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Triple Fixed Point Theorems in Complex Valued Metric Spaces, using Rational Inequality

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ABSTRACT

In this paper, we prove common tripled fixed point theorems for a pair of mapping satisfying certain rational contraction in complex valued metric spaces.

Keywords: Common fixed point, complex valued metric spaces, Triple fixed point.

1. INTRODUCTION

In 2011, Azam *et al*.[2] introduced the notion of complex valued metric spaces and obtained sufficient condition for existence of common fixed point of a pair of contractive type mappings involving rational type mappings which is a generalization of the classical Banach contraction principal[3]. In 2006 Bhaskar and Lakshmikantham[5] introduced the concept of coupled fixed points for a given partially ordered set X. There after S.M. Kang *et al*.[8] introduced the notion of coupled fixed point for a mapping in complex valued metric spaces. Recently, Berinde and Borcut[6,4] introduced the notion of tripled fixed point for nonlinear contractive mappings in partially order complete metric spaces and obtained tripled coincidence and fixed point theorems for commuting mappings. Very recently Roldan *et al*.[10] introduced the tripled fixed point in fuzzy metric spaces and proved existence and uniqueness theorem for contractive type mappings in fuzzy metric spaces. In this manner many researchers have contributed their works in coupled and tripled fixed point. For detailed development one can see in[1,7,8,9]. In order to that we consider as light modification of the concept of tripled fixed point for a mapping in complex valued metric spaces as follows. We recall some definitions and notations that will be used in our note.

2. PRELIMINARIES

Consistent with Azam, Fisher and Khan[2], the following definitions and results will be needed in the sequel.

$Let\ C\ be\ the\ set\ of\ complex\ numbers\ and\ let\ z_1, z_2 \in C. Define\ a\ partial\ order$ $\lesssim on\ C\ as\ follows: z_1 \lesssim z_2\ if\ and\ only\ if\ Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2).$ $It\ follows\ that:\ z_1 \lesssim z_2\ if\ one\ of\ the\ following\ conditions\ is\ satisfied:$

(1) $Re(z_1) = Re(z_2)$, $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$,
(2) $Re(z_1) < Re(z_2)$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$,
(3) $Re(z_1) = Re(z_2)$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$.
$In\ particular, we\ will\ write\ z_1 \lnsim z_2$ if $z_1 \neq z_2\ and\ one\ of\ (1), (2)\ and$ $(3)\ is\ satisfied\ and\ we\ will\ write\ z_1 \prec z_2\ if\ only\ (3)\ is\ satisfied.$

Definition 2.1:[2] $Let\ X\ be\ a\ nonempty\ set. Suppose\ that\ the\ mapping$ $d : X \times X \rightarrow C\quad Satisfies:$
$(a)\ 0 \lesssim d(x, y)\ for\ all\ x, y \in X\ and\ d(x, y) = 0\ if\ and\ only\ if\ x = y$
$(b)\ d(x, y) = d(y, x)\ for\ all\ x, y \in X$
$(c)\ d(x, y) \lesssim d(x, z) + d(z, y)\ for\ all\ x, y, z \in X.$
$Then\ d\ is\ called\ a\ complex\ valued\ metric\ on\ X\ and\ (X, d)\ is\ called\ a$ $complex\ valued\ metric\ space.$

Definition2.2:[2] $Let\ \{x_n\}\ be\ a\ sequence\ in\ X\ and\ x \in X. If\ for\ every$ $c \in C, with\ 0 < c\ there\ is\ n_0 \in N\ such\ that\ for\ all\ n > n_0 , d(x_n, x) < c,$ then x $is\ called\ the\ limit\ of\ \{x_n\}\ and\ we\ write\ \lim_{n\to\infty} x_n = x\ or\ x_n \rightarrow x as\ n \rightarrow \infty.$

Definition 2.3:[2] $If\ every\ Cauchy\ sequence\ is\ convergent\ in\ (X, d),$ $then\ (X, d)\ is\ called\ a\ Complete\ Complex\ valued\ metric\ space.$

Lemma 2.4:[2] $Let\ (X, d)\ be\ a\ complex\ valued\ metric\ space\ and\ \{x_n\}\ a$ $sequence\ in\ X.\ Then\ \{x_n\}\ converges\ to\ x\ if\ and\ only\ if\ |d(x_n, x)| \rightarrow 0$ $as\ n \rightarrow \infty.$

Lemma 2.5:[2] $Let\ (X, d)\ be\ a\ complex\ valued\ metric\ space\ and\ \{x_n\}\ a$ $sequence\ in\ X. Then\ \{x_n\}\ is\ a\ Cauchy\ sequence\ if\ and\ only\ if$ $|d(x_n, x_{n+m})| \rightarrow 0\ as\ n \rightarrow \infty\ where\ m, n \in N.$

Definition 2.6:[6,4] $Let\ (X, d)\ be\ a\ Complex\ valued\ metric\ space.$ $Then\ an\ element\ (x, y, z) \in X \times X \times X\ is\ said\ to\ be\ tripled\ fixed\ point$ $of\ the\ mapping\ F : X \times X \times X \rightarrow X\ if\ x = F(x, y, z), y = F(y, z, x),$ $z = F(z, x, y).$

Example 2.7 : $Let\ X = C\ and\ defined: F : X \times X \times X \rightarrow C\ by$ $d(x, y) = i\ |\ x - y\ |\ Then\ (X, d)\ is\ a\ complex\ valued\ metric\ space. Consider$ $the\ mapping\ F : X \times X \times X \rightarrow X\ with\ F(x, y, z) = \frac{xyz}{2}\ for\ all\ x, y, z \in X,$ $then\ (0, 0, 0)\ is\ tripled\ fixed\ point\ of\ F.$

3. MAIN RESULT

In this section, we prove some common fixed point theorems for contraction conditions described by rational expressions.

Theorem 3.1: *Let* (X,d) *be a complete Complex valued Metric Space. Suppose that the mapping* $F: X\times X\times X \to X$, *satisfies*

$$d(F(x,y,z),F(u,v,w)) \precsim \alpha\, max\left[d(x,u), \frac{\{1+d(x,F(x,y,z)\}d(u,F(u,v,w))}{1+d(x,u)}\right]$$
$$+\beta\, d(u,F(u,v,w))\frac{1+d\,(x,F(x,y,z)+\ d(y,F(y,z,x)+d(z,F(z,x,y))}{1+d(x,u)+d(y,v)+d(z,w)}$$

for all $x,y,z,u,v,w\in X$ *where* α,β *are non negative reals with* $\alpha+\beta<1$. *Then* F *has a unique triple fixed point.*

Proof: *Choose* $x_{0,}y_0,z_0\in X$ *and set*

$$x_1 = F\,(x_{0,}y_0,z_0),\ y_1 = F\,(y_0,z_0x_0), z_1 = F\,(z_{0,}y_0,x_0)$$
$$\ldots,$$
$$\ldots,$$
$$x_{n+1} = F\,(x_n,y_n,z_n),\ y_{n+1} = F\,(y_n,z_n,x_n)\,, z_{n+1} = F(z_{n,}y_n,x_n)$$

We have

$$d\,(x_n,x_{n+1}) = d\,(F(x_{n-1},y_{n-1},z_{n-1}),F(x_n,y_n,z_n))$$
$$\precsim \alpha\, max\left\{ d(x_{n-1},x_n), \frac{(\ \{1+d(x_{n-1},F(x_{n-1},y_{n-1},z_{n-1})\}d(x_n,F(x_n,y_n,z_n)))}{1+d(x_{n-1},x_n)}\right\}$$
$$+\beta d(x_n,F(x_n,y_n,z_n))\frac{1+d\,(x_{n-1},F(x_{n-1},y_{n-1}z_{n-1})+\ d(y_{n-1},F(y_{n-1},z_{n-1},x_{n-1})+d(z_{n-1},F(z_{n-1},x_{n-1},y_{n-1}))}{1+d(x_{n-1},x_n)+d(y_{n-1},y_n)+d(z_{n-1},z_{n,})}$$
$$= \alpha\, max\left\{ d(x_{n-1},x_n), \frac{(\{1+d(x_{n-1}x_n)\}d(x_n,x_{n+1}))}{1+d(x_{n-1},x_n)}\right\}$$
$$+\beta\, d(x_n,x_{n+1})\frac{1+d\,(x_{n-1},x_n)+\ d(y_{n-1},y_n)+d(z_{n-1},z_n))}{1+d(x_{n-1},x_n)+d(y_{n-1}y_n)+d(z_{n-1},z_n)}$$
$$= \alpha\, max\{\ d(x_{n-1},x_n),d(x_n,x_{n+1})\}\ +\ \beta\, d(x_n,x_{n+1})$$

