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Power of (3, 2) Jection Operator

Navin Kumar Singh

Research scholar (Reg. No. 1227/14/16).
V.K.S., University, Ara, Bihar, INDIA.
email:singhnavinkumar839@gmail.com

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ABSTRACT

This paper concerns with a newly introduced linear operator called (3, 2)-jection operator, In the present paper, we deal with the power of (3,2)-jection operator as well as some special properties.

Keywords: Linear operator, Projection operator, Trijection operator, (3, 2)-jection operator.

I. INTRODUCTION

The concept involved in this paper require a good knowledge of power of operators. Let us, therefore, review the concept of power of linear operator.

Let T be a linear operator on a vector space V. Then composition of T of T denoted by TOT is also a linear operator on V. We sometimes write T_1T_2 instead of T_1OT_2 where T_1 and T_2 are linear operator on V. We define power of a linear operator T as follows :

$T^0 = I$ where I is the identity operator.

$T^1 = T$

$T^2 = TT$

Since the product of linear operator is associative. Therefore if n is a positive integer, we define

$$T^n = TTT \dots \dots \text{ upto n times}$$

Obviously T^n is a linear operator on V. Moreover, if m and n are non-negative integers, it can be easily seen that

$T^mT^n = T^{m+n}$ and $(T^m)^n = T^{mn}$.

II. IMPORTANT DEFINITIONS

This section provides a brief review of all basic concepts, definitions which will be used later.

(1) **Linear operator :** It is an operator E on a linear space L such that E(ax + by) = a E(x) + bE(y) $\forall x, y \in L$ and for scalars a and b.
(2) **Projection operator :** It is an operator E on some subspace M of a linear space L such that $E^2 = E$.
(3) **Trijection operator :** It is the linear operator E on a linear space L such that $E^3 = E$.
(4) **(3, 2)-jection operator :** It is the linear operator E on a linear space L such that $E^3 = E^2$.

III. MAIN RESULTS

The analogous results are listed with the following theorems :

Theorem 1

If E be a (3, 2)-jection. Then $E^{2n}\ \forall n \in N$, is a projection.

Proof : Since E is a (3, 2)-jection, then from the definition,

$$E^3 = E^2 \tag{1.1}$$

Now, $E^4 = E^3 . E$

$= E^2 . E$ {from (1.1)}

$= E^3$

$= E^2$ {from (1.1)}

Thus, $$E^4 = E^2 \tag{1.2}$$

Here, we prove the theorem by the method of induction.
For convenience, we denote the statement
"$E^{2n}\ \forall n \in N$, is a projection." by P(n).
For n = 1, we have

$(E^2)^2 = E^4$

$= E^2$ {from (1.2)}

$\Rightarrow$ E^2 is a projection
$\Rightarrow$ P(1) is true.
Now, we suppose that P(k) be true $\forall 1 \le k < n \in N$
$\Rightarrow$ E^{2k} is a projection
$\Rightarrow$ $$(E^{2k})^2 = E^{2k} \tag{1.3}$$
For n = k + 1, we have

$\{E^{2(k+1)}\}^2 = (E^{2k}.E^2)^2$

$= (E^{2k})^2.(E^2)^2$

$= E^{2k}E^2$ {from (1.2) & (1.3)}

$= E^{2(k+1)}$

$\Rightarrow$ $E^{2(k+1)}$ is projection

Hence,P(k) is true $\Rightarrow$ P(k + 1) is true
Thus, by the method of induction

$$P(n) \text{ is true } \forall n \in N$$

$\Rightarrow$ E^{2n} is a projection $\forall n \in N$

Theorem 2

If E be a (3, 2)-jection then I – E^{2n} is a projection $\forall n \in N$

Proof : We Know

$$(E^{2n})^2 = E^{2n} \quad \{\text{See theorem (1)}\} \tag{2.1}$$

We have

$$(I - E^{2n})^2 = I + (E^{2n})^2 - 2E^{2n}$$
$$= I + E^{2n} - 2E^{2n} \quad \{\text{from (2.1)}\}$$
$$= I - E^{2n}$$

$\Rightarrow$ $I - E^{2n}$ is a projection.

