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Minimum Spacings between 2-PrimeFactors Numbers till 1 Trillion

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ABSTRACT

Recently after defining '*k*-PrimeFactors number' to be a positive integer with exactly *k* number of prime divisors, which need not be necessarily distinct, 2-PrimeFactors numbers have been analyzed in detail for their low and high densities of occurrences. This work deals with 2-PrimeFactors numbers with a view of minimum spacings between two successive 2-PrimeFactors numbers. This analysis is taken up from two perspectives, viz., for blocks of various sizes like 10, 100 and so on within fixed range of one trillion and then for various ranges like 10, 100 and so on for fixed block sizes of 10, 100, 1000 and so on. In both approaches, minimum in-block spacing, number of times it occurs, first and last numbers with minimum spacing with their successors, and number of blocks exhibiting minimum spacing between 2-PrimeFactors numbers in them are presented.

Mathematics Subject Classification 2010 : 11A51, 11N05, 11N80.

Keywords: Prime number, *k*-PrimeFactors number, 2-PrimeFactors number, Minimum spacing.

1. INTRODUCTION

The prime numbers

2, 3, 5, 7, 11, 13, 17, ⋯

happen to be building blocks of all natural numbers in the sense that every natural number, except the beginner 1, is either itself a prime of product of primes. So primes enjoy supreme importance in the study of multiplicative number theory.

The supremacy of primes is not only due to the above property, which is technically known as Fundamental Theorem of Arithmetic, but there are many other reasons. One major amongst such is that they are not regularly distributed amongst natural numbers which they generate[1]. The spacings between successive prime numbers are quite randomly occurring. We

know that there are infinitely many primes in all higher ranges having spacing of 2 with their successors and at the same time the instances of arbitrary large gaps between successive primes also confirmed. Actual verifications have endorsed these theoretical properties[3]. Not only this, but their special types like twin primes also tend to show such properties[4].

2. *K*-PRIMEFACTORS NUMBERS

Recently new types of integers, based on the number of prime divisors they have, have been defined[6].

Definition (*k*-PrimeFactors Number) : For any integer $k \geq 0$, a positive integer greater than 1 having k number of prime factors, which need not be necessarily distinct, is called as *k*-PrimeFactors number.

Theoretically it has been proved way back in 300 BC by Euclid that primes are infinite in number. So their products, taken any k of them at a time, for any positive integer, are naturally infinite, which guarantees that there are infinitely many *k*-PrimeFactors number for each $k > 0$. The case of $k = 0$ is different and there is only unique 0-PrimeFactors number which is 1.

The above definition generalized even primes themselves which it uses! Simple prime is regarded as 1-PrimeFactor prime as it has only one prime factor, viz., itself.

For any positive integer, since its number of prime factors is fixed, it fits in one and only one category of *k*-PrimeFactors numbers.

3. 2-PRIMEFACTORS NUMBERS

For particular value of k as 2, we get 2-PrimeFactor numbers[6].

Definition (2-PrimeFactors Number) : A positive integer having exactly 2 prime divisors, which need not be necessarily distinct, is called as 2-PrimeFactors number.

First few 2-PrimeFactors numbers are :

4, 6, 9, 10, 14, 21, ⋯

Each of them have 2 prime divisors : $4 = 2^2$, $6 = 2\times3$, $9 = 3\times3$, $10 = 2\times5$, $14 = 2\times7$, $21 = 3\times7$, …

No surprise that 2-PrimeFactors numbers inherit from usual prime numbers the infinitude. But in addition they also seem to inherit randomness. The sequence of 2-PrimeFactors numbers doesn't fit in any known standard types.

4. MINIMUM SPACING BETWEEN SUCCESSIVE 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZES 10^n

For this and previous analysis[6,7] of 2-PrimeFactors numbers in large range of as high as 1 trillion, first usual primes were determined by using algorithms chosen by exhaustive comparison[2].

