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Thermal Radiation and Chemical Reaction Effects on an Unsteady Stretching Sheet

Nabajyoti Dutta and B. R. Sharma

Department of Mathematics,
Dibrugarh University, Dibrugarh-786004, Assam, INDIA.
email: nabajyotidutta961@gmail.com, bishwaramsharma@yahoo.com.

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ABSTRACT

In this research paper, we have examines the unsteady flow of heat and mass transfer over a horizontal stretching sheet in presence of thermal radiation and chemical reaction effects by numerical method. The governing unsteady boundary layer equations for the momentum, heat and mass transfer were transformed to a set of ordinary differential equations by using similarity transformation. Then these set of ordinary differential equations were solved by using MATLAB's built in solver bvp4c. The velocity, temperature and concentration profiles were drawn for various values of parameters such as the Chemical reaction parameter (C_R), Thermal radiation parameter (R) and unsteadiness parameter (A) and results are discussed graphically.

Keywords: Heat transfer and Mass transfer, Thermal radiation, Chemical reaction Unsteadiness parameter.

1. INTRODUCTION

The study of nonlinear MHD boundary layer flow and heat transfer over a stretching surfaces or flat plates has achieved great attention due to its applications in lots of engineering problems such as MHD paper production, power generators, petroleum industries, plasma studies, geothermal energy extractions etc. A large amount of research work has been done in the field of chemical reaction, heat and mass transfer. There has been renewed interest in studying hydro-magnetic flow and heat transfer of continuously stretched surfaces in the presence of a weak transverse magnetic field. This is because hydro-magnetic flow and heat transfer have become more essential industrially and in different branches of science and engineering.

Most of the researchers like Vajravelu and Roper[1], Ali and Magyari[2], Sajid and Hayat[3], and Ibrahim and Makinde[4,5] investigated the heat transfer problem in a stretching sheet with a linear, power-law or exponential surface velocity and a uniform or different surface temperature conditions. These type of problem was extended by Abo–Eldahab and Aziz[6] to include space-dependent exponentially decaying with internal heat generation or absorption. Abel *et al*.[7] and Bataller[8] analysed the effects of non-uniform heat source on viscoelastic fluid flow and heat transfer over a stretching sheets. Other researchers including Pantokratoras[9] and Mukhopadhyay and Layek[10] extended the problem to include the effects of variable fluid properties on the flow over a stretching sheet. In most of these investigations, the flow and temperature fields were considered at steady state. Some other researchers such as Dandapat *et al*.,[11,12]; Seini, Y. I.[13] recently investigated the unsteady flow over stretching surface in presence of non-uniform heat source and chemical reaction effects. Also Sharma and Nath[14] have studied the effects of heat source, chemical reaction, thermal diffusion and magnetic field on demixing of a binary fluid mixture flowing over a stretched surface.

In this paper, we extend the work of Shateyi and Motsa[15]. Here, we have studied the two dimensional unsteady nonlinear MHD boundary layer flow of an incompressible viscous electrically conducting binary mixture of fluids flowing over a horizontal porous stretching surface in presence of uniform magnetic field by taking into account the Thermal radiation and Chemical reaction effects. In previous studies finite- difference, Runga-Kutta integration etc. schemes were used. Here, we are used bvp4c solver of MATLAB's software which implements a collocation method for the solution of BVPs. And this numerical method gives better approximations than most of the other numerical methods.

