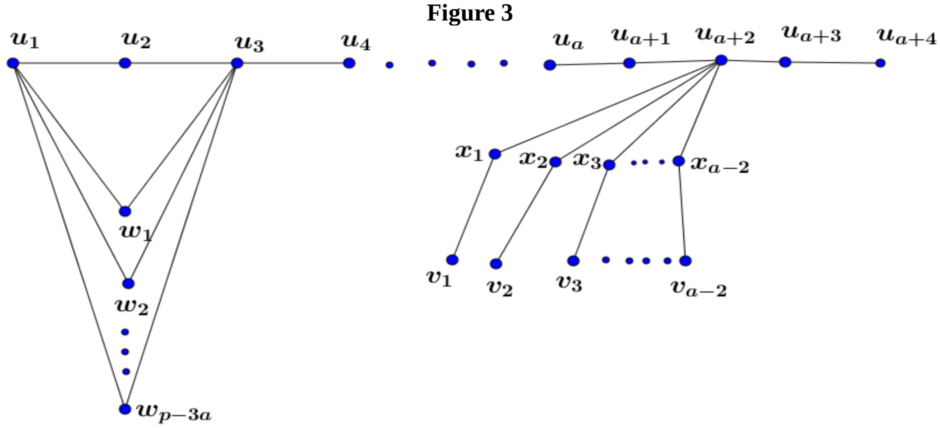

Figure 4

Case 2: Let $b = 2a$. Let $3 \leq a < b \leq p-2$. Let P_{a+4}: $u_1, u_2, u_3 \dots, u_{a+4}$ be the path of length $a+3$. Let H be the graph obtained by adding new vertices $x_1, x_2, x_3 \dots, x_{a-2}$ and $w_1, w_2, w_3 \dots, w_{p-3a}$. Now join each $x_i (1 \leq i \leq a-2)$ to u_{a+2} and also join each $w_i (1 \leq i \leq p-2a)$ to u_1 and u_3 as shown in the Figure 4. Let G be the graph obtained from H by adding new non adjacent vertices $v_1, v_2, v_3 \dots, v_{a-2}$ and join each $x_i (1 \leq i \leq a-2)$ to each $v_i (1 \leq i \leq a-2)$. Then G has order p. Let $N = \{u_{a+4}, v_1, v_2, v_3 \dots, v_{a-2}\}$ be the set of all end vertices of G. The set $M_3 = N \cup \{u_1\}$ is the edge detour monophonic set of G and so $edm(G) = a-2+2 = a$. Let $M_4 = M_3 \cup \{u_2, x_1, x_2, x_3 \dots, x_{a-2}, u_{a+4}\}$ is the total edge detour monophonic set of G and so $edm_t(G) = a + a - 2 + 2 = 2a = b$.

Theorem 3.2. If p and k are positive integers such that $3 \leq k \leq p$, then there exists a connected graph G of order p, $edm_t(G) = k$.

Proof : We prove this result by considering two cases.

Case 1: Let $k = 3$. Let P_3: v_1, v_2, v_3 be the path of order 3. Now add $p-3$ new vertices $w_1, w_2, w_3 \dots, w_{p-3}$ to P_3. Let G be the graph obtained by joining each $w_i (1 \leq i \leq p-3)$ to v_1 and v_3. Then G has order p. The graph G is shown in Figure 5. Take the set $M = \{v_1, v_3\}$. It is easily verified that M is the unique edge detour monophonic set of G. Clearly $M_1 = M \cup \{v_3\}$ is the unique total edge detour monophonic set of G and so that $edm_t(G) = 3 = k$.

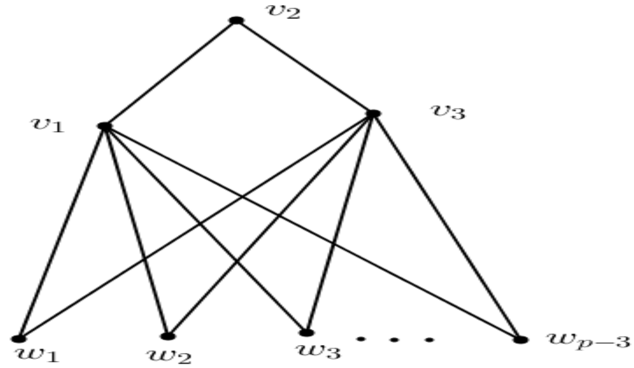

Figure 5

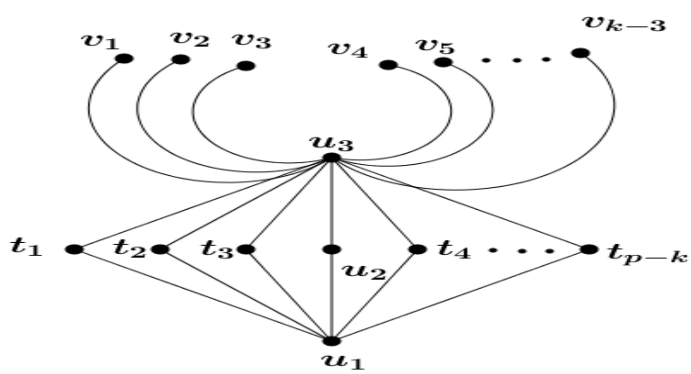

Figure 6

Case 2: Let $k > 3$. Let P_3: u_1, u_2, u_3 be the path of order 3. Let H be the graph obtained from P_3 by adding the new vertices $t_1, t_2, t_3 \dots, t_{p-k}$ and joining each $t_i (1 \leq i \leq p-k)$ to u_1 and u_3. Let G be the graph obtained from H by adding new vertices $v_1, v_2, v_3 \dots, v_{k-3}$ and also joining each $v_i (1 \leq i \leq k-3)$ to u_3. Then the graph G has order $p - k + 3 + k - 3 = p$. The graph G is shown in Figure 6. From the graph G we have that $M_2 = \{v_1, v_2, v_3 \dots, v_{k-3}, u_1\}$ be the edge detour monophonic set and $edm(G) = k - 3 + 1 = k - 2$. Also $M_3 = M_2 \cup \{u_2, u_3\}$ be the total edge detour monophonic set of G and $edm_t(G) = k - 2 + 2 = k$.

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