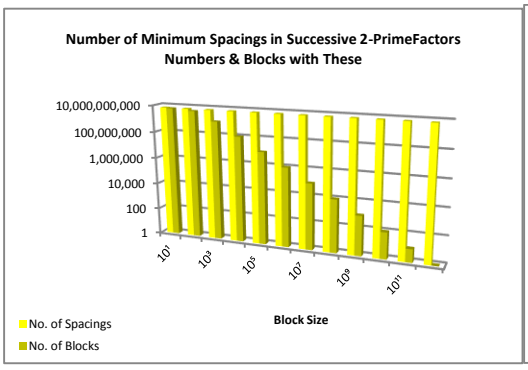
Advanced computers running in parallel and evolved programming languages like Java[5] have been used for all this work.

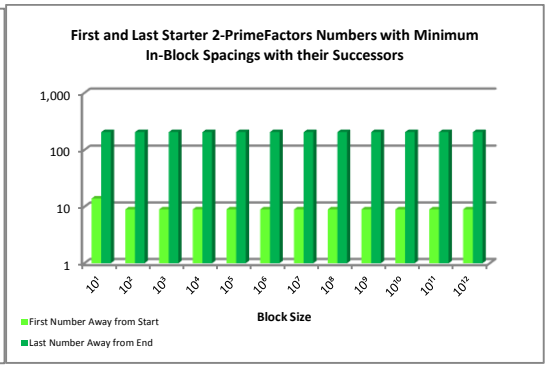
In complete range of 1 trillion, blocks of sizes of 10^1, 10^2, 10^3, till 10^{12} are selected and minimum spacing between successive 2-PrimeFactors numbers within blocks of each size, number of times this minimum spacing occurs in pairs of successive 2-PrimeFactors numbers in these blocks, first starter number of minimum spacing pair and last starter number of minimum spacing pair, and number of blocks in which such minimum spacing pairs occur are all determined.

Sr. No.	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	10^1	1	6,646,644,718	14	999,999,999,793	6,277,981,456
2	10^2	1	6,646,644,719	9	999,999,999,793	4,820,112,453
3	10^3	1	6,646,644,719	9	999,999,999,793	998,705,033
4	10^4	1	6,646,644,719	9	999,999,999,793	100,000,000
5	10^5	1	6,646,644,719	9	999,999,999,793	10,000,000
6	10^6	1	6,646,644,719	9	999,999,999,793	1,000,000
7	10^7	1	6,646,644,719	9	999,999,999,793	100,000
8	10^8	1	6,646,644,719	9	999,999,999,793	10,000
9	10^9	1	6,646,644,719	9	999,999,999,793	1,000
10	10^{10}	1	6,646,644,719	9	999,999,999,793	100
11	10^{11}	1	6,646,644,719	9	999,999,999,793	10
12	10^{12}	1	6,646,644,719	9	999,999,999,793	1

The minimum spacing between two successive 2-PrimeFactors numbers between blocks of all sizes under consideration is uniformly 1. This reflects that these numbers frequently occur closest possible to each other, i.e., are successive, in blocks of all sizes.

While the number of times the minimum spacing occurs is constant, with increasing block size, the number of blocks containing successive 2-PrimeFactors numbers with minimum spacing goes on decreasing consistently as block of higher size contains more such numbers.





Except for the first block size, the first and last starter 2-PrimeFactors number having minimum spacing with its successor is also same for all sizes.

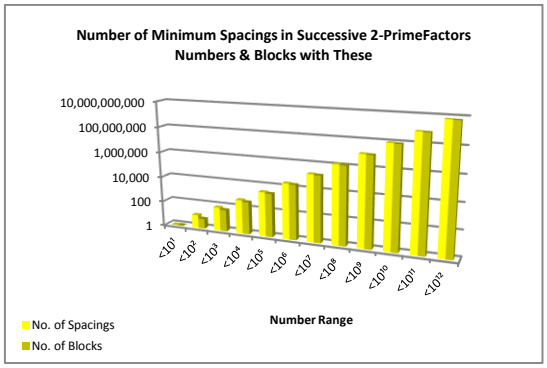
We start specific sized block-wise analysis inside increasing ranges till 1 trillion.

