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# An Inventory Model for Inverse Chain Deteriorating Items with Price Dependent Demand Rate and Partial Backlogging

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**ABSTRACT**

In this paper, we have developed a deterministic inventory model for deteriorating items which follows Inverse chain distribution function. Demand rate is taken as a function of selling price. Also, time varying holding cost is taken in to account. Shortages are allowed here and are partially backlogged. The model is illustrated with the help of numerical example and verified graphically. The sensitivity analysis is also carried out based on numerical example.

**Keywords:** Inventory, Deterioration, Inverse Chain distribution, Shortages, Demand.

## 1. INTRODUCTION

Inventory models create a lot of interest due to their applicability at various places like market, ware houses, production process, transportation systems and cargo handling etc. Most of inventory models have been developed and analyzed to study various inventory systems. The most important factors influencing the inventory systems are demand, nature of goods and shortages etc. Traditional inventory systems had considered the deterioration rate constant but in reality deterioration rate are time dependent, stock dependent and randomly variable.

Mostly physical goods undergo deteriorate or decay over time. Fruits, vegetables and food items suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film etc. deteriorate through a gradual loss of potential or utility with the passage of time. So, many inventory researchers felt the necessity to use the deterioration factor into consideration.

Whiten (1957) considered the deterioration of the fashion goods at the end of a prescribed shortage period. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. An order level inventory model for deteriorating items at a constant rate was presented by Shah and Jaiswal (1977), Aggarwal(1978), Dave and Patel(1981). Inventory model with a time-dependent rate of deterioration were considered by Deb and Chaudhry (1986). Some of the recent work in this field has been done by Chung and Ting(1993), Giri and Chaudhry(1997), Jalan and Chaudhry(1999).

Displayed stock (inventory) level plays an important role to attract consumers. For certain type of goods, particularly for consumer goods, the demand is proportional to the stock level. Gupta and Vrat (1986) developed an inventory model where demand rate is replenishment size (initial stock) dependent. They analyzed the model through cost minimization. Urban (1992) developed an inventory model with instantaneous inventory level-dependent demand rate. Mandal and Phaujdar (1989) analyzed two inventory models considering linearly stock dependent demand rate, first with instantaneous replenishment without shortages and second with finite replenishment rate allowing shortages.

In most of the inventory model, holding cost is known and is considered as constant. But in real life situation holding cost need not be a constant as because other parameters may govern the holding cost. In generalization of EOQ (Economic Order Quantity) models, various functions describing holding cost were considered by several researchers like Naddor (1996), Muhlemann and Valtis-Spanopoulos (1980) and Gho (1994). Giri and Chaudhry (1997) treated the holding cost as a non-linear function of the length of the time for which the item is held in stock and as a functional form of the amount of the on-hand inventory. Roy (2008) developed an EOQ model for deteriorating items where deterioration rate and holding cost are expressed as linearly increasing functions of time and demand rate is a function of selling price and shortages are allowed and completely backlogged.

In this paper, we have developed a deterministic inventory model for deteriorating items which follows Inverse chain distribution function. Demand rate is taken as a function of selling price. Also, time varying holding cost is taken in to account. Shortages are allowed here and are partially backlogged. The model is illustrated with the help of numerical example and verified graphically. Also sensitivity analysis is carried out based on numerical example.

## 2. ASSUMPTIONS AND NOTATIONS

The fundamental assumptions and notations of this model are as follows

(i) Demand rate is considered as selling price dependent of the items in linear form $D(s) = a - s$ where $a > 0$

(ii) The ordering cost A is constant

(iii) The deterioration of items as follows by inverse chain distribution $\theta(t) = \alpha\beta t^{-(\beta+1)} e^{t^{-\beta}} \quad , 0 < \alpha < 1 \text{ and } \beta > 0$