Suppose that $max\{\ d(x_{n-1},x_n),d(x_n,x_{n+1})\} = d(x_n,x_{n+1})$ *for some* $n\geq 1$. *then the inequality turns into*

$$d(x_n,x_{n+1}) \leq \alpha\, d(x_n,x_{n+1})+\beta\, d(x_n,x_{n+1}).$$

This is contradiction. Thus

$$max\{\ d(x_{n-1},x_n),d(x_n,x_{n+1})\} = d(x_{n-1},x_n)\ for\ some\ n\geq 1$$

Hence the inequality yields

$$d(x_n,x_{n+1}) \precsim \alpha\ d(x_{n-1},x_n)+\beta\, d(x_n,x_{n+1})$$
$$(1-\beta)\,d(x_n,x_{n+1}) \precsim \alpha\ d(x_{n-1},x_n)$$
$$d(x_n,x_{n+1}) \precsim \frac{\alpha}{(1-\beta)}\,d(x_{n-1},x_n)$$
$$|\,d(x_n,x_{n+1})\,| \leq \frac{\alpha}{(1-\beta)}\,|\,d(x_{n-1},x_n)\,| \tag{3.1.1}$$

Similarly also obtain

$d\ (y_n, y_{n+1})\ =\ d\ (F(y_{n-1}, z_{n-1}, x_{n-1}), F(y_n, z_n, x_n))$

$\lesssim\ \alpha\ max\left\{ d(y_{n-1}, y_n), \frac{\left(\ \{1+d(y_{n-1}, F(y_{n-1}, z_{n-1}, x_{n-1})\} d\left(y_n, F(y_n, z_n x_n)\right)\right)}{1+d(y_{n-1}, y_n)}\right\}$

$+\ \beta d(\ y_n, F(y_n, z_n, x_n))\ \frac{1 + d\ (y_{n-1}, F(y_{n-1} z_{n-1} x_{n-1}) +\ d(z_{n-1}, F(z_{n-1}, x_{n-1}, y_{n-1}) + d(x_{n-1}, F(x_{n-1}, y_{n-1} z_{n-1}))}{1 + d(y_{n-1}, y_n) + d(z_{n-1}, z_n) + d(x_{n-1}, x_n)}$

$= \alpha\ max\left\{\ d(y_{n-1}, y_n), \frac{\left(\{1+d(y_{n-1}, y_n)\} d(y_n, y_{n+1})\right)}{1+d(y_{n-1}, y_n)}\right\}$

$+\ \beta\ d\left(y_n, y_{n+1}\right) \frac{1+d\ (y_{n-1}, y_n) + d(z_{n-1}, z_n) + d(x_{n-1}, x_n))}{1+d(y_{n-1}, y_n) + d(z_{n-1}, z_n) + d(x_{n-1}, x_n)}$

$=\ \alpha\ max\{\ d(y_{n-1}, y_n), d(y_n, y_{n+1})\}\ +\ \beta\ d(y_n, y_{n+1})$

Which implies that

$d(y_n, y_{n+1})\ \lesssim\ \ \alpha\ d(y_{n-1}, y_n) + \beta\ d(y_n, y_{n+1})$

$(1 - \beta) d(y_n, y_{n+1})\ \lesssim\ \ \alpha\ d(y_{n-1}, y_n)$

$d(y_n, y_{n+1})\ \lesssim\ \frac{\alpha}{(1-\beta)}\ d(y_{n-1}, y_n)$

$|\ d(y_n, y_{n+1})\ |\ \leq\ \frac{\alpha}{(1-\beta)}\ |\ d(y_{n-1}, y_n)\ |$ (3.1.2)

$d\ (z_n, z_{n+1})\ =\ d\ (F(z_{n-1}\ , x_{n-1}\ , y_{n-1}), F(z_n, x_n, y_n))$

$\lesssim \alpha\ max\left\{\ d(z_{n-1}, z_n), \frac{\{1+d(z_{n-1}, F(z_{n-1} x_{n-1}, y_{n-1})\} d(z_n, F(z_n, x_n, y_n))}{1+d(z_{n-1}, z_n)}\right\}$

$+\beta d\left(z_n, F(z_n, x_n, y_n)\right) \frac{1+d\ \left(z_{n-1}, F\left(z_{n-1}\ , x_{n-1}, y_{n-1}\right) + d(x_{n-1}, F(x_{n-1}, y_{n-1}, z_{n-1},\ ) + d(y_{n-1}, F(y_{n-1}, z_{n-1}\ x_{n-1}\ \right)}{1+d(z_{n-1}, z_n) + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)}$

$= \alpha\ max\left\{\ d(z_{n-1}, z_n), \frac{1+\{1+d(z_{n-1} z_n)\} d(z_n, z_{n+1})}{1+d(z_{n-1}, z_n)}\right\}$

$+\ \beta\ d(z_n, z_{n+1}) \frac{1+d(z_{n-1}, z_n) + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)}{1+d(z_{n-1}, z_n) + d(x_{n-1}, x_n) + d(y_{n-1} y_n)}$

$=\ \alpha\ max\{\ d(z_{n-1}, z_n), d(z_n, z_{n+1})\}\ +\ \beta\ d(z_n, z_{n+1})$

Which implies that

$d(z_n, z_{n+1})\ \lesssim\ \ \alpha\ d(z_{n-1}, z_n) + \beta\ d(z_n, z_{n+1})$

$d(z_n, z_{n+1})\ \lesssim\ \frac{\alpha}{(1-\beta)}\ d(z_{n-1}, z_n)$

$|\ d(z_n, z_{n+1})\ |\ \leq\ \frac{\alpha}{(1-\beta)}\ |\ d(z_{n-1}, z_n)\ |$ (3.1.3)

Adding (3.1.1) (3.1.2) *and* (3.1.3)

$|d\ (x_n, x_{n+1})\ | + |\ d(y_n, y_{n+1})\ | + |\ d(z_n, z_{n+1})\ | \lesssim\ \frac{\alpha}{(1-\beta)}\ |\ d(x_{n-1}, x_n)\ |$

$+|\ d(y_{n-1}, y_n)\ | +|\ d(z_{n-1}, z_n)\ |$

$d_n \lesssim \frac{\alpha}{(1-\beta)} d_{n-1}$

where $d_n =\ |\ d\ (x_n, x_{n+1})\ |\ \ +|\ d(y_n, y_{n+1})\ \ | + |\ d(z_n, z_{n+1})\ |$

and $d_{n-1} =|\ \ d(x_{n-1}, x_n)\ | +|\ d(y_{n-1}, y_n)\ | + d(z_{n-1}, z_n)\ |$

i.e. $d_n \lesssim$ p d_{n-1} where p = $\frac{\alpha}{(1-\beta)}$

in general , we have $d_n\ \leq\ \ p d_{n-1} \leq\ p^2\ d_{n-2}\ \leq \cdots \leq\ p^n\ d_0$.

$Now, for\ all\ m > n$

$| d(x_n, x_m) | + | d(y_n, y_m) | + | d(z_n, z_m) | \leq \ | d(x_n, x_{n+1})$
$+ \ | d(x_{n+1}, x_{n+2}) | \ + . . . + | d(x_{m+n-1}, x_m) | + | d(y_n, y_{n+1}) |$
$+ \ | d(y_{n+1}, y_{n+2}) \ | + \cdots + | d(y_{m+n-1}, y_m) | + | d(z_n, z_{n+1}) |$
$+ \ d(z_{n+1}, z_{n+2}) | + . . . + | d(z_{m+n-1}, z_m) |$

Therefore we have

$| d(x_n, x_m) | + | d(y_n, y_m) | + | d(n_m, z_m) | \leq p^n \ d_0 + p^{n+1} \ d_0 + . . . + p^{m+n-1} \ d_0,$

Thus

$|d(x_n, x_m) | + | d(y_n, y_m) | + | d(z_n, z_m)| \leq \frac{p^n}{1-p} | d_0 | \to 0 \ as\ n \to \infty.$

$which\ implies\ that\ \ | d(x_n, x_m) | \to 0, | d(y_n, y_m) | \to 0 \ , | d(z_n, z_m) | \to 0 \ ,$
$hence\ \{x_n\}, \{y_n\}\ and\ \{z_n\}\ are\ Cauchy\ sequence\ in\ X. Since\ X\ is$
$complete. Threrfore\ there\ exist\ x^*, y^*, z^* \in \ X\ such\ that$
$\lim_{n\to\infty} x_n = x^*, \lim_{n\to\infty} y_n = y^*, \lim_{n\to\infty} z_n = z^*.$