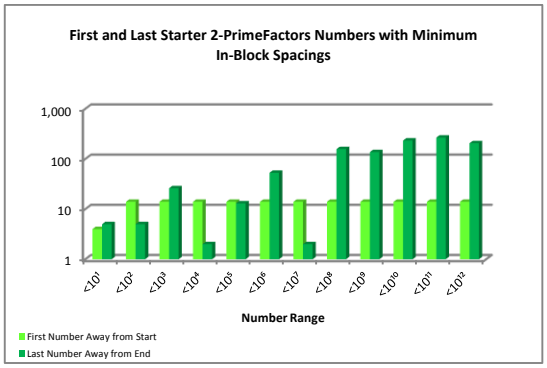
4.1 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10

Till now, within fixed range of 1 trillion, we have analyzed different sized blocks. Now we reverse the roles. For fixed block size of 10, we inspect different ranges of increasing 10 powers for occurrences of minimum spacing between successive 2-PrimeFactors numbers. Here, block 0 stands for number range 0 to 9, block 10 for range 10 to 19 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	10^1	2	1	4	4	1
2	10^2	1	11	14	94	6
3	10^3	1	73	14	973	52
4	10^4	1	467	14	9,997	363
5	10^5	1	3,160	14	99,986	2,641
6	10^6	1	23,171	14	999,946	20,161
7	10^7	1	175,219	14	9,999,997	156,575
8	10^8	1	1,376,400	14	99,999,842	1,252,779
9	10^9	1	11,146,606	14	999,999,862	10,278,170
10	10^{10}	1	92,226,179	14	9,999,999,766	85,891,401
11	10^{11}	1	777,140,724	14	99,999,999,733	729,438,111
12	10^{12}	1	6,646,644,718	14	999,999,999,793	6,277,981,456

Except for the first range 0f 0 to 9, the minimum in-block spacing between consecutive 2-PrimeFactors numbers in blocks of size 10 in all ranges is 1. The number of occurrences of such minimum spacings and number of blocks containing pairs of successive 2-PrimeFactors numbers with such spacing in-between them keep increasing. The first and particularly last minimum spacing pair starters in respective ranges are at different distances from start and end, respectively.



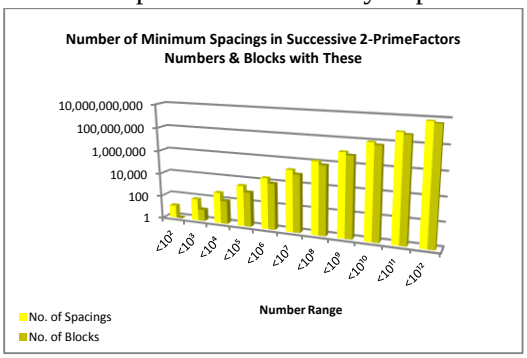


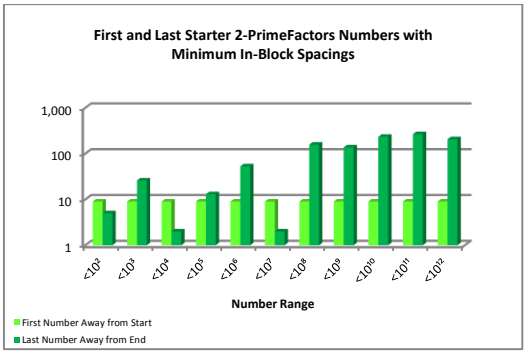
4.2 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^2

Second number is of block size 10^2, i.e., 100, where block 0 is range 0 to 99, block 100 is 100 to 199 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^2					
	Block-Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^2$	1	12	9	94	1
2	$<10^3$	1	74	9	973	10
3	$<10^4$	1	468	9	9,997	100
4	$<10^5$	1	3,161	9	99,986	960
5	$<10^6$	1	23,172	9	999,946	8,994
6	$<10^7$	1	175,220	9	9,999,997	82,470
7	$<10^8$	1	1,376,401	9	99,999,842	746,230
8	$<10^9$	1	11,146,607	9	999,999,862	6,697,729
9	$<10^{10}$	1	92,226,180	9	9,999,999,766	59,949,419
10	$<10^{11}$	1	777,140,725	9	99,999,999,733	536,947,613
11	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	4,820,112,453

The patterns of our analysis parameters here are as follows.