Theorem 3

Let p and q be two scalars such that p + q = 1 then $pE + qE^2$ is a (3, 2)-jection where E is a (3, 2)-jection

Proof : Since E is a (3, 2)-jection then from the definition,

$$E^3 = E^2 \tag{3.1}$$

Now,

$$E^4 = E^3.E$$
$$= E^2.E \quad \{\text{from (3.1)}\}$$
$$= E^3$$
$$= E^2 \quad \{\text{from (3.1)}\}$$

So,

$$E^4 = E^2 \tag{3.2}$$

Here,

$$(pE + qE^2)^2 = p^2E^2 + q^2E^4 + 2pqE^3$$
$$= p^2E^2 + q^2E^2 + 2pqE^2 \quad \{\text{from (3.1\& (3.2)}\}$$
$$= (p^2 + q^2 + 2pq)E^2$$
$$= (p+q)^2E^2$$
$$= E^2 \quad \{\text{ puttingp + q = 1}\} \tag{3.3}$$

and,

$$(pE + qE)^3 = (pE + qE^2)^2(pE + qE^2)$$
$$= E^2(pE + qE^2) \quad \{\text{from (3.3)}\}$$

$$= pE^3 + qE^4$$
$$= pE^2 + qE^2 \quad \{\text{from (3.1) and (3.2)}\}$$
$$= (p+q)E^2$$
$$= E^2 \text{ (putting } p+q=1) \tag{3.4}$$

From (3.3) and (3.4), we have

$$(pE+qE^2)^3 = (pE+qE^2)^2$$

Hence, $pE+qE^2$ is a (3, 2)-jectionprovided $p+q=1$ and E being a (3, 2)-jection

Theorem 4

If E be a (3, 2)-jection, then

$(E-E^2)^n = 0 \forall 1 < n \in N.$

Proof : Let the statement $"(E-E^2)^n = 0"$ be denoted by P(n).

For n = 2, we have

$$(E-E^2)^2 = E^2 + (E^2)^2 - 2EE^2$$
$$= E^2 + E^2 - 2EE^3 \ \{\text{from theorem (2.1)}\}$$
$$= 2E^2 - 2E^2$$

{from the definition of (3, 2)-jection}

$$= 0$$

Let P(k) be true $\forall\ 2 \le k < n \in N$

$$\Rightarrow \quad (E-E^2)^k = 0 \tag{4.1}$$

For n = k + 1, we have

$$(E-E^2)^{k+1} = (E-E^2)^k.(E-E^2)$$
$$= 0\,(E-E^2) \quad \{\text{from (4.1)}\}$$
$$= 0$$

So, P(k) is true $\Rightarrow$ P(k + 1) is true

Hence,

P(n) is true $\forall 2 \le n \in N$

$$\Rightarrow \quad (E-E^2)^n = 0 \forall 1 < n \in N$$

Theorem 5

Let E be a (3, 2)-jection then $-E^{2n} \forall n \in N$ is a trijection.

Proof : Since E is a (3, 2) – jection then from the definition

$$E^3 = E^2 \tag{5.1}$$

Here, we prove the theorem by the method of induction.

For convenience, we denote the statement "$-E^{2n} \forall n \in N$ is a trijection" by P(n).

For n = 1, we have

$$
\begin{aligned}
(-E^2)^3 &= (-1)^3 (E^2)^3 \\
&= -(E^3)^2 \\
&= -(E^2)^2 && \{\text{from } (5.1)\} \\
&= -E^4 \\
&= -E^3.E \\
&= -E^2.E && \{\text{from } (5.1)\} \\
&= -E^3 \\
&= -E^2 && \{\text{from } (5.1)\}
\end{aligned}
$$

$\Rightarrow$ $-E^2$ is a trijection

$\Rightarrow$ P(1) is true

Now, we suppose P(k) is true $\forall \ 1 \le k < n \in N$ then $-E^{2k}$ is trijection

$$\Rightarrow \quad (-E^{2k})^3 = -E^{2k} \tag{5.2}$$

For n = k + 1, we have

$$
\begin{aligned}
\{-E^{2(k+1)}\}^3 &= (-E^{2k}.E^2)^3 \\
&= (-E^{2k})^3.(E^2)^3 \\
&= -E^{2k}.(E^3)^2 && \{\text{from } (5.2)\} \\
&= -E^{2k}.(E^2)^2 && \{\text{from } (5.1)\} \\
&= -E^{2k}.E^4 \\
&= -E^{2k}.E^3.E \\
&= -E^{2k}.E^2.E && \{\text{from } (5.1)\} \\
&= -E^{2k}.E^3 \\
&= -E^{2k}.E^2 && \{\text{from } (5.1)\} \\
&= -E^{2k+2} \\
&= -E^{2(k+1)}
\end{aligned}
$$

$\Rightarrow$ $-E^{2(k+1)}$ is a trijection

$\Rightarrow$ P(k + 1) is true when P(k) is true

Hence,

by the method of induction P(n) is true $\forall n \in E$

$\Rightarrow$ $-E^{2(n+1)}$ istrijection.