$Thus\ we\ have$

$$d(F(x^*, y^*, z^*), x^*) \ \lesssim \ d(F(x^*, y^*, z^*), x_{r+1}) \ + \ d(x_{r+1}, x^*)$$
$$= \ d\big(F(x^*, y^*, z^*), F(x_r y_r, z_r)\big) \ + \ d(x_{r+1}, x^*)$$
$$\lesssim \alpha\ max \left\{ d(x^*, x_r), \frac{\{1+d(x^*, F(x^*, y^*, z^*)\} d\big(x_r, F(x_r, y_r, z_r)\big)}{1+d(x^*, x_r)} \right\}$$
$$+ d\big(x_r, F(x_r, y_r, z_r)\big) \frac{1 + d\ (x^*, F(x^*, \ y^*, z^*) + \ d(y^*, F(y^*, \ z^*, \ x^*) + d(z^*, F(z^*, \ x^*, y^*))}{1 + d(x^*, x_r) + d(y^*, y_r) + d(z^*, z_r)} + \ d(x_{r+1}, x^*)$$
$$= \ \alpha\ max \left\{ d(x^*, x_r), \frac{\{1+d\big(x^*, F(x^*, y^*, z^*)\} d(x_r, x_{r+1})\big)}{1+d(x^*, x_r)} \right\}$$
$$+ \beta\ d(x_r, x_{r+1}) \frac{1+d\ (x^*, x^*) + \ d(y^*, y^*) + d(z^*, z^*)}{1+d(x^*, x_r) + d(y^*, y_r) + d(z^*, z_r)} + \ d(x_{r+1}, x^*)$$

$| F(x^*, y^*, z^*), x^* | \to \ 0\ as\ n \ \to \infty.$
$hence\ \ F(x^*, y^*, z^*) = x^*.$

$Similarly\ we\ have\ F(y^*, z^*, x^*) = y^*. F(z^*, x^*, y^*) = z^*.$
$Hence\ (x^*, y^*, z^*) is\ tripled\ fixed\ point\ of\ F.$
$Uniqueness: Now\ if\ \ (x', y', z')\ is\ another\ tripled\ fixed\ point\ of\ F\ , then$

$$d(x', x^*) = \ d\big(F(x', y', z'), F(x^*, y^*, z^*)\big)$$
$$\lesssim \alpha\ max \left\{ d(x', x^*) \frac{\{1+d(x', F(x', y', z')\} d\big(x^*, \ \ F(x^*, y^*, z^*)\big)}{1+d(x', x^*)} \right\}$$
$$+ \beta d(x^*, F(x^*, y^*, z^*)) \frac{1+d\ \big(x', F(x', y' z') + \ d(y', F(y', z', x') + d(z', F(z', x', y')\big)}{1+d(x', x^*) + d(y', y^*) + d(z', z^*)}$$
$$= \ \alpha\ max \left\{ d(x', x^*), \frac{1+d\big(x', x'), d(x^*, x^*)\big)}{1+d(x', x^*)} \right\}$$
$$+ \ \beta\ d(x^*, x^*)) \frac{1+d\ (x', x') + \ d(y' y') + d(z', z')}{1+d(x', x^*) + d(y', y^*) + d(z', z^*)}$$

$|d(x', x^*) \ \ | \leq \alpha \ \ | \ d(x', x^*)|$

$Similarly$

$| \, d(y', y^*) \, | \leq \alpha \, | \, d(y', y^*) \, |$
$| \, d(z', z^*) \, | \leq \alpha \, | \, d(z', z^*) \, |$
which implies
$| \, (1-\alpha) \, [\, | \, d(x', x^*) \, | + | \, d(y', y^*) \, | + | \, d(z', z^*) \, |] \leq 0. \, i.e.$
$| \, d(x', x^*) \, | + | \, d(y', y^*) \, | + | \, d(z', z^*) \, | = 0.$
$ie \; d(x', x^*) = 0, \quad d(y', y^*) = 0, \; d(z', z^*) = 0 \,.$
$ie \;\; x' = x^*, y' = y^*, z' = z^* \; Thus\ we\ have (x', y', z') = (x^*, y^*, z^*).$
$Therefore\ F\ has\ a\ unique\ tripled\ fixed\ point.$

Theorem 3.2: *Let* (X, d) *be a complete Complex valued Metric Space. Suppose that the mapping* $F : X \times X \times X \to X$, *satisfies*

$$d(F(x,y,z), F(u,v,w)) \lesssim \alpha d(F(x,y,z), x) + \beta d(F(u,v,w), u) + \gamma \left\{ \frac{d(u, F(x,y,z)) d(x, F(u,v,w)) + d(F(x,y,z), x)) \, d(F(u,v,w), u))}{1 + d(x,u)} \right\}$$

for all $x, y, z, u, v, w \in X$ *where* α, β *and* γ *are non negative reals with* $\alpha + \beta + \gamma < 1$.*Then* F *has a unique common fixed point.*

Proof: *Choose* $x_{0,} y_0, z_0 \in X$ *and set*

$$x_1 = F(x_0, y_0, z_0), \quad y_1 = F(y_0, z_0, x_0), z_1 = F(z_0, y_0, x_0)$$
$$\ldots,$$
$$\ldots,$$
$$x_{n+1} = F(x_n, y_n, z_n), \; y_{n+1} = F(y_n, z_n, x_n), z_{n+1} = F(z_n, y_n, x_n)$$

We have

$$d(x_n, x_{n+1}) = d(F(x_{n-1}, y_{n-1}, z_{n-1}), F(x_n, y_n, z_n))$$
$$\lesssim \alpha d(F(x_{n-1}, y_{n-1}, z_{n-1}), x_{n-1}) + \beta d(F(x_n, y_n, z_n), x_n)$$
$$+ \gamma \left\{ \frac{d(x_n, F(x_{n-1}, y_{n-1}, z_{n-1})) d(x_{n-1}, F(x_n, y_n, z_n)) + d(F(x_{n-1}, y_{n-1,} z_{n-1}), x_{n-1})) \, d(F(x_n, y_n, z_n), x_n))}{1 + d(x_{n-1,}, x_n)} \right\}$$
$$= \alpha d(x_n, x_{n-1}) + \beta d(x_{n+1}, x_n)$$
$$+ \gamma \left\{ \frac{d(x_{n,}, x_n) d(x_{n-1,}, x_{n+1}) + d(x_n, x_{n-1}) \, d(x_{n+1}, x_n)}{1 + d(x_{n-1}, x_n)} \right\}$$
$$= \alpha \, d(x_n, x_{n-1,}) + \beta \, d(x_{n+1}, x_n) + \gamma \left\{ \frac{d(x_n, x_{n-1}) \, d(x_{n+1}, x_n)}{1 + d(x_{n-1}, x_n)} \right\}$$
$$= \alpha \, d(x_n, x_{n-1}) + \beta \, d(x_{n+1}, x_n) + \gamma \, d(x_{n+1}, x_n)$$
$$| \, d(x_{n+1}, x_{n,}) \, | \leq \frac{\alpha}{(1-\beta-\gamma)} \, | \, d(x_n, x_{n-1}) \, | \qquad (3.2.1)$$
$$d(y_{n,}, y_{n+1,}) = d(F(y_{n-1}, z_{n-1} x_{n-1}), F(y_n, z_n x_n))$$
$$\lesssim \alpha d(F(y_{n-1,}, z_{n-1,} x_{n-1,}), y_{n-1,}) + \beta d(F(y_n, z_n x_{n,}), y_{n,})$$
$$+ \gamma \left\{ \frac{d(y_n, F(y_{n-1}, z_{n-1} x_{n-1})) d(y_{n-1}, F(y_n, z_n x_n)) + d(F(y_{n-1,} z_{n-1}, {}_{x_{n-1}}), y_{n-1})) \, d(F(y_n, z_{n,} x_{n,}), y_n))}{1 + d(y_{n-1}, y_n)} \right\}$$
$$= \alpha \, d(y_n, y_{n-1,}) + \beta \, d(y_{n+1}, y_n)$$
$$+ \gamma \left\{ \frac{d(y_n, y_n) d(y_{n-1}, y_n) + d(y_n, y_{n-1}) \, d(y_{n+1}, y_n)}{1 + d(y_{n-1}, y_n)} \right\}$$

$= \alpha d(y_n, y_{n-1}) \; + \beta d(y_{n+1}, y_n) + \; \gamma \left\{\frac{1+d(y_n ,y_{n-1})\, d(y_{n+1},y_n)}{1+d(y_{n-1},y_n)}\right\}$

$\lesssim \alpha d(y_n, y_{n-1}) \; + \beta d(y_{n+1}, y_n) + \gamma\, d(y_{n+1}, y_n)$

$\mid d\big(y_{n+1}, y_{n,}\big) \mid \; \leq \frac{\alpha}{(1-\beta-\gamma)} \mid d(y_n, y_{n-1}) \mid$ (3.2.2)

$d\,(z_n, z_{n+1}) = \; d\; \Big(F\big(z_{n-1}x_{n-1,}y_{n-1}\big), F(z_n x_n, y_n)\Big)$

$\lesssim \alpha d\big(F\big(z_{n-1}, x_{n-1,}y_{n-1,}\big), y_{n-1}\big) + \; \beta d\big(F\big(z_{n,}x_{n,}y_n\big), y_n\big)$

$+\gamma \left\{ \frac{d\left(z_n, F(z_{n-1,}x_{n-1,}y_{n-1,}))d\left(z_{n-1,}, F\big(z_{n,}x_{n,}, y_{n,}\big)\right) + \; d\big(F\big(z_{n-1x_{n-1,}y_{n-1,}}\big), z_{n-1,}\big)\, d(F\big(z_{n,}x_{n,}, y_{n,}\big), z_n)\right)}{1 + d(z_{n-1}, z_n)} \right\}$