2. MATHEMATICAL FORMULATION

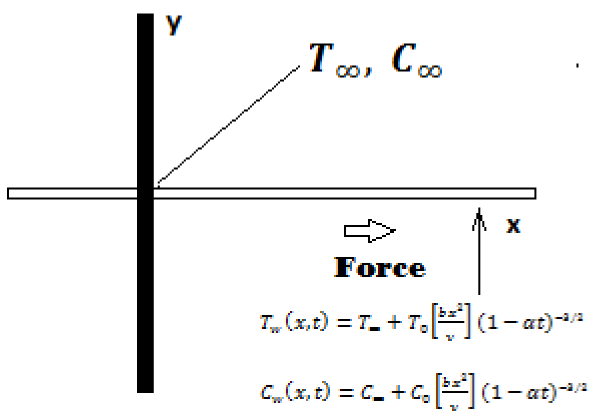

Fig 1: Schematic Diagram of the flow problem

Let us consider an unsteady two dimensional incompressible and viscous flow on a horizontal porous stretching sheet which comes from a narrow slot at the origin. The fluid flow over the unsteady stretching sheet is composed of a reacting chemical species. The fluid motion arises due to the stretching of the elastic sheet. The continuous sheet aligned with the x axis at $y = 0$ moves in its own plane with a velocity $U_w(x,t) = \frac{bx}{(1-at)}$, (where both a and

b are positive constants with dimension reciprocal time and $t < \frac{1}{a}$) in the positive x-direction. The fluid is considered to be Newtonian with constant temperature (T_∞) and concentration (C_∞) away from the surface. The temperature $T_w(x,t)$ of the sheet is different from that of the ambient medium and $C_w(x,t)$ is concentration distribution near the sheet and both vary with time t and that distance x along the sheet. The fluid is assumed to be gray, emitting and absorbing but non-scattering medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x direction is negligible in comparison with that in the y direction. It is assumed that the external electric field is zero and Hall effects are negligible. Here, the induced magnetic field is negligibly small. The fluid velocity and thermal conductivity are assumed to vary linearly with temperature. The system influenced by an external transverse magnetic field of strength B defined as $B(t) = B_0(1-at)^{-1/2}$. It also assumed that the chemically reactive species undergo first order chemical reaction.

Under the above assumptions, the governing equations of continuity, momentum, energy and concentration are given by

Equation of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Equation of Momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\vartheta}{k}u \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Concentration equation

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1(C - C_\infty) \tag{4}$$

Where u and v are the velocity components in the x and y axes respectively. ϑ is the kinematic coefficient of viscosity, g is the acceleration due to gravity, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, σ is the electrical conductivity, B_0 is the uniform magnetic field, k is the thermal conductivity of the fluid, T is the temperature of the fluid mixture, C is the fluid concentration, T_∞ is the temperature far away from the sheet, C_∞ is the species concentration far away from the sheet, α is the thermal diffusivity of the fluid mixture, ρ is the density of the fluid mixture, C_p is the specific heat at constant pressure, D is the molecular diffusion coefficient, K_1 is the chemical reaction coefficient.

The radiative heat flux term is simplified by using the Rosseland approximation as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the Mean absorption coefficient, respectively. We assume that the temperature differences within the flow are sufficiently small so that T^4 can be expressed as a linear function of temperature, we expand T^4 in a Taylor's series about T_∞ as follows:

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \ \dots \dots$$

And neglecting higher order terms beyond the first degree in $(T - T_\infty)$ we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

In view of equation (5) and (6), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

So from equation (3), we have

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k_1\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} u(x,0) = U_w(x,t),\ v(x,0) = 0,\ T(x,0) = T_w(x,t),\ C(x,0) = C_w(x,t), \\ u(x,\infty) \longrightarrow 0,\ (x,\infty) \longrightarrow T_\infty\ ,\ C(x,\infty) \longrightarrow C_\infty \end{array}\right\} \tag{9}$$

We assume that both the surface temperature $T_w(x,t)$ and surface concentration $C_w(x,t)$ of the stretching sheet to vary with the distance x along the sheet and time in the following form:

$$T_w(x,t) = T_\infty + T_0\left[\frac{bx^2}{\vartheta}\right](1 - \alpha t)^{-3/2} \tag{10}$$

$$C_w(x,t) = C_\infty + C_0\left[\frac{bx^2}{\vartheta}\right](1 - \alpha t)^{-3/2} \tag{11}$$

Where T_0 is a heating or cooling reference temperature and C_0 is a positive concentration reference.