(iv) q is the ordering quantity

(v) h is the inventory holding cost per unit item per unit time

(vi) s is the selling price per unit item

(vii) $\gamma$ is partial backlogging rate, $0<\gamma<1$

(viii) Q is the optimal inventory model

(ix) The length of the cycle is T

(x) $C_1$ is the shortage cost per unit item per unit time

(xi) $C_2$ is the deterioration cost per unit item per unit time

(xii) $C_3$ is the lost sale cost per unit item per unit time

## 3. MATHEMATICAL FORMULATION

During time $t_1$ inventory is depleted due to deterioration and demand of item. At time $t_1$ the inventory becomes zero and shortages start occurring. Let I(t) be the inventory level at time $t$ $0 \le t \le T$ . The differential equations to describe instantaneous state over (0,T) are given by

$$\frac{dI(t)}{dt}+\alpha\beta t^{-(\beta+1)}e^{t^{-\beta}}I(t)=-(a-s) \tag{1}$$

and

$$\frac{dI(t)}{dt}=-(a-s), \quad t_1 \le t \le T \tag{2}$$

The boundary condition of above inventory is taken as

$I(t_1)=0 \; at \; t=t_1$

## 4. MATHEMATICAL SOLUTION

Since the differential equation (1) and (2) are linear differential equation, therefore to solve equation (1) and (2) first we find integrating factor (I.F.) as

$$I.F.=e^{\int \alpha\beta t^{-1+\beta} e^{t^{-\beta}} dt}=e^{-\alpha e^{t^{-\beta}}}=1-\alpha e^{t^{-\beta}}+\frac{\alpha^2}{2!}e^{t^{-\beta}}-..................$$

Neglegting the higher order of $\alpha$ the solutions of $1$ is given as

$$I(t)\; 1-\alpha e^{t^{-\beta}} =- \; a-s \; \int 1-\alpha e^{t^{-\beta}} \; dt+C$$

$$I(t)\; 1-\alpha t^{-\beta} =- \; a-s \; \left(t+\frac{\alpha}{\beta}t^{1+\beta}e^{t^{-\beta}}\right)+C$$

Using Boundary condition I(t)=0 at t=$t_1$, we get

$$C= \; a-s \; \left(t_1+\frac{\alpha}{\beta}t_1^{1+\beta}e^{t_1^{-\beta}}\right)$$

Therefore, we have

$$I(t)\ 1-\alpha t^{-\beta}\ =(a-s)\left\{\ t_1-t\ +\frac{\alpha}{\beta}\ t_1^{1+\beta}e^{t_1^{-\beta}}-t^{1+\beta}e^{t^{-\beta}}\ \right\}$$

$$I(t)=(a-s)\left\{(t_1-t)(1+\alpha t^{-\beta})+\frac{\alpha}{\beta}(t_1^{1+\beta}e^{t_1^{-\beta}}-t^{\beta+1}e^{t^{-\beta}})\right\},\ 0\le t\le t_1 \tag{3}$$

The solution of equation (2) is given as

$$dI(t)=-\ a-s\ dt$$

$$\int dI(t)=-\ a-s\ \int dt+C$$

$$I(t)=-(a-s)t+C$$

Using boundary condition I(t)=0 at $t=t_1$ in above equation, we get

$$C=\ a-s\ t_1$$

Therefore, we have

$$I(t)=(a-s)(t_1-t),\qquad t_1\le t\le T \tag{4}$$

Total holding cost during the time $0\ to\ t_1$ is

$$H=h\int_0^{t_1}I(t)dt$$

$$H=h\ a-s\ \int_0^{t_1}\left\{\ t_1-t\ \ 1+\alpha t^{-\beta}\ +\frac{\alpha}{\beta}\ t_1^{1+\beta}e^{t_1^{-\beta}}-t^{1+\beta}e^{t^{-\beta}}\ \right\}dt$$

$$H=h(a-s)\left\{\frac{t_1^2}{2}+\frac{\alpha}{1-\beta\ \ 2-\beta}t_1^{2-\beta}+\frac{\alpha}{\beta}e^{t_1^{-\beta}}(t_1^{\beta+2}-\frac{t_1^{\beta+2}+t_1^{\beta}}{\beta+1})\right\} \tag{5}$$

Total deterioration cost for the period of 0 to $t_1$ is given by

$$D=C_2\int_0^{t_1}\theta(t)I(t)dt$$

$$D=C_2\alpha\beta(a-s)\int_0^{t_1}t^{-\ 1+\beta}\ e^{t^{-\beta}}\left\{\ t_1+\alpha t_1t^{-\beta}-t-\alpha t^{1-\beta}\ +\frac{\alpha}{\beta}\ t_1^{1+\beta}e^{t_1^{-\beta}}-t^{1+\beta}e^{t^{-\beta}}\ \right\}dt$$