$= \; \alpha d\big(z_n, z_{n-1,}\big) \; + \beta d(z_{n+1}, z_n)$

$+ \; \gamma \left\{\frac{d(z_n,z_n) d(z_{n-1},z_n) + \; d(z_n ,z_{n-1}))\; d(z_{n+1},,z_n))}{1+d(z_{n-1},z_n)}\right\}$

$\lesssim \alpha d(z_n, z_{n-1}) \; + \beta d(z_{n+1}, z_n) + \gamma\; d\big(z_{n+1,}z_n\big)$

$\mid d\big(z_{n+1}, z_{n,}\big) \mid \; \leq \frac{\alpha}{(1-\beta-\gamma)} \mid d(z_n, z_{n-1}) \mid$ (3.2.3)

Adding (3.2.1) (3.2.2) *and* (3.2.3)

$\mid d\;(x_n, x_{n+1}) \mid \; + \mid d(y_n, y_{n+1}) \mid \; + \mid d(z_n, z_{n+1}) \mid$

$\leq \; \frac{\alpha}{(1-\beta-\gamma)} \; [\; \mid d(x_{n-1}, x_n) \mid + \mid d(y_{n-1}, y_n) \mid + \mid d(z_{n-1}, z_n)$

$\mid d_n \mid \leq \frac{\alpha+\gamma}{(1-\beta)} \mid d_{n-1} \mid$

where $d_n = \; \mid \mathrm{d}\;(x_n, x_{n+1}) \; \mid + \mid d(y_n, y_{n+1}) \; \mid + \mid d(z_n, z_{n+1}) \mid$

and $\mid d_{n-1} \mid = \mid d(x_{n-1}, x_n) \mid + \mid d(y_{n-1}, y_n) \mid + \mid d(z_{n-1}, z_n) \mid$

i.e. $\mid d_n \mid \leq \; \mathrm{p} \mid d_{n-1} \mid$ where $\mathrm{p} = \frac{\alpha}{(1-\beta-\gamma)}$

in general ,

$d_n \leq \mathrm{p}\; p d_{n-1} \leq p^2\; d_{n-2} \leq \cdots \leq p^n\; d_0$.

Now, for all $m \; > \; n$

$\mid d(x_n, x_m) \mid + \mid d(y_n, y_m) \mid + \mid d\big(z_{n,}z_m\big) \mid \leq \quad \mid d\big(x_{n,}, x_{n+1,}\big)$

$+ \; \mid d\big(x_{n+1,}, x_{n+2}\big) \mid + \ldots + \mid d(x_{m+n-1}, x_m) \mid + \mid d(y_n, y_{n+1}) \mid$

$+ \; \mid d(y_{n+1}, y_{n+2}) \; \mid + \cdots + \mid d(y_{m+n-1}, y_m) \mid \; + \mid d\big(z_{n,}, z_{n+1}\big) \mid$

$+ \; d(z_{n+1}, z_{n+2}) \mid + \ldots + \mid d(z_{m+n-1}, z_m) \mid$

Therefore we have

$\mid d(x_n, x_m) \mid + \mid d(y_n, y_m) \mid + \mid d(n_m, z_m) \mid \leq p^n\; d_0 + p^{n+1}\; d_0 + \ldots + p^{m+n-1}\; d_0,$

Thus

$|d(x_n, x_m) | + \mid d(y_n, y_m) \mid + \mid d(z_n, z_m)| \leq \frac{p^n}{1-p} \mid d_0 \mid \to 0 \; as\; n \to \infty. which$

implies that $|d(x_n, x_m) | \to 0, \mid d(y_n, y_m) \mid \to 0\;, \mid d(z_n, z_m)| \to 0$,

hence $\{x_n\}, \{y_n\}$ *and* $\{z_n\}$ *are Cauchy sequence in X. Since X is Complete.*

Threrfore there exist $x^*, y^*, z^* \; \in \; X$ *such that*

$\lim_{n\to\infty} x_n = \; x^*, \lim_{n\to\infty} y_n = y^*, \lim_{n\to\infty} z_n = z^*.$

Thus we have

$d(F(x^*,y^*,z^*),x^*) \precsim d(F(x^*,y^*,z^*),x_{r+1}) + d(x_{r+1},x^*)$
$= d\big(F(x^*,y^*,z^*),F(x_r, y_r, z_r)\big) + d(x_{r+1},x^*)$
$= \alpha d(F(x^*,y^*,z^*),x^*) + \beta d(F(x_r, y_r, z_r),x_r)$
$+ \gamma\left\{\frac{d(x_r,F(x^*,y^*,z^*))d(x^*,F(x_r,y_r,z_r)) + d(F(x^*,y^*,z^*),x^*)\, d(F(x_r,y_r,z_r),x_r))}{1+d(x^*,x_r)}\right\} + d(x_{r+1},x^*)$
$d(F(x^*,y^*,z^*),x^*) = \alpha d(x^*,x^*) + \beta d(x_r,x_r)$
$+ \gamma\left\{\frac{d(x_r,x^*)d(x^*,x_r) + d(x^*,x^*))\, d(x_r,x_r))}{1+d(x^*,x_r)}\right\} + d(x_{r+1},x^*)$
$d(F(x^*,y^*,z^*),x^*) \precsim \gamma\{ d(x_r, x^*)\} + d(x_{r+1},x^*)$
$| d(F(x^*,y^*,z^*),x^*) | \to 0 \; as\, n \to \infty.$
$Thus\ we\ have\ | d(F(x^*,y^*,z^*),x^*) | = 0\ and\ hence\ F(x^*,y^*,z^*) = x^*.$
$Similarly\ we\ have\ F(,y^*,z^*,x^*) = y^*. F(,z^*x^*,y^*) = z^*.$
$Hence\ (x^*,y^*,z^*)\ is\ tripled\ fixed\ point\ of\ F.$
$Now\ if\ (x',y',z')\ is\ another\ tripled\ fixed\ point\ of\ F, then$
$d(x',x^*) = d\big(F(x',y',z'),F(x^*,y^*,z^*)\big)$
$\precsim \alpha d(F(x',y',z'),x') + \beta\, d(F(x^*,y^*,z^*),x^*)$
$+\gamma\left\{\frac{d\big(x^*,F(x',y',z')\big)d\big(x',F(x^*,y^*,z^*)\big) + d(F(x',y',z),x'))\, d(F(x^*,y^*,z^*),x^*)\big)}{1+d(x',x^*)}\right\}$
$= \alpha d(x',x') + \beta d(x^*,x^*) + \gamma\left\{\frac{d(x^*,x')d(x',x^*) + d(x',x'))\, d(x^*,x^*))}{1+d(x',x^*)}\right\}$
$| d(x',x^*) | \le \gamma\ | d(x',x^*) |$
$Similarly$
$| d(y',y^*) | \le \gamma\ | d(y',y^*) |$
$| d(z',z^*) | \le \gamma\ | d(z',z^*) |$
$which\ implies$
$| (1-\gamma)\,[\, d(x'^{,x^*}) + d(y',y^*) + d(z',z^*)] | \le 0. i.e.$
$d(x',x^*) + d(y',y^*) + d(z',z^*) = 0.$
$Thus\ we\ have\ (x',y',z') = (x^*,y^*,z^*).$
$Therefore\ F\ has\ a\ unique\ tripled\ fixed\ point$

Theorem 3.3: $Let\ (X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$
$Suppose\ that\ the\ mapping\ F : X \times X \times X \to X, satisfies$
$$d\big(F(x,y,z),F(u,v,w)\big) \precsim \alpha\, d(F(x,y,z),x) + \beta d(F(u,v,w),u)$$
$$+\gamma\left\{ d(x,u)\frac{d(F(x,y,z),u)d(F(u,v,w),u))}{1+d(F(x,y,z),x)+d(F(u,v,w),x)}\right\}$$
$for\ all\ x,y,z,u,v,w \in X\ where\ \alpha,\beta\ are\ non\ negative\ reals\ with$
$\alpha+\beta<1. Then\ F\ has\ a\ unique\ common\ fixed\ point.$
$Proof: Choose\ x_{0,}y_0, z_0 \in X\ and\ set$
$x_1 = F(x_{0,}y_0,z_0), \quad y_1 = F(y_0,z_0x_0), z_1 = F(z_{0,}y_0,x_0)$
. . .
. . .