The equation for the temperature increases (reduces) if T_0 is positive (negative) from T_0 at the leading edge in proportion to x^2 and such that the amount of temperature increase (reduction) along the sheet increases with time. Similarly, it is same for the concentration equation.

In order to reduce $(1) - (4)$ into a set of ordinary differential equations, we introduce the similarity variable η and the dimensionless variables f, θ and ψ.

$$\left.\begin{array}{l} \psi = (\vartheta b)^{1/2}(1 - \alpha t)^{-1/2} x f(\eta),\ \eta = (\frac{b}{\vartheta})^{1/2}(1 - \alpha t)^{-1/2} y, \\ T = T_\infty + T_0\left[\frac{bx^2}{2\vartheta}\right](1 - \alpha t)^{-3/2}\,\theta(\eta),\ \ C = C_\infty + C_0\left[\frac{bx^2}{2\vartheta}\right](1 - \alpha t)^{-3/2}\,\theta(\eta) \end{array}\right\} \tag{12}$$

Where $\psi(x,y)$ is the physical stream function which automatically satisfies the continuity equation.

The velocity components are then derived from the stream function expression and obtained as

$$u = \frac{\partial\psi}{\partial y} = U_w f'(\eta),\ v = -\frac{\partial\psi}{\partial x} = -(\vartheta b)^{1/2}(1-\alpha t)^{-1/2} f(\eta) \quad (13)$$

Using (12) in equations $(2)-(4)$, the governing equations are reduces to

$$f''' + ff'' - f'^2 - A\frac{\eta}{2}f'' - (A+M+K)f' + Gr\theta + Gm\phi = 0 \quad (14)$$

$$(3R+4)\theta'' + 3RPr\left[f\theta' - 2f'\theta - \frac{A}{2}(3\theta + \eta\theta')\right] = 0 \quad (15)$$

$$\phi'' + Sc\left[f\phi' - 2f'\phi - \frac{A}{2}(3\phi - \eta\phi') - C_R\phi\right] = 0 \quad (16)$$

Boundary conditions are given below

$$\left.\begin{array}{l} f(0)=0,\ f'(0)=1,\ \theta(0)=\phi(0)=1 \\ f'(\infty)\longrightarrow 1,\ \theta(\infty)\longrightarrow 0,\ \phi(\infty)\longrightarrow 0 \end{array}\right\} \quad (17)$$

Using Parameters are

$A = \frac{\alpha}{b}$ = Unsteadiness parameter, $M = \frac{\sigma B_0^2(1-\alpha t)}{\rho b}$ = Magnetic parameter,

$K = \frac{\vartheta(1-\alpha t)}{kb}$ = Permeability parameter, $Gr = \frac{g\beta(T_w - T_\infty)(1-\alpha t)^2}{b^2 x}$ = Thermal Grashof number,

$Gm = \frac{g\beta^*(C_w - C_\infty)(1-\alpha t)^2}{b^2 x}$ = Mass Grashof number, $R = \frac{kk_1}{3\sigma^* T_\infty^3}$ = Radiation parameter,

$Pr = \frac{\alpha}{D}$ = Prandtl number, $Sc = \frac{\vartheta}{D}$ = Schmidt number, $C_R = \frac{k_1(1-\alpha t)}{b}$ = Instantaneous reaction rate parameter.

3. NUMERICAL ANALYSIS AND DISCUSSIONS

Solutions of non-linear coupled ordinary differential equations (14) to (16) under boundary condition (17) cannot be obtained in closed form. Hence these equations are solved numerically by using MATLAB's built in solver bvp4c. Graphical representations of these solutions are shown below for various values of parameters like unsteadiness parameter (A), Chemical reaction parameter (C_R) and Thermal radiation parameter (R).