4.3 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^3

Next turn is of block size 10^3, i.e., 1000. Block 0 gives number range 0 to 999, block 1000 gives range 1000 to 1999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^3					
	Block-Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^3$	1	74	9	973	1
2	$<10^4$	1	468	9	9,997	10
3	$<10^5$	1	3,161	9	99,986	100
4	$<10^6$	1	23,172	9	999,946	1,000
5	$<10^7$	1	175,220	9	9,999,997	10,000
6	$<10^8$	1	1,376,401	9	99,999,842	100,000
7	$<10^9$	1	11,146,607	9	999,999,862	999,985
8	$<10^{10}$	1	92,226,180	9	9,999,999,766	9,999,031
9	$<10^{11}$	1	777,140,725	9	99,999,999,733	99,958,465
10	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	998,705,033

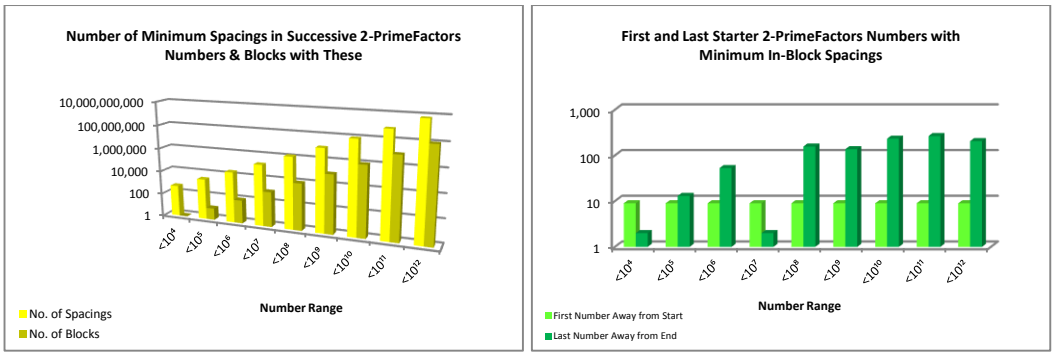
Many figures remain same, except of course, number of blocks.

4.4 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^4

10^4, i.e., 10000 will be 4th block size with block 0 denoting number range 0 to 9999, block 10000 denoting number range 10000 to 19999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^4					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^4$	1	468	9	9,997	1
2	$<10^5$	1	3,161	9	99,986	10
3	$<10^6$	1	23,172	9	999,946	100
4	$<10^7$	1	175,220	9	9,999,997	1,000
5	$<10^8$	1	1,376,401	9	99,999,842	10,000
6	$<10^9$	1	11,146,607	9	999,999,862	100,000
7	$<10^{10}$	1	92,226,180	9	9,999,999,766	1,000,000
8	$<10^{11}$	1	777,140,725	9	99,999,999,733	10,000,000
9	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	100,000,000

Each block of size 10^4 in every range till 1 trillion contains at least one pair of successive 2-PrimeFactors numbers with minimum spacing 1 in it.

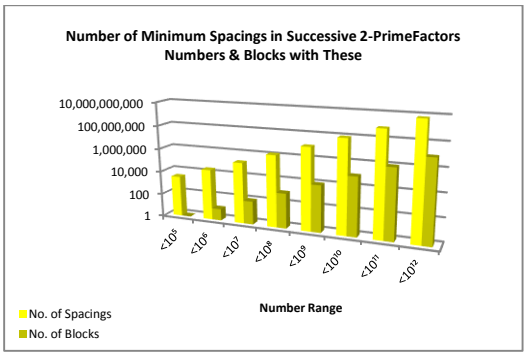


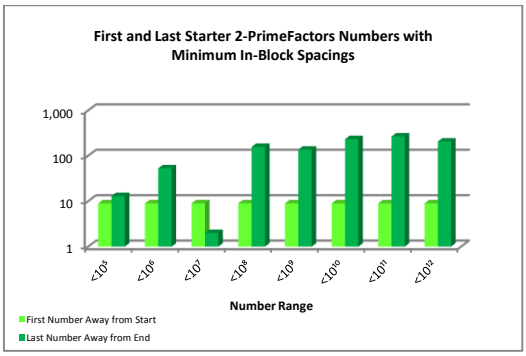
4.5 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^5

