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Some Fractional Derivative of H-Function in one Variable

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ABSTRACT

In this paper,we derive the some Fractional derivative of H-function in one variable defined by (3) and (4). The results established are of general character and include some known results.

Keywords : Fractional Calculus, H-function , Hypergeometric function.

INTRODUCTION

Fractional calculus is a field of Applied mathematics that deals with derivatives and integrals of arbitrary orders during the left some decades Fractional Calculus has been applied to almost every field of science and technology .Many applications of fractional can be found in turbulence and fluid dynamics,Stochastic dynamical system,Plasam Physics and Controlled Thermonuclear fusion,Non linear control theory,Image Processing,Non linear biological system,Astrophysics etc. The fractional derivative,extension of the familiar derivative $\frac{d^n f(x)}{dx^n}$ to no integral values of n,is of immense utility in finding the solution of ordinary, partial and integral equation as well as in other contents.

The Gauss hypergeometric function is defined[4].

$$ {}_2F_1[a,b;c;z] = \sum_{k=1}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \tag{1}$$

$|z| < 1, c \neq ,1,2,...$

The Pochhammers symbol defined in term of the Gamma function by

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1, & when \quad n=0 \\ \lambda(\lambda+1)(\lambda+2)(\lambda+3)....(\lambda+n-1), & when \quad n\in N \quad for \quad \lambda\neq 0,1,2,.... \end{cases} \tag{2}$$

Binomial expression[7]

$$(x+\xi)^{\lambda} = \xi^{\lambda} \sum_{r=0}^{\infty} \binom{\lambda}{r} \left(\frac{x}{\xi}\right)^r ; \quad \left(\frac{x}{\xi}\right) < 1$$

$$D_x^{\mu}(x^{\lambda}) = \frac{\Gamma(\lambda+\mu)}{\Gamma(\lambda)} x^{\lambda-\mu}; \quad Re(\lambda) > -1 \tag{3}$$

$${}_{z}D_{\infty}^{\mu} z^{\lambda-\mu} = (-1)^{\mu} \frac{\Gamma(\lambda+\mu)}{\Gamma(\lambda)} z^{-\mu-\lambda}, \tag{4}$$

μ *is arbitrary*

Oldham & Sapnier[2] and Shrivastav & Goyal[10] the fractional derivative of a function f(x) of complex order γ (or alternatively, a $-\gamma^{th}$ order) fractional integrals of f(x)is defined as

$${}_{\alpha}D_{t}^{-\gamma}(f(x)) = \begin{cases} \frac{1}{\Gamma(-\gamma)} \int_{\alpha}^{t} (t-x)^{-\gamma-1} f(x)dx & Re(\gamma)<0, \\ \frac{d^m}{dx^m} {}_{\alpha}D_{t}^{\gamma-m} f(t); & 0 \le Re(\gamma) < m \end{cases} \tag{5}$$

where m is a positive integer.

For simply we use $D_{t}^{-\gamma} = {}_{0}D_{t}^{-\gamma}$ When the lower terminal limit $\alpha \to 0$ (5) reduce to the Riemann-Liouville representation used for differ integral of arbitrary order which $\alpha \to \infty$ gives the differ integral of arbitrary order in weyl sense.

The Leibnitz rule for the n^{th} derivative of a Product can be generalized to a derivative of arbitrary order (see Ross and Northover[6])

$${}_{\alpha}D_{x}^{\gamma}[f(x)g(x)] = \sum_{m=0}^{\infty} \binom{\gamma}{m} {}_{\alpha}D_{x}^{m} f(x)_{\alpha} D_{x}^{\gamma-m} g(x), \forall \alpha \tag{6}$$

According to a standard notation of one variable which is introduced by Fox,C.[7] will be represented as follows-

$$H[x] = H_{p,q}^{m,n}[x] = H_{p,q}^{m,n}\left[x/{}^{(a_p,\alpha_p)}_{(b_q,\beta_q)}\right] = H_{p,q}^{m,n}\left[x/{}^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] = \frac{1}{2\pi\iota}\int_{l} \theta(s) x^{s} ds \tag{7}$$

Where (a) ι is a square root of -1 and (a_j, α_j) represent the set of p-order pairs $(a_1,\alpha_1),(a_2,\alpha_2),(a_3,\alpha_3),......,(a_p,\alpha_p)$ and similarly for (b_q, β_q).

(b) $\theta(s) = \frac{\Pi_{j=1}^{m}\Gamma(b_j - \beta_j s)\Pi_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\Pi_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\Pi_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}$,

(c) m,n,p,and q are integers satisfying $1 \le m \le q, 0 \le n \le p, \alpha_j (1 \le j \le p), \beta_j (1 \le j \le q)$ are positive numbers and $a_j (1 \le j \le p), b_j (1 \le m \le q)$ are complex numbers.