$$D=C_2\alpha\beta(a-s)\left\{\begin{array}{l}\frac{t_1}{\beta}(1-e^{t_1^{-\beta}})-\frac{\alpha}{\beta}t_1^{1-\beta}(e^{t_1^{-\beta}}-1)+\frac{1}{(\beta-1)}t_1^{-(1+\beta)}e^{t_1^{-\beta}}+\frac{\alpha}{(2\beta-1)}e^{t^{-\beta}}\\+\frac{\alpha}{2\beta}t_1^{-(\beta+1)}e^{2t_1^{-\beta}}+\frac{\alpha}{\beta^2}t_1^{1+\beta}(e^{t_1^{-\beta}}-e^{2t_1^{-\beta}})\end{array}\right\} \tag{6}$$

The total shortage cost for the period $t_1\ to\ T$ is given by

$$S = -C_1(a-s)\int_{t_1}^{T} t_1 - t \; dt$$

$$S = -C_1 \; a-s \; \left( t_1 T - \frac{T^2}{2} - t_1^2 + \frac{t_1^2}{2} \right)$$

$$S = \frac{C_1}{2} \; a-s \quad T-t_1 \; ^2 \tag{7}$$

The lost sale cost is given by

$$\text{Lost sale cost} = -C_3 \int_{t_1}^{T} (1-\gamma)(a-s)dt \; = -C_3(1-\gamma)(a-s)(T-t_1) \tag{8}$$

Now total profit per unit time is given by

$$P(T,s) = s(a-s) - \frac{1}{T}\begin{bmatrix} Ordering\cos t + Holding\cos t + Deterioration\,Cost \\ +Shortage\,Cost + Lost\;sale\cos t \end{bmatrix} \tag{9}$$

The necessary conditions for maximizing the total profit function are

$\dfrac{\partial P(T,s)}{\partial T} = 0 \; and \; \dfrac{\partial P(T,s)}{\partial s} = 0,$ which gives

$$\frac{\partial P(T,s)}{\partial T} = \frac{1}{T^2}\left[\begin{array}{l} A + h(a-s)\left\{\begin{array}{l} \dfrac{t_1^2}{2} + \dfrac{\alpha}{(1-\beta)(2-\beta)} t_1^{2-\beta} \\ + \dfrac{\alpha}{\beta} e^{t_1^{-\beta}} (t_1^{\beta+2} - \dfrac{t_1^{\beta+2} + t_1^{\beta}}{\beta+1}) \end{array}\right\} \\ + C_2\alpha\beta(a-s)\left\{\begin{array}{l} \dfrac{t_1}{\beta}(1-e^{t_1^{-\beta}}) - \dfrac{\alpha}{\beta} t_1^{1-\beta}(e^{t_1^{-\beta}} - 1) + \dfrac{1}{(\beta-1)} t_1^{-(1+\beta)} e^{t_1^{-\beta}} \\ + \dfrac{\alpha}{(2\beta-1)} e^{t^{-\beta}} + \dfrac{\alpha}{2\beta} t_1^{-(\beta+1)} e^{2t_1^{-\beta}} + \dfrac{\alpha}{\beta^2} t_1^{1+\beta}(e^{t_1^{-\beta}} - e^{2t_1^{-\beta}}) \end{array}\right\} \\ + \dfrac{C_1}{2}(a-s)(T-t_1)^2 - C_3(1-\gamma)(a-s)(T-t_1) \end{array}\right]$$

$$-\frac{1}{T} \; C_1(a-s)(T-t_1) - C_3(1-\gamma)(a-s) \; = 0 \tag{10}$$

$$\frac{\partial P(T,s)}{\partial s} = (a-2s) + \frac{1}{T}\left[\begin{array}{l} h\left\{\dfrac{t_1^2}{2} + \dfrac{\alpha}{(1-\beta)(2-\beta)} t_1^{2-\beta} + \dfrac{\alpha}{\beta} e^{t_1^{-\beta}} (t_1^{2+\beta} - \dfrac{t_1^{2+\beta}}{1+\beta} + \dfrac{t_1^{\beta}}{1+\beta})\right\} \\ + C_2\alpha\beta\left\{\begin{array}{l} (\dfrac{t_1}{\beta} + \dfrac{\alpha}{\beta} t_1^{1-\beta})(1-e^{t_1^{-\beta}}) + \dfrac{1}{(\beta-1)} t_1^{-\beta} e^{t_1^{-\beta}} \\ + \dfrac{\alpha}{(2\beta-1)} t_1^{1-2\beta} e^{t_1^{-\beta}} + \dfrac{\alpha}{2\beta} t_1^{-(1+\beta)} e^{2t_1^{-\beta}} + \dfrac{\alpha}{\beta^2} t_1^{1+\beta}(e^{t_1^{-\beta}} - e^{2t_1^{-\beta}}) \end{array}\right\} \\ + \dfrac{C_1}{2}(T-t_1)^2 - C_3(1-\gamma)(T-t_1) \end{array}\right] = 0 \tag{11}$$