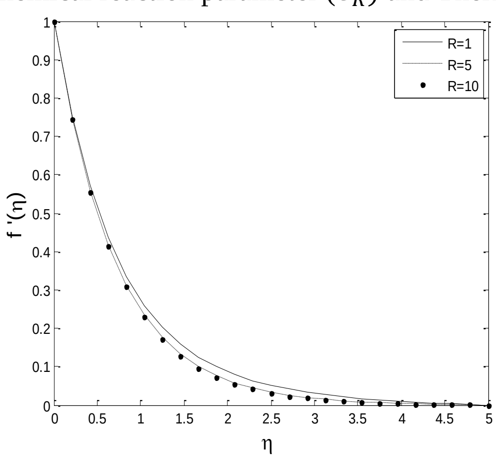

Figure 2: Effects of Thermal radiation (*R*) on velocity profiles.

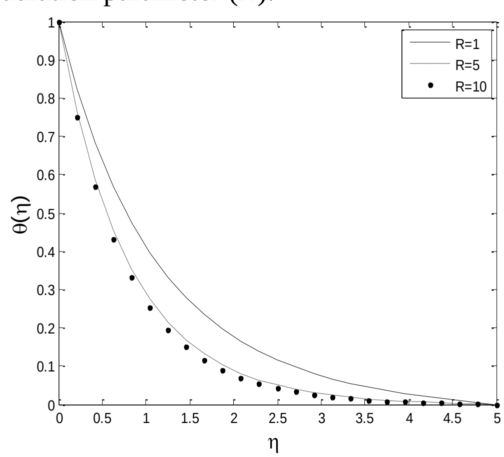

Figure 3: Effects of Thermal radiation (*R*) on temperature profiles.

The effects of thermal radiation parameter (R) on the velocity profiles and temperature profiles in the boundary layer are illustrated in figure 2 and figure 3. We observe in these figures that increasing the thermal radiation parameter produces significant decrease in the thermal condition of fluid and its boundary layer. It can be explained in such a way that decrease in the values of $R\ (= kk_1/3\sigma^* T_\infty^3)$ for the given values of k and T_∞ means a decrease in the Rosseland radiation absorptivity k_1.

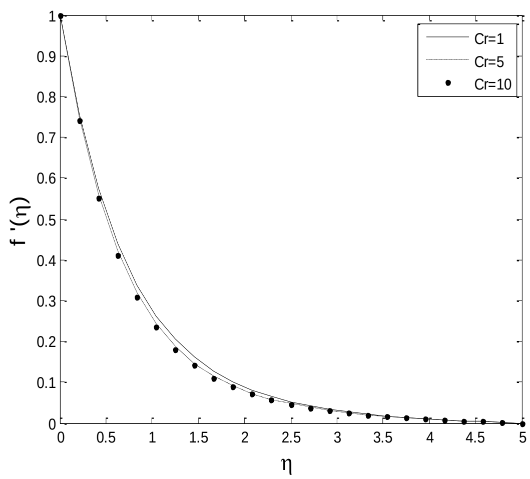

Figure 4: Effects of Chemical reaction (C_R) on velocity profiles.

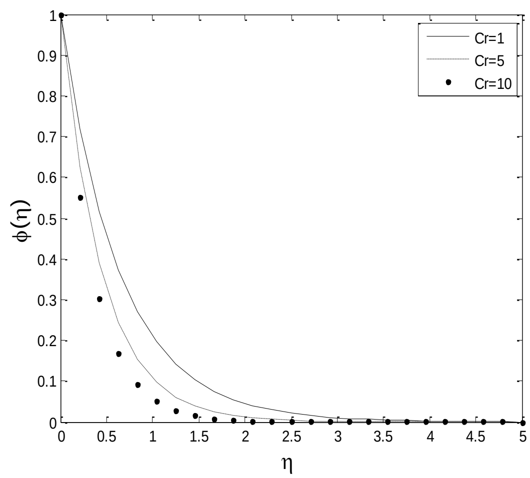

Figure 5: Effects of Chemical reaction (C_R) on concentration profiles.