$x_{n+1} = F(x_{n,} y_n, z_n),\ y_{n+1} = F(y_n, z_n x_n), z_{n+1} = F(z_{n,} y_n, x_n)$

We have

$d(x_n, x_{n+1}) = d(F(x_{n-1}, y_{n-1}, z_{n-1}), F(x_n, y_n, z_n))$

$\lesssim \alpha d(F(x_{n-1}, y_{n-1}, z_{n-1}), x_{n-1}) + \beta d(F(x_n, y_n, z_n), x_n)$

$+ \gamma \left\{ d(x_{n-1}, x_n) \frac{d(F(x_{n-1}, y_{n-1}, z_{n-1}, x_n))\, d(F(x_n, y_n, z_n), x_n)}{1 + d(F(x_{n-1}, y_{n-1} z_{n-1}), x_{n-1}) + d(F(x_n, y_n, z_n), x_{n-1})} \right\}$

$= \alpha\, d(x_n, x_{n-1}) + \beta d(x_{n+1}, x_n)$

$+\gamma \left\{ d(x_{n-1}, x_n) \frac{d(x_n, x_n)\, d(, x_{n+1}, x_n))}{1 + d(x_{n,}, x_{n-1})) + d(x_{n+1}, x_{n-1})} \right\}$

$= \alpha d(x_n, x_{n-1}) + \beta d(x_{n+1}, x_n)$

$\lesssim \alpha d(x_n, x_{n-1}) + \beta d(x_{n+1}, x_n)$

$|\, d(x_{n+1}, x_n) \,| \leq \frac{\alpha}{(1-\beta)} \,|\, d(x_n, x_{n-1}) \,|$ (3.3.1)

$d(y_{n,}, y_{n+1,}) = d(F(y_{n-1}, z_{n-1} x_{n-1}), F(y_n, z_n, x_n))$

$\lesssim \alpha d(F(y_{n-1}, z_{n-1} x_{n-1}), y_{n-1}) + \beta d(F(y_n, z_n, x_n), y_n)$

$+ \gamma \left\{ d(y_{n-1}, y_n) \frac{1 + d(F(y_{n-1}, z_{n-1,} x_{n-1}), y_n) + d(F(y_n, z_n x_n), y_n))}{1 + d(F(y_{n-1}, z_{n-1,} x_{n-1}), y_{n-1}) d(F(y_n, z_n x_n), y_{n-1})} \right\}$

$= \alpha d(y_n, y_{n-1}) + \beta d(y_{n+1}, y_n)$

$+ \gamma \left\{ d(y_{n-1}, y_n) \frac{d(y_n, y_n)\, d(y_{n+1}, y_n)}{1 + d(y_{n,} y_{n-1}) + d(y_{n+1}, y_{n-1})} \right\}$

$= \alpha d(y_n, y_{n-1}) + \beta d(y_{n+1}, y_n)$

$\lesssim \alpha d(y_n, y_{n-1,}) + \beta d(y_{n+1}, y_n)$

$|\, d(y_{n+1}, y_n) \,| \leq \frac{(\alpha)}{(1-\beta)} \,|\, d(y_n, y_{n-1}) \,|$ (3.3.2)

$and\ d(z_{n,}, z_{n+1}) = d\left(F(x_{n-1,} y_{n-1,}), F(z_n x_n, y_n)\right)$

$\lesssim \alpha d\left(F(z_{n-1} x_{n-1,} y_{n-1}), y_{n-1}\right) + \beta d\left(F(z_n x_{n,} y_n), y_n\right)$

$+ \gamma \left\{ d(z_{n-1,}, z_n) \frac{d(F(z_{n-1,} x_{n-1,} y_{n-1,} z_n)\, d(F(z_{n,} x_{n,} y_{n,}), z_{n,}))}{1 + d(F(z_{n-1,} x_{n-1,} y_{n-1}), z_{n-1} + d(F(z_{n,} x_{n,} y_n), z_{n-1})} \right\}$

$= \alpha\, d(z_n, z_{n-1}) + \beta\, d(z_{n+1}, z_n)$

$+ \gamma \left\{ d(z_{n-1}, z_n) \frac{d(z_{n,} z_n)\, d(z_{n+1}, z_{n,}))}{1 + d(z_{n\,,} z_{n-1}) + d(z_{n+1}, z_{n-1})} \right\}$

$= \alpha\, d(z_n, z_{n-1}) + \beta\, d(z_{n+1}, z_n)$

$\lesssim \alpha\, d(z_n, z_{n-1}) + \beta\, d(z_{n+1}, z_n)$

$\lesssim \alpha\, d(z_n, z_{n-1}) + \beta\, d(z_{n+1}, z_n)$

$|\, d(z_{n+1}, z_{n,}) \,| \leq \frac{(\alpha)}{(1-\beta)} \,|\, d(z_n, z_{n-1}) \,|$ (3.3.3)

Adding (3.3.1) (3.3.2) and (3.3.3)

$|\, d(x_{n,}, x_{n+1}) \,| + |\, d(y_n, y_{n+1}) \,| + |\, d(z_{n,}, z_{n+1}) \,|$

$\leq \frac{\alpha}{(1-\beta)} \,|\, d(x_{n-1,}, x_n) \,| \;\; |\, d(y_{n-1,}, y_{n,}) \,| + |\, d(z_{n-1,}, z_{n,}) \,|$

$d_n \lesssim \frac{\alpha}{(1-\beta)} d_{n-1}\ where$

$d_n = |\, \mathrm{d}\,(x_n, x_{n+1})\,| + |\, d(y_n, y_{n+1})\,| + |\, d(z_n, z_{n+1})\,|$ *and*
$d_{n-1} = |\, d(x_{n-1}, x_n)\,| + |\, d(y_{n-1}, y_n)\,| + |\, d(z_{n-1}, z_n)\,|$

$i.e.\, d_n \lesssim p\, d_{n-1}$ *where* $p = \dfrac{\alpha}{(1-\beta)}$

in general, we have
$d_n \leq p d_{n-1} \leq p^2 d_{n-2} \leq \cdots \leq p^n d_0$.
Now, for all $m > n$
$|\, d(x_n, x_m)\,| + |\, d(y_n, y_m)\,| + |\, d(z_n, z_m)\,| \leq |\, d(x_n, x_{n+1})$
$+ |\, d(x_{n+1}, x_{n+2})\,| + \ldots + |\, d(x_{m+n-1}, x_m)\,| + |\, d(y_n, y_{n+1})\,|$
$+ |\, d(y_{n+1}, y_{n+2})\,| + \cdots + |\, d(y_{m+n-1}, y_m)\,| + |\, d(z_n, z_{n+1})\,|$
$+ d(z_{n+1}, z_{n+2})\,| + \ldots + |\, d(z_{m+n-1}, z_m)\,|$

Therefore we have

$|\, d(x_n, x_m)\,| + |\, d(y_n, y_m)\,| + |\, d(n_m, z_m)\,| \leq p^n d_0 + p^{n+1} d_0 + \ldots + p^{m+n-1} d_0,$

Thus $|d(x_n, x_m)\,| + |\, d(y_n, y_m)\,| + |\, d(z_n, z_m)| \leq \frac{p^n}{1-p} |\, d_0\,| \to 0$ as $n \to \infty$.

which implies that $|\, d(x_n, x_m)\,| \to 0, |\, d(y_n, y_m)\,| \to 0, |\, d(z_n, z_m)\,| \to 0$
hence $\{x_n\}, \{y_n\}$ *and* $\{z_n\}$ *are Cauchy sequence in* X. *Since* X *is complete. Threrfore there exist* $x^*, y^*, z^* \in X$ *such that*
$lim_{n\to\infty} x_n = x^*, lim_{n\to\infty} y_n = y^*, lim_{n\to\infty} z_n = z^*$.
Thus we have

$$d(F(x^*, y^*, z^*), x^*) \lesssim d(F(x^*, y^*, z^*), x_{r+1}) + d(x_{r+1}, x^*)$$
$$= d\big(F(x^*, y^*, z^*), F(x_r, y_r, z_r)\big) + d(x_{r+1}, x^*)$$
$$= \alpha\, d(F(x^*, y^*, z^*), x^*) + \beta\, d(F(x_r, y_r, z_r), x_r)$$
$$+ \gamma \left\{ d(x^*, x_r) \frac{d(F(x^*, y^*, z^*), x_r)\, d(F(x_r, y_r, z_r), x_r)}{1 + d(F(x^*, y^*, z^*), x^*) + d(F(x_r, y_r, z_r), x^*)} \right\} + d(x_{r+1}, x^*)$$
$$= \alpha\, d(x^*, x^*) + \beta\, d(x_r, x_r)$$
$$+ \gamma \left\{ d(x^*, x_r) \frac{d(x^*, x_r)\, d(x_{r+1}, x_r)}{1 + d(x^*, x^*) + d(x_r, x^*)} \right\} + d(x_{r+1}, x^*)$$
$$\to 0 \text{ as } n \to \infty.$$

Thus we have $d(F(x^*, y^*, z^*), x^*) = 0$ *and hence* $F(x^*, y^*, z^*) = x^*$.
Similarly we have $F(y^*, z^*, x^*) = y^*, F(z^*, x^*, y^*) = z^*$.
Hence (x^*, y^*, z^*) *is tripled fixed point of* F.
Now if (x', y', z') *is another tripled fixed point of* F, *then*

$$d(x^*, x') = d\big(F(x', y', z'), F(x^*, y^*, z^*)\big)$$
$$\lesssim \alpha\, d(F(x', y', z'), x') + \beta\, d(F(x^*, y^*, z^*), x^*)$$
$$+ \gamma \left\{ d(x', x^*) \frac{d(F(x', y', z'), x^*)\; d(F(x^*, y^*, z^*), x^*)}{1 + d(F(x', y', z'), x') + d(F(x^*, y^*, z^*), x')} \right\}$$
$$= \alpha\, d(x', x') + \beta d(x^*, x^*) + \gamma \left\{ d(x', x^*) \frac{d(x', x^*)\, d(x^*, x^*)}{1 + d(x', x') + d(x^*, x')} \right\}$$
$$= \alpha\, d(x', x') + \beta d(x^*, x^*)$$