MAIN RESULTS

The fractional derivative to be evaluate here is

$$D_{x}^{\gamma}\{(x^{m_1}+a)^{\lambda}(b-x^{m_2})^{-\delta} H[zx^{-\rho_1}(x^{m_1}+a)^{\sigma_1}(b-x^{m_2})^{-\delta_1}]\} \tag{8}$$

$$= a^{\lambda} b^{-\delta} x^{-\gamma} \sum_{r_1,r_2=0}^{\infty} \frac{(\frac{x^{m_1}}{a})^{r_1} (\frac{x^{m_2}}{b})^{r_2}}{r_1! r_2!}$$

$$H_{p+3,q+3}^{m+1,n+2}\left[zx^{\rho_1} a^{\sigma_1} b^{-\delta} / {}^{(-\lambda,\sigma_1)(1-\delta-r_2,s)(a_j,\alpha_j)_{1,p}(1-\gamma+r_1 m_1+r_2 m_2,\rho_1)}_{(1+r_1 m_1+r_2 m_2,\rho_1)(b_j,\beta_j)_{1,q}(-\lambda+m_1,\sigma_1)(1-\delta,\delta_1)}\right]$$

$${}_{x}D_{\infty}^{\gamma}\{(x^{m_1}+a)^{\lambda}(b-x^{m_2})^{-\delta} H[zx^{\rho_1}(x^{m_1}+a)^{-\sigma_1}(b-x^{m_2})^{-\delta_1}]\} \tag{9}$$

$$= a^{\lambda} b^{-\delta} x^{-\gamma} (-1)^{\gamma} \sum_{r_1,r_2=0}^{\infty} \frac{(\frac{x^{m_1}}{a})^{r_1} (\frac{x^{m_2}}{b})^{r_2}}{r_1! r_2!}$$

$$H^{m+2,n+1}_{p+3,q+3}[zx^{\rho_1}a^{-\sigma_1}b^{\delta_1}/ {}^{(1-\delta-r_2,s)(a_j,\alpha_j)_{1,p}(\lambda-r_1,\sigma_1)(-m_1r_1-m_2r_2,\rho_1}_{(1+\lambda,\sigma_1)(b_j,\beta_j)_{1,q})(\gamma-m_1r_1-m_2r_2,\rho_1)(-\delta,\sigma_1)}]$$

To Prove of (8), we first express the H-function occurring on the L.H.S. of equation (8) in terms of Mellin Barnes type of contour integral given by equation(7) and collected the power of x, $(x^{m_1}+a)$ and $(b-x^{m_2})$ and appply binomial expansion, we get

$$a^{\lambda}b^{-\delta}\frac{1}{2\pi\iota}\int_{-\iota\infty}^{+\iota\infty}\frac{\Pi_{j=1}^{m}\Gamma(b_j-\beta_j s)\Pi_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\Pi_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\Pi_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}.\sum_{r_1,r_2=0}^{\infty}$$

$$\frac{\Gamma(1-(-\lambda)+\sigma_1 s)\Gamma(1-\delta-\delta_1 s)}{\Gamma(1-(\lambda)-r_1+\sigma_1 s)(\Gamma(1-\delta-r_2-\delta_1 s))}.$$

$$\frac{(\frac{1}{a})^{r_1}(\frac{x^1}{b})^{r_2}}{r_1!r_2!}.D_x^{\gamma}\{x^{m_1r_1+m_2r_2-\rho_1 S}\}$$

Further using the result(4) the above equation becomes R.H.S. of (8).

$$a^{\lambda}b^{-\delta}x^{-\gamma}\sum_{r_1,r_2=0}^{\infty}\frac{(\frac{x^{m_1}}{a})^{r_1}(\frac{x^{m_2}}{b})^{r_2}}{r_1!r_2!}H^{m+1,n+2}_{p+3,q+3}[zx^{\rho_1}a^{\sigma_1}b^{-\delta}/ {}^{(-\lambda,\sigma_1)(1-\delta-r_2,s)(a_j,\alpha_j)_{1,p}(1-\gamma+r_1m_1+r_2m_2,\rho_1)}_{(1+r_1m_1+r_2m_2,\rho_1)(b_j,\beta_j)_{1,q})(-\lambda+m_1,\sigma_1)(1-\delta,\delta_1)}]$$

On the same parallel lines,second formula given by equation (9) can be easily obtained by using the results (4) and (5) respectively.

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