Journal of Computer and Mathematical Sciences, *Vol.10(1),215-221 January 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Effect of Magnetic Field on the Flow of a Dusty Fluid in An Inclined Parallel Plate Vertical Channel

Monika Srivastava[1] and Rajeev Khare[2]

Department of Mathematics & Statistics,
Sam Higginbottom University of Agriculture,
Technology and Sciences, Prayagraj (Allahabad), (U. P.), INDIA.
email: monikasrivastava945@gmail.com

(Received on: January 7, 2019)

ABSTRACT

The two phase flow of fluid through a channel in an inclined magnetic field is of very importance as such flows occur in industries using metals. Therefore in this work an attempt has been made to study the flow of a dusty fluid through a vertical channel filled with porous medium placed under a magnetic field. Assuming suitable boundary conditions and governing equations have been solved analytically by variable separable method and a graphical study has been made to express the result with physical nature of the problem. The focus has been given on velocity profile with respect to magnetic field.

Keywords: MHD flow, dusty fluid, magnetic field, porous medium.

INTRODUCTION

The oscillatory dusty fluid through a channel in an inclined magnetic is of great importance as such flows occur in industries producing metallic items and petroleum transport etc. Fluid flow under the effect of magnetic field and heat transfer occur in MHD accelerators, pumps and generators.

The fluid flow through porous media has very important because of recovery of crude oil from pores of reservoir rocks. This type of fluid is very useful in plasma studies, aerodynamics and geothermal energy extraction etc. Many researchers have worked this type of problem. Saffiman (1962) proposed equation of motion for binary mixture of fluid and dust particles. Kulshetra and and Puri (1981) studied wave structure in oscillatory couette flow of a dusty gas. Han *et al*. (1991) analyzed the heat transfer in pipe carrying two-phase gas particle suspension. Meanwhile, the thermal radiation is a characteristic of any flow system at temperature above the absolute zero and can strongly interact with convection in many situations of engineering interest. The differential approximation for radiative transfer in a non- gray gas near equilibrium was analyzed by Cogley *et al*. (1968). Forced convection-radiation on free convection flow through porous medium. Rapits et al (2004) investigated the effect of thermal radiation on MHD flow. Unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer was discussed by Mbeledogu *et al*. (2007). Giressha *et al*. (2010) analyzed unsteady flow and heat transfer of a dusty fluid through a rectangular channel under the effect of pulsatile pressure gradient and uniform magnetic field.

In this present paper, we extend the study of OM Prakash, Makinde and Diwedi (2015) on heat transfer to MHD oscillatory dusty fluid flow in a channel filled with a porous medium by introducing the fluid as binary mixture of fluid and suspended particles. In the following sections, the problem is formulated analyzed and solved with graphical result. The aim of the present paper is to consider the effect of magnetic field on the flow of a dusty fluid in an inclined parallel plate vertical channel.

FORMULATION OF THE PROBLEM

Consider the two-phase flow of fluid through a channel in an inclined magnetic field .the flow of a dusty fluid through a vertical channel filled with porous medium. Take a less electrically conducting fluid in channel and also consider Cartesian co-ordinate system (x,y). Where x-axis lies along the centre of the channel y-axis is distance measured in the normal direction.

The solid dust particles uniformly distributed in the flow region and their number density N_0 is constant throughout the motion and temperature acts between the dust particles is uniformly during the motion. The flow region has uniformly applied magnetic field. The dust particles are uniformly distributed and transported within the fluid such that continuity equation of motion is satisfied. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are as follows:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \vartheta\frac{\partial^2 u}{\partial y^2} - \frac{\vartheta}{K}u + \frac{N_\circ K_\circ}{\rho}(u_P - u) - \frac{\sigma_{eB_\circ^2 sin^2\varphi}}{\rho}u + g\beta(T - T_\circ) \quad (1.1)$$

$$\frac{\partial u_P}{\partial t} = K_\circ(u - u_p) \quad (1.2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_P}\frac{\partial q}{\partial y} \quad (1.3)$$

Where

u, u_P	:	Velocity of fluid and dust particles in x-direction of motion.
t	:	Time
ω	:	Frequency of oscillations
T	:	Fluid temperature
T_f	:	Fluid initial temperature
$T_\circ$	:	Left force

q : Radiative heat flux
β : Expansion due to temperature
$K_\circ (= 6\pi\rho\vartheta D)$: Stokes constant
D : Average radius of dust particles
C_P : Specific heat of constant pressure
k : Thermal conductivity
K : Porous medium permeability coefficient
σ_e : Fluid conductivity
ρ : Fluid density
ϑ : Kinematic viscosity
$\beta_\circ (= \mu_e H_\circ)$: Electromagnetic induction
μ_e : Magnetic permeability
$H_\circ$: Intensity of magnetic field