$| \, d(x^*, x') \, | \leq 0$
$Similarly \ | \, d(y^*, y') \ | \leq 0$
$| \, d(z^*, z') \ | \leq 0$
$which\ implies \ || \, [\, d(x^*, x') + d(y^*, y') + d(z^*, z')] \, | \leq 0.$
$i.e. \, d(x^*, x') + d(y^*, y') + d(z^*, z') = 0 \, .$
$Thus\ we\ have (x', y', z') = (x^*, y^*, z^*).$
$Therefore\ F\ has\ a\ unique\ tripled\ fixed\ point$

Theorem 3.4: $Let\ (X, d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$
$Suppose\ that\ the\ mapping\ S, T : X \times X \times X \to X, satisfies$

$$d(S(x,y,z), T(u,v,w)) \precsim \alpha \max\left\{ d(x,u), d\big(u, T(u,v,w)\big) \frac{1+d\big(x, S(x,y,z)\big),}{1+d(x,u)} \right\}$$
$$+\beta \frac{d\,(x, S(x,y,z)) d(u, T(u,v,w))}{1+d(x,u)} + \gamma \frac{d\,(u, S(x,y,z)\ d(x, T(u,v,w))}{1+d(x,u)+d(y,v)+d(z.w)} \qquad (3.4.1)$$

$for\ all\ x, y, z, u, v, w \in X\ where\ \alpha, \beta\ and\ \gamma\ are\ non\ negative\ reals\ with$
$\alpha + \beta < 1 \, . Then\ S, T\ have\ a\ unique\ common\ fixed\ point.$
$Proof: Choose\ x_0, y_0, z_0 \in X\ and\ set\ x_{2n+1} = S\left(x_{2n,} y_{2n}, z_{2n}\right),$
$y_{2n+1} = S\,(y_{2n}, z_{2n}, x_{2n}), z_{2n+1} = S(z_{2n}, x_{2n}, y_{2n})$
$x_{2n+2} = T(x_{2n+1}, y_{2n+1}, z_{2n+1}),\ y_{2n+2} = T(y_{2n+1}, z_{2n+1}, x_{2n+1})\ ,$
$z_{2n+2} = T(z_{2n+1,} x_{2n+1}, y_{2n+1})\ for\ all\ n = 0,1,2,3 \ldots$
$We\ have$

$$d(x_{2n+1}, x_{2n+2}) = d(S(x_{2n}, y_{2n}, z_{2n}), T(x_{2n+1}, y_{2n+1}, z_{2n+1}))$$
$$\precsim \alpha \max\left\{ d(x_{2n}, x_{2n+1}), d\big(x_{2n+1}, T(x_{2n+1}, y_{2n+1}, z_{2n+1})\big) \frac{1+d\big(x_{2n}, S(x_{2n}, y_{2n}, z_{2n})\big)}{1+d(x_{2n}, x_{2n+1})} \right\}$$
$$+ \beta \frac{d\left(x_{2n}, S(x_{2n}, y_{2n}, z_{2n})\right) d\big(x_{2n+1}, T\big(x_{2n+1}, y_{2n+1,} z_{2n+1}\big)\big)}{1+d(x_{2n}\,, x_{2n+1})}$$
$$+ \gamma \frac{d\left(x_{2n+1,}, S(x_{2n}, y_{2n}, z_{2n})\ d(x_{2n}, T(x_{2n+1}, y_{2n+1}, z_{2n+1})\right)}{1+d\big(x_{2n}, x_{2n+1,}\big)+d\big(y_{2n}, y_{2n+1,}\big)+d(z_{2n}, z_{2n+1})}$$
$$= \alpha \max\left\{ d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}) \frac{1+d\big(x_{2n} x_{2n+1,}\big)}{1+d(x_{2n}, x_{2n+1})} \right\}$$
$$+ \beta \frac{d\left(x_{2n}, x_{2n+1}\right)\ d\big(x_{2n+1,} x_{2n+2}\big)}{1 + d(x_{2n}, x_{2n+1})} + \gamma \frac{d\left(x_{2n+1,}, x_{2n+1}\right) d\big(x_{2n}, x_{2n+2}\big)}{1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) + d(z_{2n}, z_{2n+1})}$$
$$= \alpha \max\{ d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}) \} + \beta \frac{d\left(x_{2n}, x_{2n+1}\right)\ d\big(x_{2n+1\,}, x_{2n+2}\big)}{1+d\big(x_{2n,}, x_{2n+1}\big)}$$

$Suppose\ that\ \max\{d\,(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n+1}, x_{2n+2})$
then
$| \, d(x_{2n+1}, x_{2n+2}) \, | \leq \alpha \, | \, d(x_{2n+1}, x_{2n+2}) \, | + \beta \, | \, d(x_{2n+1}, x_{2n+2}) \, |.$
$This\ is\ contradiction. Thus$
$\max\big\{\{d\,(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n}, x_{2n+1})\,\big\}$
Hence the inequality yields
$|d(x_{2n+1}, x_{2n+2}) \, | \leq \alpha | \, d(x_{2n}, x_{2n+1}) \, | + \beta \ | \, d(x_{2n+1} x_{2n+2}) \, |$

$$(1-\beta)\ \mid d(x_{2n+1},x_{2n+2})\mid \leq\ \alpha\ \mid d(x_{2n},x_{2n+1})\mid \tag{3.4.2}$$

Similarly also obtain

$d(y_{2n+1},y_{2n+2}) = \mathrm{d}\,(F(y_{2n},z_{2n}x_{2n}),F(y_{2n+1},z_{2n+1},x_{2n+1}))$

$\lesssim \alpha\, max\left\{ d(y_{2n},y_{2n+1}), d\big(y_{2n+1},T(y_{2n+1},z_{2n+1},x_{2n+1})\big)\ \frac{1+d\left(y_{2n},S(y_{2n},z_{2n},x_{2n,})\right)}{1+d(x_{2n},x_{2n+1})}\right\}$

$+\beta\,\frac{d\,(y_{2n},S(y_{2n},z_{2n}x_{2n}))d(y_{2n+1},T(y_{2n+1},z_{2n+1},x_{2n+1}))}{1+d(y_{2n},y_{2n+1})}$

$+\gamma\,\frac{d\,(y_{2n+1},S(y_{2n},z_{2n},x_{2n})\ d(x_{2n},T(y_{2n+1},z_{2n+1}x_{2n+1}))}{1+d(y_{2n},y_{2n+1})}$

$= \alpha\, max\left\{ d(y_{2n},y_{2n+1}), d(y_{2n+1},y_{2n+2})\ \frac{1+d(y_{2n},y_{2n+1})}{1+d(y_{2n},y_{2n+1})}\right\}$

$+\beta\,\frac{d\,(y_{2n},y_{2n+1})d(y_{2n+1},y_{2n+2,})}{1+d(y_{2n},\ y_{2n+1}\,)} + \gamma\,\frac{d\,\left(y_{2n+1},y_{2n+1,}\right)\,d(y_{2n,},y_{2n+2,})}{1+d(y_{2n},y_{2n+1}) + +d(z_{2n},z_{2n+1}) + d(x_{2n},x_{2n+1})}$

$= \alpha\, max\{\, d(y_{2n},y_{2n+1}), d(y_{2n+1},y_{2n+2})\,\}$

$+\beta\ \frac{d\,(y_{2n},y_{2n+1})d(y_{2n+1},y_{2n+2})}{1+d(y_{2n},\,y_{2n+1}\,)}$

Suppose that

$max\{d\,(y_{2n},y_{2n+1}),d(y_{2n+1},y_{2n+2})\} = d(y_{2n+1},y_{2n+2})$ *then*

$d(y_{2n+1},y_{2n+2}) \lesssim \alpha\, d(y_{2n+1},y_{2n+2}) + \beta\, d(y_{2n+1},y_{2n+2})$.

This is contradiction.