Next number is of block size 10^5, i.e., 100000, block 0 giving number range 0 to 99999, block 100000 giving 100000 to 199999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^5					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^5$	1	3,161	9	99,986	1
2	$<10^6$	1	23,172	9	999,946	10
3	$<10^7$	1	175,220	9	9,999,997	100
4	$<10^8$	1	1,376,401	9	99,999,842	1,000
5	$<10^9$	1	11,146,607	9	999,999,862	10,000
6	$<10^{10}$	1	92,226,180	9	9,999,999,766	100,000
7	$<10^{11}$	1	777,140,725	9	99,999,999,733	1,000,000
8	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	10,000,000

For size 10^5 also, every block in each range does contain pairs of most closely placed successive 2-PrimeFactors numbers.



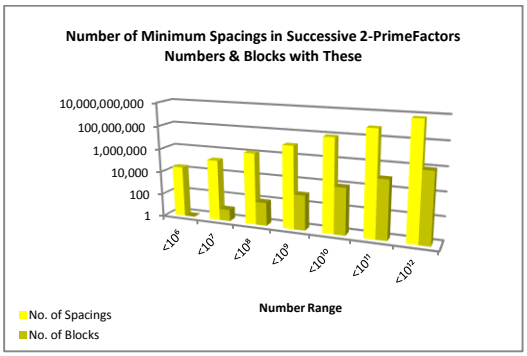


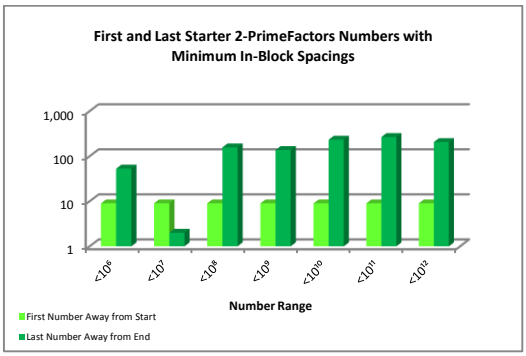
4.6 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^6

Next turn is of block of size 10^6, i.e., 1000000, for which block 0 represents number range 0 to 999999, block 1000000 represents number range 1000000 to 1999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^6					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^6$	1	23,172	9	999,946	1
2	$<10^7$	1	175,220	9	9,999,997	10
3	$<10^8$	1	1,376,401	9	99,999,842	100
4	$<10^9$	1	11,146,607	9	999,999,862	1,000
5	$<10^{10}$	1	92,226,180	9	9,999,999,766	10,000
6	$<10^{11}$	1	777,140,725	9	99,999,999,733	100,000
7	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	1,000,000

Pattern continues with no change in values except the values of number of blocks.



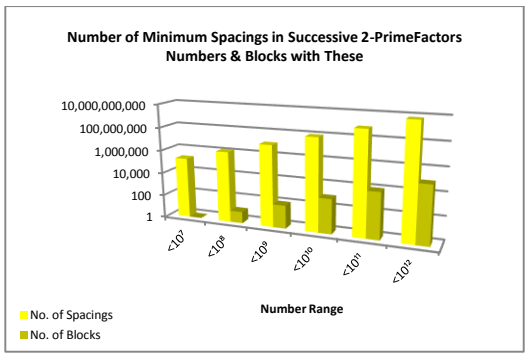


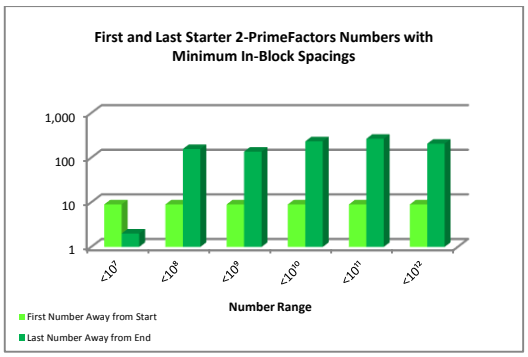
4.7 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^7

Next higher block size is 10^7, i.e., 10000000, block 0 gives range 0 to 9999999, block 10000000 gives range 10000000 to 19999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^7					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^7$	1	175,220	9	9,999,997	1
2	$<10^8$	1	1,376,401	9	99,999,842	10
3	$<10^9$	1	11,146,607	9	999,999,862	100
4	$<10^{10}$	1	92,226,180	9	9,999,999,766	1,000
5	$<10^{11}$	1	777,140,725	9	99,999,999,733	10,000
6	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	100,000

Apart from the reduction of number blocks containing minimally spaced successive 2-PrimeFactors numbers, the other values are as in earlier sized block.