From the equations (10) and (11), we can calculate the optimum value of $T^*$ and $s^*$ simultaneously and the optimum value $P^*(T,s)$ of the average net profit is determined by (9) provided they satisfy the sufficiency conditions for maximizing P(T,s)

$$\frac{\partial^2 P(T,s)}{\partial T^2} < 0, \frac{\partial^2 P(T,s)}{\partial s^2} < 0 \tag{12}$$

and $$\frac{\partial^2 P(T,s)}{\partial T^2}\frac{\partial^2 P(T,s)}{\partial s^2} - \left(\frac{\partial^2 P(T,s)}{\partial T \partial s}\right)^2 > 0 \tag{13}$$

If the solution obtained from equations (10) and (11) do not satisfy the sufficiency conditions (12) and (13), we conclude that that no feasible solution will be optimal for the set of parameter values taken to solve equations (10) and (11). Such a situation will imply that the parameter values are inconsistent and there is some error in estimation.

## 5. NUMERICAL EXAMPLE

Let us consider A=$ 475, a=210, $C_1$=$1.3, $C_2$=$ 0.5, $C_3$=$ 0.2, h=2, α=0.01, β=1.5, $\gamma$ =0.2, t=1.25 year.

Based on above input data and using the software R programming, we calculate the optimal value of $P^*(T,s)$, $s^*$ and $T^*$ by equations number (9), (10), (11).

$P^*(T,s)$=10659.28, $s^*$=100 and $T^*$=2.256 year,

## 6. SENSITIVITY ANALYSIS

To study the effects of changes of the parameters on the optimal profit $P^*(T,s)$, $s^*$ and $T^*$, a sensitivity analysis is executed considering the numerical example given above. Sensitivity analysis is executed by changing (increasing or decreasing) the parameters by 10% and 20% taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in the following table:

| Parameters | Change (%) | Change in | | |
|---|---|---|---|---|
| | | $P^*(T,s)$ | $s^*$ | $T^*$ |
| A | -20 | 10702.11 | 100 | 2.179 |
| | -10 | 10680.51 | 100 | 2.210 |
| | +10 | 10638.39 | 100 | 2.290 |
| | +20 | 10617.82 | 100 | 2.326 |
| a | -20 | 6747.62 | 86.642 | 2.365 |
| | -10 | 8608.63 | 95.118 | 2.316 |
| | +10 | 12734.97 | 100 | 2.195 |
| | +20 | 14811.52 | 100 | 2.148 |
| α | -20 | 10659.14 | 100 | 2.256 |
| | -10 | 10659.21 | 100 | 2.256 |
| | +10 | 10659.34 | 100 | 2.255 |
| | +20 | 10659.41 | 100 | 2.255 |

| | | | | |
|---|---|---|---|---|
| β | -20 | 10659.37 | 100 | 2.251 |
| | -10 | 10658.99 | 100 | 2.255 |
| | +10 | 10660.09 | 100 | 2.255 |
| | +20 | 10662.20 | 100 | 2.251 |
| h | -20 | 10674.23 | 100 | 2.229 |
| | -10 | 10666.73 | 100 | 2.243 |
| | +10 | 10651.87 | 100 | 2.268 |
| | +20 | 10644.50 | 100 | 2.281 |
| $\gamma$ | -20 | 10660.16 | 100 | 2.259 |
| | -10 | 10659.72 | 100 | 2.257 |
| | +10 | 10658.84 | 100 | 2.254 |
| | +20 | 10658.39 | 100 | 2.252 |
| $C_1$ | -20 | 10675.63 | 100 | 2.388 |
| | -10 | 10666.95 | 100 | 2.316 |
| | +10 | 10652.42 | 100 | 2.203 |
| | +20 | 1066.24 | 100 | 2.157 |
| $C_2$ | -20 | 10659.52 | 100 | 2.256 |
| | -10 | 10659.40 | 100 | 2.256 |
| | +10 | 10659.16 | 100 | 2.256 |
| | +20 | 10659.04 | 100 | 2.256 |
| $C_3$ | -20 | 10655.76 | 100 | 2.242 |
| | -10 | 10657.51 | 100 | 2.249 |
| | +10 | 10661.05 | 100 | 2.262 |
| | +20 | 10662.84 | 100 | 2.269 |
| t | -20 | 10653.16 | 100 | 2.069 |
| | -10 | 10657.39 | 100 | 2.160 |
| | +10 | 10659.36 | 100 | 20354 |
| | +20 | 10657.95 | 100 | 2.456 |