With initial boundary conditions,

$u(y,0) = u_P(y,0) = 0, T(y,0) = T_f$

$u(a,t) = u_P(a,t) = 0, T(a,t) = T_w = T_\circ + (T_f - T_\circ)e^{i\omega t},$

$u(0,t) = u_P(0,t) = 0, T(0,t) = T_\circ$

The fluid is assumed to be optically thin with a relatively low density and the radiative heat flux is given by-

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_\circ - T) \qquad (1.4)$$

Where

α : Mean radiation absorption coefficient

Dimensionless variables and parameters are-

$$x' = \frac{x}{a}, y' = \frac{y}{a}, u' = \frac{u}{a}, \ \theta = \frac{(T - T_\circ)}{(T_f - T_\circ)}, \ t' = \frac{tU}{a}, \ D_a = \frac{K}{a^2}, \ M = \frac{\vartheta}{K_\circ a^2},$$

$$l = \frac{N_\circ K_\circ a^2}{\rho\vartheta}, \ R_e = \frac{Ua}{\vartheta}, P_r = \frac{\vartheta\rho C_P}{k}, \ N^2 = \frac{4\alpha^2 a^2}{k}, u'_P = \frac{u_P}{U}.$$

$$G_r = \frac{g\beta(T_f - T_\circ)a^2}{\vartheta U}, \ P' = \frac{aP}{\vartheta\rho U}, H^2 = \frac{a^2\sigma_e B_\circ{}^2}{\vartheta\rho}, \ s^2 = \frac{1}{Da}, \qquad (1.5)$$

Where

U : Flow mean flow velocity
s : Porous medium shape factor parameter
D_a : Darcy number
G_r : Grashof number
H : Hartmann number
l : Particle concentration parameter
M : Particle mass parameter
N : Radiation parameter
R_e : Flow Reynolds number
P_r : Prandtl number

The dimensionless governing equations together with the suitable boundary conditions (removing the $(\,'\,)$ can be written as

$$R_e \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2 sin^2\varphi + l)u + lu_P + G_r\theta \quad (1.6)$$

$$R_e M \frac{\partial u_P}{\partial t} = (u - u_P) \quad (1.7)$$

$$R_e P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \quad (1.8)$$

Initial boundary conditions

$\text{u}(y, 0) = u_P(y, 0) = 0, \theta(y, 0) = 1$

$u(1, t) = u_P(1, t) = 0, \theta(1, t) = e^{i\omega t}$,

$u(0, t) = u_P(0, t) = 0, \theta(0, t) = 0$

Solution of the Problem

Solving equations (1.6), (1.7) and 1.8) for pure Oscillatory flow. Let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}\ ,\quad \text{u}(y, t) = u_0(y)e^{i\omega t}\ ,\quad u_P(y, t) = u_{P_0}(y)e^{i\omega t}\ ,\quad \theta_0(y, t) = \theta_0(y)e^{i\omega t} \quad (1.9)$$

Where

λ : Constant Oscillation amplitude for pressure gradient.

Substituting the values from equations (1.9) into equations (1.6) to (1.8) and above boundary conditions, we obtain

$$\frac{d^2 u}{dy^2} - {m_2}^2 u_\circ = -\lambda - G_r\theta_\circ \quad (1.10)$$

Where

$${m_2}^2 = s^2 + H^2 sin^2\varphi + i\omega t R_e + \frac{l}{1 + i\omega R_e M} \quad (1.11)$$

$$u_{P_0} = \frac{u_0}{(1 + i\omega R_e M)} \quad (1.12)$$

$$\frac{d^2\theta_0}{dy^2} + {m_1}^2\theta_0 = 0 \quad (1.13)$$

Where

${m_1}^2 = N^2 - i\omega R_e P_r$

With boundary condition

$$u_0 = u_{P_0} = 0\ ,\ \theta_0 = 1\ on\ y = 1 \quad (1.14)$$

$$u_0 = u_{P_0} = 0\ ,\ \theta_0 = 0\ on\ y = 0 \quad (1.15)$$

Solving equation (1.13) with boundary conditions (1.14) and (1.15),

We obtain temperature for fluid

$$\theta(y, t) = \frac{sin(m_1 y)}{sin(m_1)} e^{i\omega t} \quad (1.16)$$

Using equation (1.10) and boundary condition (1.14) and (1.15), we obtain the solution for the dusty fluid velocity

$$u(y, t) = \left\{\frac{G_r}{{m_1}^2 + {m_2}^2}\left[\frac{sin(m_1 y)}{sin(m_1)} - \frac{sinh(m_2 y)}{sinh(m_2)}\right] + \frac{\lambda}{{m_2}^2}\left[\begin{matrix}\frac{sinh(m_2 y)}{sinh(m_2)}(cosh(m_2) - 1) \\ +\left(1 - cosh(m_2 y)\right)\end{matrix}\right]\right\} e^{i\omega t} \quad (1.17)$$