Theorem 6

If E be a (3, 2)-jection then $E^n = E^2 \; \forall 2 \le n \in N$

Proof : We prove the theorem by the method of induction.

For convenience, we denote the statement "$E^n = E^2 \; \forall 2 \le n \in N$, by P(n).

For n = 2, we have

$E^2 = E^2$ which is clearly true

$\Rightarrow$ P(2) is true

Now, we suppose p(k) is true i.e.

$$E^k = E^2 \tag{6.1}$$

$\forall \; 2 \le k < n \in N$

For n = k + 1, we have

$$\begin{aligned} E^{k+1} &= E^k.E \\ &= E^2.E \quad \{\text{from (6.1)}\} \\ &= E^3 \\ &= E^2 \end{aligned}$$

{from the definition of (3, 2)-jection}

Hence, P(k) is true

$\Rightarrow$ P(k + 1) is true

So, by the induction, p(n) is true

$\Rightarrow E^n = E^2 \; \forall 2 \le n \in N$

Theorem 7

If E be a (3, 2)-jection then $E^{2n} - I$ is always a trijectionfor $n \in N$.

Proof : We have

$$\begin{aligned} (E^{2n} - I)^2 &= (E^{2n})^2 + I - 2E^{2n} \\ &= E^{2n} + I - 2E^{2n} \end{aligned}$$

{from theorem (1)}

$$\begin{aligned} &= I - E^{2n} \\ &= -(E^{2n} - I) \end{aligned} \tag{7.1}$$

Now,

$$\begin{aligned} (E^{2n} - I)^3 &= (E^{2n} - I)^2 (E^{2n} - I) \\ &= -(E^{2n} - I)(E^{2n} - I) \quad \{\text{from (7.1)}\} \\ &= -(E^{2n} - I)^2 \\ &= -\{-(E^{2n} - I)\} \quad \{\text{from (7.1)}\} \\ &= E^{2n} - I \end{aligned}$$

Hence $E^{2n} - I$ is a trijection.

Theorem 8

If E be a trijection then E^{2n} is projection and (3, 2)-jection both, for $n \in N$.

Proof : Since E is a trijection then from the definition, we have

$$E^3 = E \tag{8.1}$$

Now, $(E^{2n})^2 = E^{4n}$

$$= (E^4)^n$$

$$= (E^3.E)^n$$

$$= (E.E)^n \quad \{\text{from } 8.1\}$$

$$= (E^2)^n$$

$$= E^{2n} \tag{8.2}$$

From (8.2), it is clear that E^{2n} is projection.

Again,

$$(E^{2n})^3 = (E^{2n})^2(E^{2n})$$

$$= E^{2n}E^{2n} \quad \{\text{from } 8.2\}$$

$$= (E^{2n})^2 \tag{8.3}$$

From (8.3), it is clear that E^{2n} is a(3, 2)-jection.

Hence, E^{2n} is a projection as well as a(3, 2)-jection where E being a trijectionand $n \in N$.

Theorem 9

If E be a (3, 2)-jection then $2E^{2n} - I$ is always a trijection $\forall n \in N$.

Proof : We have

$$(2E^{2n} - I)^2 = 4(E^{2n})^2 - 4E^{2n} + I$$

$$= 4E^{2n} - 4E^{2n} + I \quad \{\text{by theorem 1}\}$$

$$= I \tag{9.1}$$

Now,

$$(2E^{2n} - I)^3 = (2E^{2n} - I)^2(2E^{2n} - I)$$

$$= I(2E^{2n} - I) \quad \{\text{from (9.1)}\}$$

$$= 2E^{2n} - I$$

Hence, $2E^{2n} - I$ is a trijection.

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