After the careful study of above table, we observe that

(i) If the ordering cost 'A' decreases the optimal profit function $P^*$ increases and if the ordering cost increases the profit function $P^*$ decreases while the optimal selling price '$s^*$' remains constant.

(ii) To decrease in the value of the parameter a, the values of optimal profit function $P^*$, $s^*$, $T^*$ decreases while to increase in the value of the parameter a, the values of optimal profit function $P^*$, $s^*$, $T^*$ increases.

(iii) To increase or decrease in the value of the parameter 'α', $P^*$, $s^*$ and $T^*$ are slightly sensitive.

(iv) $P^*$ and $T^*$ are moderately sensitive to changes in the value of the parameter β.

(v) To decrease in the value of the parameter 'h', $P^*$ increases and $T^*$ decreases while to increase in the value of the parameter 'h', $P^*$ decreases and $T^*$ increases.

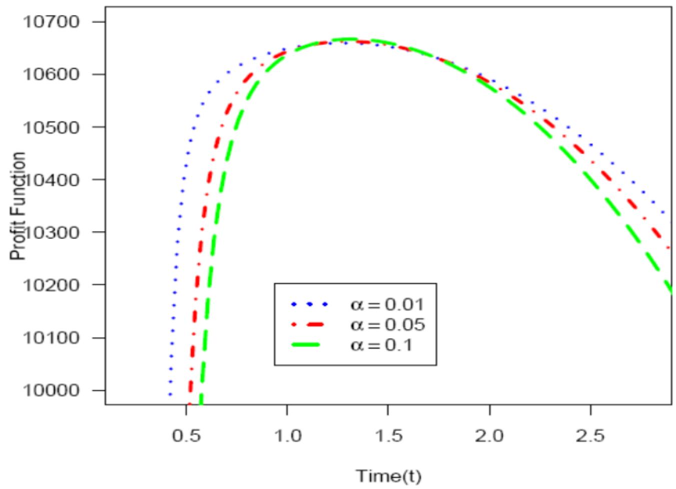
(vi) $P^*$ and $T^*$ are moderately sensitive to changes in the value of the parameter $\gamma$, while $s^*$ remains constant.

(vii). To decrease in the value of the parameter '$C_1$', $P^*$ increases and $T^*$ decreases while to increase in the value of the parameter '$C_1$', $P^*$ and $T^*$ decreases.

(viii) $P^*$ and $T^*$ are slightly sensitive to changes in the value of the parameter $C_2$ while $s^*$ remains constant.

(ix) To decrease in the value of the parameter '$C_3$', $P^*$ and $T^*$ decreases while to increase in the value of the parameter '$C_3$', $P^*$ and $T^*$ increases.

(x) $P^*$ and $T^*$ are moderately sensitive to changes in the value of the parameter 't' while $s^*$ remains constant.




We could generalize this model to allow for completely backlogging, shortages at starting and others.

## 7. CONCLUSION

In this paper, a deterministic inventory model is developed and analyzed for deteriorating items which follows inverse chain distribution deterioration rate of items. Demand rate is taken as a function of selling price and holding cost is time variable. Here, we derived the total optimal profit function, optimal selling price and optimal length of cycle. The model is solved analytically by maximizing the total profit function. Also the sensitivity analysis is carried out based on above numerical example. The sensitivity analysis of the model reveals that the optimal profit function and optimal cycle length are influenced by shortages and others parameters. The proposed model is useful for managers to obtain best optimal policies of the inventory system by estimating demand, deterioration and holding cost. The proposed model is illustrated with numerical example and verified by graphically.

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