In figures 4 and 5, we observed that the velocity and concentration are decreases with increase in Chemical reaction (C_R) parameter. This is caused by the destructive nature of the chemical reaction within the boundary layer.

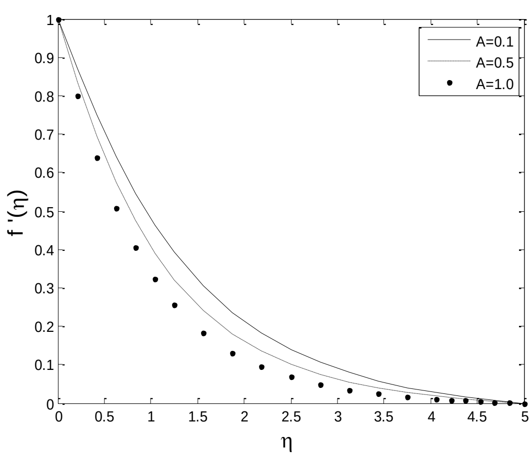

Figure 6: Effects of Unsteadiness parameter(A) on velocity profiles.

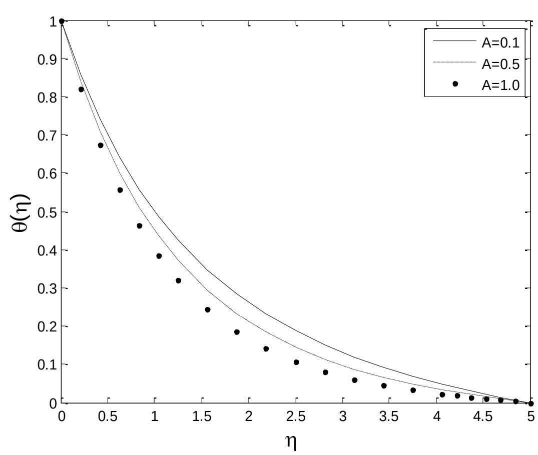

Figure 7: Effects of Unsteadiness parameter(A) on temperature profiles.

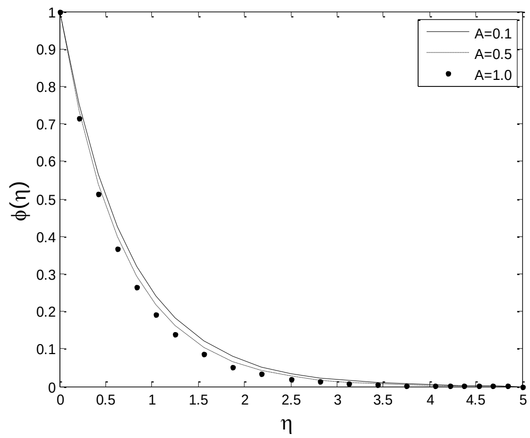

Figure 8: Effects of Unsteadiness parameter (A) on concentration profiles.

In figures (6) – (8), we show the velocity, temperature and concentration profiles for different values of Unsteadiness parameter (A) and all other values are constant. In figure (6) we observe that the velocity decreases with the distance from the stretching sheet for all values of A. Also increasing the values of A, it decreases the velocity in the boundary layer. The flow will be steady for $A = 0$ and for $A > 0$, the flow will be unsteady. Similarly in figures (7) and (8), concentration and temperature decreases due to increase the unsteadiness parameter (A). When the values of A are increased in the system, the boundary layer thicknesses are reduced and this inhibits the development of transition of laminar to turbulent flow. Which implies that the stretching of surfaces can be used as a flow stabilizing mechanism.

4. CONCLUSIONS

The velocity and temperature of the binary fluid mixture decreases with the increase of Thermal radiation parameter. It can also be concluded that the velocity of the binary fluid mixture decreases near the surface due to increase of chemical reaction parameter. Concentration of the rarer and lighter components of the binary fluid mixture decreases with increase of dimensionless chemical reaction parameter. Also with the increase of Unsteadiness parameter, the velocity, temperature and concentration of the binary fluid mixture decreases.

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