Thus $max\left\{\{d\,(y_{2n},y_{2n+1}),d(y_{2n+1},y_{2n+2})\} = d\big(y_{2n,},y_{2n+1}\big)\right\}$

$\mid d\,(y_{2n+1},y_{2n+2})\mid \leq \alpha\ \mid d(y_{2n},y_{2n+1})\ \mid +\beta\mid\ d\big(y_{2n+1,}y_{2n+2}\big)\mid$

$$(1-\beta)\mid d(y_{2n+1},y_{2n+2})\mid \leq\ \alpha\ \mid d(y_{2n},y_{2n+1})\mid \tag{3.4.3}$$

$d\,(z_{2n+1},z_{2n+2}) = d\,\left(F\big(z_{2n,}x_{2n,}y_{2n}\big),F(z_{2n+1,}x_{2n+1,}y_{2n+1})\right)$

$\lesssim \alpha\, max\left\{ d(z_{2n},z_{2n+1}), d\big(z_{2n+1},T(z_{2n+1},x_{2n+1},y_{2n+1})\big)\ \frac{1+d\left(z_{2n,},S(z_{2n,}x_{2n,}y_{2n})\right)}{1+d(z_{2n,},z_{2n+1})}\right\}$

$+\beta\,\frac{d\,\left(z_{2n,},S(z_{2n,}x_{2n,}y_{2n})\right)d\left(z_{2n+1,}T(z_{2n+1,}x_{2n+1,}y_{2n+1})\right)}{1+d(z_{2n,}z_{2n+1})}$

$+\gamma\ \frac{d\,\left(z_{2n+1,},S(,z_{2n,}x_{2n,},y_{2n,})\ d(z_{2n,},T\big(z_{2n+1,}x_{2n+1,},y_{2n+1}\big)\right)}{1+d(z_{2n\,,}\,z_{2n+1})+d(x_{2n\,,}\,x_{2n+1})+d(y_{2n\,,}\,y_{2n+1})}$

$= \alpha\, max\ \left\{ d(z_{2n},z_{2n+1}), d(z_{2n+1},z_{2n+2})\ \frac{1+d(z_{2n,}z_{2n+1})}{1+d(z_{2n,}z_{2n+1})}\right\}$

$+\beta\,\frac{d\,\left(z_{2n,},z_{2n+1,}\right)d(z_{2n+1,}z_{2n+2,},)}{1+d\big(y_{2n,},y_{2n+1,}\big)} + \gamma\,\frac{d\,\left(z_{2n+1},z_{2n+1,}\right)\,d(z_{2n,},z_{2n+2,})}{1+d\big(z_{2n\,,}\,z_{2n+1}\big)+d\big(x_{2n\,,}\,x_{2n+1}\big)+d\big(y_{2n\,,}\,y_{2n+1}\big)}$

$= \alpha\, max\{\, d(z_{2n},z_{2n+1}), d(z_{2n+1},z_{2n+2})\,\}$

$+\beta\ \frac{d\,(z_{2n,}z_{2n+1})d(z_{2n+1,}z_{2n+2})}{1+d(y_{2n,}y_{2n+1})}$

$+\gamma\,\frac{d\,(z_{2n+1,}z_{2n+1})\ d(z_{2n,}z_{2n+2})}{1+d(z_{2n\,,}\,z_{2n+1})+d(x_{2n\,,}\,x_{2n+1})+d(y_{2n\,,}\,y_{2n+1})}$

$= \alpha\, max\{\, d(z_{2n},z_{2n+1}), d(z_{2n+1},z_{2n+2})\,\}\ +\beta d(z_{2n+1,}z_{2n+2})$

$Suppose\ that$ max $\{d\ (z_{2n}, z_{2n+1}), d(z_{2n+1}, z_{2n+2})\} = d(z_{2n+1}, z_{2n+2})$
$then\ d(z_{2n+1}, z_{2n+2}) \ \leq\ \alpha\ d(z_{2n+1}, z_{2n+2}) + \beta\ d(z_{2n+1}, z_{2n+2})$.
$This\ is\ contradiction.$
$Thus\ max\big\{\{d\ (z_{2n}, z_{2n+1}), d(z_{2n+1}, z_{2n+2})\} = d(z_{2n}, z_{2n+1})\big\}$
$|\ d\ (z_{2n+1}, z_{2n+2}) \leq \alpha\ |\ d(z_{2n}, z_{2n+1})\ | + \beta\ d(z_{2n+1}, z_{2n+2})\ |$

$$(1-\beta)d(z_{2n+1}, z_{2n+2}) \ \leq\ \ \alpha\ d(z_{2n}, z_{2n+1}) \qquad (3.4.4)$$

Adding (3.4.2),(3.4.3) and (3.4.4)

$$|\ d\ (x_{2n+1}, x_{2n+2})\ | + |\ d(y_{2n+1}, y_{2n+2})\ | + |\ d(z_{2n+1}, z_{2n+2})\ |$$
$$\leq \frac{\alpha}{(1-\beta)}\ |\ d(x_{2n}, x_{2n+1})\ | + |\ d(y_{2n}, y_{2n+1})\ | + |\ d(z_{2n}, z_{2n+1})\ |$$
$$|\ d_n\ | \leq \frac{\alpha}{(1-\beta)}\ |\ d_{n-1}\ |$$

$where$

$|\ d_n\ | = |\ \mathrm{d}\ (x_{2n+1}, x_{2n+2})\ | + |\ d(y_{2n+}, y_{2n+2})\ | + |\ d(z_{2n+1}, z_{2n+2})\ |$
And $|\ d_{n-1}\ | = |\ d(x_{2n}, x_{2n+1})\ | + |\ d(y_{2n}, y_{2n+1})\ | + |\ d(z_{2n}, z_{2n+1})\ |$
$i.e.\ |\ d_n\ | \leq\ p\ |\ d_{n-1}\ |\ where\ p = \frac{\alpha}{(1-\beta)}$
$in\ general$,
$d_n \ \leq\ p\, pd_{n-1} \leq p^2\ d_{n-2} \leq \cdots \leq p^n\ d_0$.
$Now, for\ all\ m > n$

$$|\ d(x_n, x_m)\ | + |\ d(y_n, y_m)\ | + |\ d(z_n z_m)\ | \leq \ |\ d(x_n, x_{n+1})$$
$$+\ |\ d(x_{n+1}, x_{n+2})\ | + \ldots + |\ d(x_{m+n-1}, x_m)\ | + |\ d(y_n, y_{n+1})\ |$$
$$+\ |\ d(y_{n+1}, y_{n+2})\ | + \cdots + |\ d(y_{m+n-1}, y_m)\ |$$
$$+ |\ d(z_n, z_{n+1})\ | + \ d(z_{n+1}, z_{n+2})\ | + \ldots + |\ d(z_{m+n-1}, z_m)\ |$$

$Therefore\ we\ have$

$$|\ d(x_n, x_m)\ | + |\ d(y_n, y_m)\ | + |\ d\big(z_{m,} z_m\big)\ | \ \leq p^n\ d_0 + p^{n+1}\ d_0 + \ldots + p^{m+n-1}\ d_0,$$

Thus

$$|\ d(x_n, x_m)\ | + |\ d(y_n, y_m)\ | + |\ d(z_n, z_m)\ | \leq \frac{p^n}{1-p}\ |\ d_0\ | \to 0\ as\ n \to \infty.$$

$which\ implies\ that\ \ |\ d(x_n, x_m)\ | \to 0, |\ d(y_n, y_m)\ | \to 0\ , |\ d(z_n, z_m)\ |$
$\to 0\ , hence\ \{x_n\}, \{y_n\}\ and\ \{z_n\}\ are\ Cauchy\ sequence\ in\ X. Since\ X\ is$
$complete. Threrfore\ there\ exist\ x^*, y^*, z^* \in\ X\ such\ that$
$lim_{n\to\infty} x_n = x^*, \lim_{n\to\infty} y_n = y^*, lim_{n\to\infty} z_n = z^*$.
$Thus\ we\ have$
$Now\ we\ show\ that\ x^* = S(x^*, y^*, z^*)\ , y^* = S(y^*, z^*, x^*)\ and$
$z^* = S(z^*, x^*, y^*)$

Thus we have

$$d\big(x^*, S(x^*, y^*, z^*)\big) \lesssim d(x^* x_{2n+2}) + d\big(x_{2n+2}\ , S(x^*, y^*, z^*)\big)$$
$$= d(x^* x_{2n+2}) + d\big(T(x_{2n+1} y_{2n+1} z_{2n+1})\ , S(x^*, y^*, z^*)\big)$$
$$= d(x^* x_{2n+2})$$
$$+\ \alpha\ max\left\{ d(x^*, x_{2n+1}), d\big(x_{2n+1}, T(x_{2n+1}, y_{2n+1}, z_{2n+1})\big), \frac{1+d\big(x^*, S(x^*, y^*, z^*)\big)}{1+d(x^*, x_{2n+1})}\right\}$$

$+\beta \frac{d\left(x^*,S(x^*,y^*,z^*)\right)d\left(x_{2n+1},T(x_{2n+1},y_{2n+1},z_{2n+1})\right)}{1+d(x^*,x_{2n+1})}$

$+\gamma \frac{d\left(x_{2n+1},S(x^*,y^*,z^*)\ d(x^*,T(x_{2n+1},y_{2n+1},z_{2n+1})\right)}{1+d(x^*,\ x_{2n+1})+d(y^*,\ y_{2n+1})+d(z^*,\ z_{2n+1})}$

$= d(x^* x_{2n+2}) + \ \alpha\ max\left\{ d(x^*,x_{2n+1}), d(x_{2n+1},x_{2n+2}), \frac{1+d(x^*,x^*)}{1+d(x^*,x_{2n+1})}\right\}$