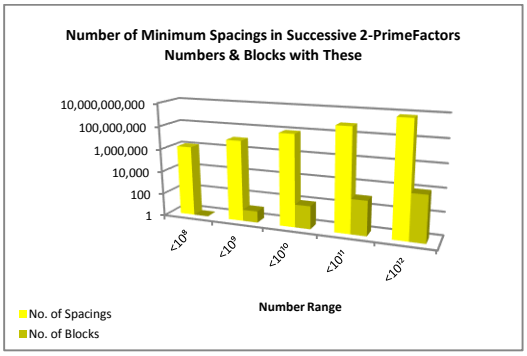


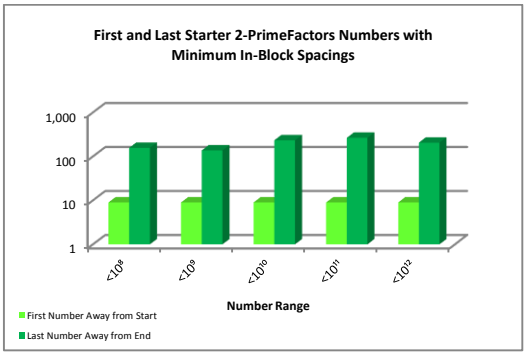
4.8 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^8

We now deal with block size of 10^8, i.e., 100000000, with block 0 indicating number range 0 to 99999999, block 100000000 indicating number range 100000000 to 199999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^8					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^8$	1	1,376,401	9	99,999,842	1
2	$<10^9$	1	11,146,607	9	999,999,862	10
3	$<10^{10}$	1	92,226,180	9	9,999,999,766	100
4	$<10^{11}$	1	777,140,725	9	99,999,999,733	1,000
5	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	10,000

Similar 10 times decrease of number of blocks compared to that of earlier sized blocks is the only change.



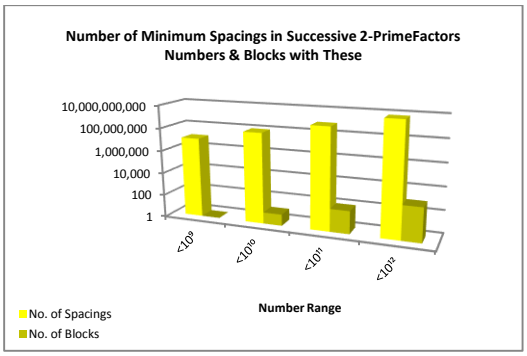


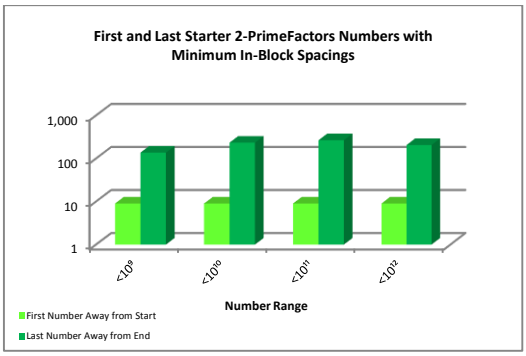
4.9 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^9

Then comes 10^9, i.e., 1000000000, as higher block size. For this size, 0 is number range 0 to 999999999, block 1000000000 is range 1000000000 to 1999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^9					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^9$	1	11,146,607	9	999,999,862	1
2	$<10^{10}$	1	92,226,180	9	9,999,999,766	10
3	$<10^{11}$	1	777,140,725	9	99,999,999,733	100
4	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	1,000

Similar values give similar graphs except those for number of blocks as mentioned earlier also.