$+\beta \frac{d\ (x^*,x^*)d(x_{2n+1},x_{2n+2})}{1+d(x^*,x_{2n+1})} + \gamma\ \frac{d\ (x_{2n+1},\ \ x^*)\ d(x^*,x_{2n+2})}{1+d(x^*,\ x_{2n+1})+d(y^*,\ y_{2n+1})+d(z^*,\ z_{2n+1})}$

$|\ d\left(x^*,S(x^*,y^*,z^*)\right)\ | \to\ 0\ as\ n\ \to \infty.$

$Thus\ we\ have\ d(S(x^*,y^*,z^*),x^*)\ =\ 0\ and\ hence\ \ S(x^*,y^*,z^*) = x^*.$

$Similarly\ we\ have\ S(y^*,z^*,x^*) = y^*. S(z^*,x^*,y^*) = z^*.$

$It\ follows\ similarly\ that\ \ T(x^*,y^*,z^*) = x^*, T(y^*,z^*,x^*) = y^*$

$T(z^*,x^*,y^*) = z^*.$

$Hence\ (x^*,y^*,z^*)\ is\ common\ tripled\ fixed\ point\ of\ S\ and\ T.$

$now\ we\ show\ that\ S\ and\ T\ have\ a\ unique\ common\ tripled\ fixed\ point. For$

$this\ assume\ (x',y',z') \in X\ is\ another\ tripled\ fixed\ point\ of\ F\ , then$

$d(x^*,x')\ =\ d\left(T(x',y',z'),S(x^*,y^*,z^*)\right)$

$\lesssim \alpha\ max\left\{ d(x^*,x'), d\left(x',T(x',y',z')\right)\ \frac{1+d\left(x^*,S(x^*,y^*,z^*)\right),}{1+d(x^*,x')}\right\}$

$+\beta\ \frac{d\left(x^*,S(x^*,y^*,z^*)\right)d\left(x',T(x',y',z')\right)}{1+d(x^*,x')} +\ \gamma \frac{d\left(x',S(x^*,y^*,z^*)\ d(x^*,T(x',y',z')\right)}{1+d(x^*,x')+d(y^*,y')+d(z^*,z')}$

$= \alpha\ max\left\{ d(x^*,x'), d(x',x')\ \frac{1+d(x^*,x^*),}{1+d(x^*,x')}\right\}$

$+\beta \frac{d\ (x^*,x^*)d(x',x')}{1+d(x^*,x')} + \gamma\ \frac{d\ (x',x^*)\ d(x^*,x')}{1+d(x^*,x')+d(y^*,y')+d(z^*,z')}$

$|\ d(x^*,x') \le \alpha\ \ |\ d(x',x^*) + \gamma\ \ |\ d(x',x^*)|$

Similarly$|\ d(y^*,y')\ | \le \alpha\ \ |\ d(y',y^*)\ |\ \ +\gamma\ \ |\ d(y',y^*)\ |$

$|\ d(z^*,z')\ | \le \alpha\ \ |\ d(z',z^*)\ \ | + \gamma\ |\ \ d(z',z^*)\ |$

which implies

$|\ (1 -\ \alpha -\ \gamma)\ |\ |\ [\ d(x^*,x') +\ d(y^*,y') +\ d(z^*,z')]\ | \le\ 0. i.e.$

$d(x^*,x') +\ d(y^*,y') +\ d(z^*,z')\ =\ 0. \Longrightarrow\ x^* = x', y^* = y', z^*\ = z'.$

$Therefore\ S\ and\ T\ \ have\ a\ unique\ tripled\ fixed\ point.$

$If\ we\ put\ \beta = 0\ in\ Theorem\ 3.1\ we\ get\ the\ following\ corollary$

Corollary 3.5: $Let\ (X,d) be\ a\ complete\ Complex\ valued\ Metric\ Space.$

$Suppose\ that\ the\ mapping\ F: X \times X\ \times X\ \to X, satisfies$

$d(F(x,y,z),F(u,v,w))\ \lesssim\ \alpha\ max\ \left[d(x,u), \frac{\{1+d(x,F(x,y,z)\} d\left(u,F(u,v,w)\right)}{1+d(x,u)}\right]$

$for\ all\ x,y,z,u,v,w \in X\ where\ \alpha\ is\ \ non\ negative\ reals\ with$

$\alpha < 1\ . Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

$If\ we\ put\ \alpha = \beta\ in\ Theorem\ 3.1\ we\ get\ the\ following\ corollary\ .$

Corollary 3.6: $Let\ (X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$ $Suppose\ that\ the\ mapping\ F: X\times X\ \times X\ \to X, satisfies$

$$d(F(x,y,z),F(u,v,w)) \precsim \alpha\ max\ \left[d(x,u),\frac{\{1+d(x,F(x,y,z)\}d(u,F(u,v,w))}{1+d(x,u)}\right]$$
$$+\alpha\ d(u,F(u,v,w))\frac{1+d\ (x,F(x,y,z)+\ d(y,F(y,z,x)+d(z,F(z,x,y))}{1+d(x,u)+d(y,v)+d(z,w)}$$

$for\ all\ x,y,z,u,v,w\in X\ where\ \alpha,\beta\ are\ non\ negative\ reals\ with$ $\alpha<\dfrac{1}{2}$ $.Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

$If\ we\ put\ \ \gamma\ =0\ in\ Theorem\ 3.2\ \ we\ get\ the\ following\ corollary$

Corollary 3.7: $Let(X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$ $Suppose\ that\ the\ mapping\ F: X\times X\ \times X\ \to X, satisfies$

$$d(F(x,y,z),F(u,v,w)) \precsim \alpha d(F(x,y,z),x)\ +\beta d(F(u,v,w),u)$$

$for\ all\ x,y,z,u,v,w\in X\ where\ \alpha,\beta\ are\ non\ negative\ reals\ with$ $\alpha+\beta<1.Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

$If\ we\ put\ \alpha=\beta=\gamma\ in\ Theorem\ 3.3\ we\ get\ the\ following\ corollary$

Corollary 3.8: $Let(X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$ $Suppose\ that\ the\ mapping\ \ F: X\times X\ \times X\ \to X, satisfies$

$$d(F(x,y,z),F(u,v,w)) \precsim \ \alpha\ \{d(F(x,y,z),x)\ +d(F(u,v,w),u)$$
$$+\ d(x,u)\frac{1+d(u,F(x,y,z))d(x,F(u,v,w))+\ d(F(x,y,z),x))\ d(F(u,v,w),u))}{1+d(x,u)}\}$$

$for\ all\ x,y,z,u,v,w\in X\ where\ \alpha\ is\ non\ negative\ real\ with$ $\alpha<\dfrac{1}{3}$ $.Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

$If\ we\ put\ \gamma\ =0\ in\ Theorem\ 3.3\ \ we\ get\ the\ following\ corollary$

Corollary 3.9: $Let\ (X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$ $Suppose\ that\ the\ mapping\ F: X\times X\ \times X\ \to X, satisfies$

$$d(F(x,y,z),F(u,v,w)) \precsim \alpha d(F(x,y,z),x)\ +\beta d(F(u,v,w),u)$$

for $all\ x,y,z,u,v,w\in X\ where\ \alpha,\beta\ are\ non\ negative\ reals\ with$ $\alpha+\beta<1.Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

$If\ we\ put\ \alpha=\beta=\gamma\ in\ Theorem\ 3.3\ \ we\ get\ the\ following\ corollary$

Corollary 3.10: $Let(X,d)\ be\ a\ complete\ Complex\ valued\ Metric\ Space.$ $Suppose\ that\ the\ mapping\ F: X\times X\ \times X\ \to X, satisfies$

$$d(F(x,y,z),F(u,v,w)) \precsim \alpha\ \{d(F(x,y,z),x)+d(F(u,v,w),u)$$
$$+\ d(x,u)\frac{d(F(x,y,z),u)d(F(u,v,w),u))}{1+d(F(x,y,z),x)+d(F(u,v,w),x)}\}$$

$for\ all\ x,y,z,u,v,w\in X\ where\ \alpha\ are\ non\ negative\ reals\ with$ $\alpha<\dfrac{1}{2}.Then\ \ F\ has\ a\ unique\ common\ fixed\ point.$

Example 3.11 :L$et\ X = [0,1], Define\ d: X \times X \longrightarrow C\ by\ d(x,y) = |\ x-y\ |\ e^{\frac{i\pi}{6}}$
$Consider\ the\ mapping\ F: X^3 \to X\ defined\ by\ F(x,y,z) = \frac{xyz}{6}\ \ \forall x,y,z \in X.$
$Fix\ \alpha = 0.3, \beta = 0.1, \gamma = 0.2\ one\ can\ easily\ see\ that\ theorem\ (3.3.3)\ holds\ for$
$all\ x,y,z,u,v,w \in X$. Thus (0,0,0) is $the\ unique\ tripled\ fixed\ point\ of\ F$.

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