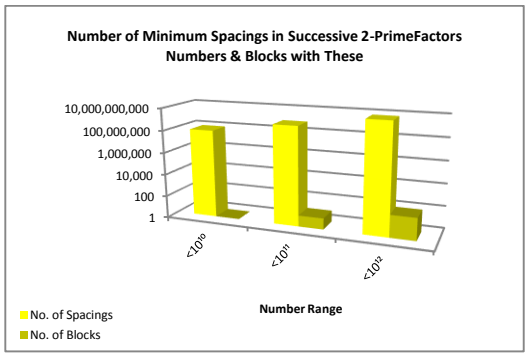


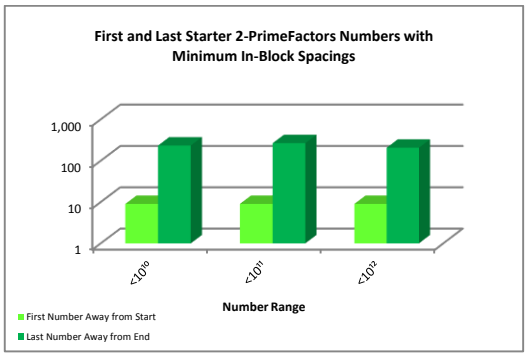
4.10 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^{10}

Block-size taken up this time is 10^{10}, i.e., 10000000000, for which block 0 denotes range 0 to 9999999999, block 10000000000 denotes number range 10000000000 to 19999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^{10}					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^{10}$	1	92,226,180	9	9,999,999,766	1
2	$<10^{11}$	1	777,140,725	9	99,999,999,733	10
3	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	100

The trend has become quite stable.

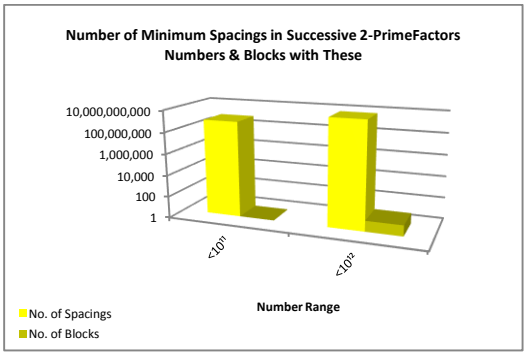


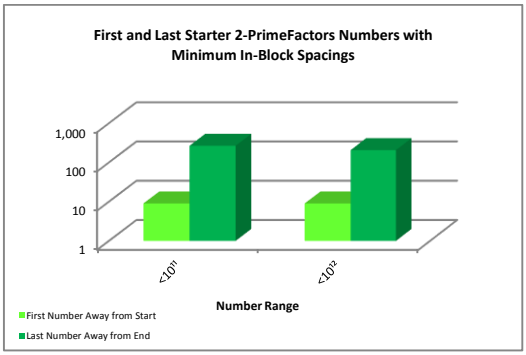


4.11 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^{11}

Further we consider blocks of size 10^{11}, i.e., 100000000000, with block 0 giving number range 0 to 99999999999, block 100000000000 giving range 100000000000 to 199999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 2-PrimeFactors Numbers for Block of Size 10^{11}					
	Block -Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^{11}$	1	777,140,725	9	99,999,999,733	1
2	$<10^{12}$	1	6,646,644,719	9	999,999,999,793	10





4.12 Minimum Spacings between Successive 2-PrimeFactors Numbers in Blocks of Size 10^{12}

As analysis range is till 1 trillion only, there happens to be unique block of 1 trillion size, which is giving everything, viz., number of successive 2-PrimeFactors numbers in it is minimum till range of its own size, first and last starter numbers of minimum spacings being actually occurring first and last starters of successive pairs of our numbers.

The analysis done in this work has shown that successive 2-PrimeFactors numbers come maximum close to each other, giving minimum spacing 1, quite frequently. This property is not shown by 1-PrimeFactor numbers, i.e., usual primes except for the unique starting pair of 2 and 3. For blocks of all sizes, till the range of 1 trillion, there is decrease in the percentage of number of successive 2-PrimeFactors numbers with minimum spacing 1 in them.

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