Solving equation (1.12) and we obtain dust particles velocity

$$u_P(y,t) = \frac{e^{i\omega t}}{1+i\omega R_e M}\left\{\frac{G_r}{{m_1}^2+{m_2}^2}\left[\frac{sin(m_1 y)}{sin(m_1)} - \frac{sinh(m_2 y)}{sinh(m_2)}\right] + \frac{\lambda}{{m_2}^2}\left[\frac{sinh(m_2 y)}{sinh(m_2)}(cosh(m_2)-1) + \left(1-cosh(m_2 y)\right)\right]\right\}e^{i\omega t} \quad (1.18)$$

RESULT AND DISCUSSION

Table (A)

Layer distance (y)	Magnetic field ($H_\circ$=1)	Magnetic field ($H_\circ$=2)	Magnetic field ($H_\circ$=3)	Magnetic field ($H_\circ$=4)
0.0	0.532272	0.268930	0.147393	0.090275
0.1	0.528300	0.279622	0.161848	0.104815
0.2	0.522928	0.289246	0.175474	0.118686
0.3	0.514769	0.296744	0.187449	0.131222
0.4	0.502465	0.301078	0.196966	0.141776
0.5	0.484701	0.301246	0.203247	0.149717
0.6	0.460212	0.296282	0.205542	0.15444
0.7	0.427805	0.285276	0.203143	0.155375
0.8	0.386362	0.267376	0.195389	0.151986
0.9	0.334859	0.241798	0.181672	0.143783
1.0	0.272370	0.207838	0.161443	0.130324
1.1	0.198079	0.164872	0.134218	0.111218
1.2	0.111290	0.112369	0.099586	0.086132

Table (B)

Layer distance (y)	Magnetic field ($H_\circ$ =1)	Magnetic field ($H_\circ$ =2)	Magnetic field ($H_\circ$ =3)	Magnetic field ($H_\circ$ =4)
0.0	0.425818	0.215144	0.117914	0.072220
0.1	0.422640	0.223698	0.129479	0.083852
0.2	0.418342	0.231397	0.140379	0.094948
0.3	0.411815	0.237395	0.149959	0.104978
0.4	0.401972	0.240863	0.157573	0.113421
0.5	0.387761	0.240997	0.162597	0.119773
0.6	0.368170	0.237026	0.164434	0.123552
0.7	0.342244	0.228221	0.162515	0.124300
0.8	0.309090	0.213901	0.156311	0.121589
0.9	0.267887	0.193439	0.145337	0.115027
1.0	0.217896	0.16627	0.129154	0.104259
1.1	0.158463	0.131898	0.107375	0.088974
1.2	0.089032	0.089895	0.079669	0.068905

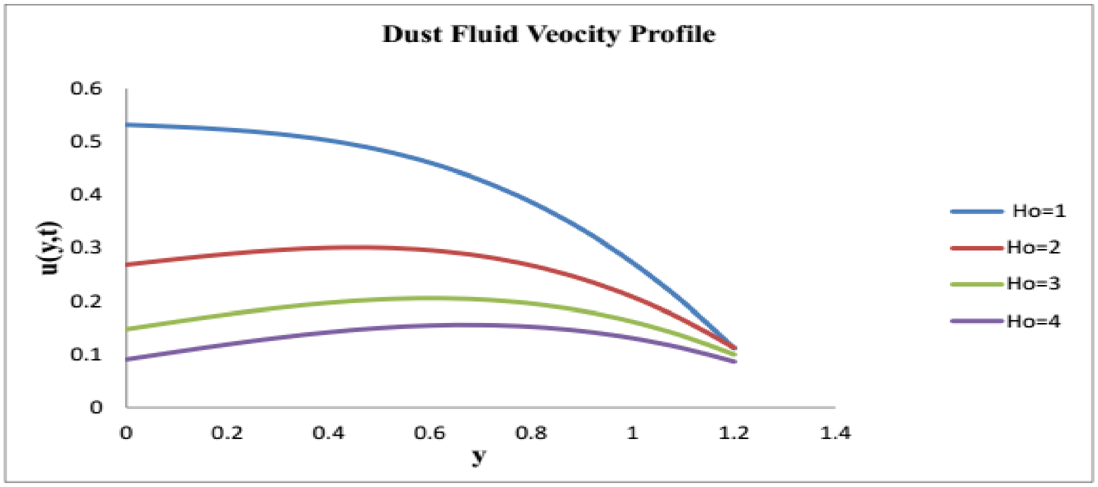

Figure (A)

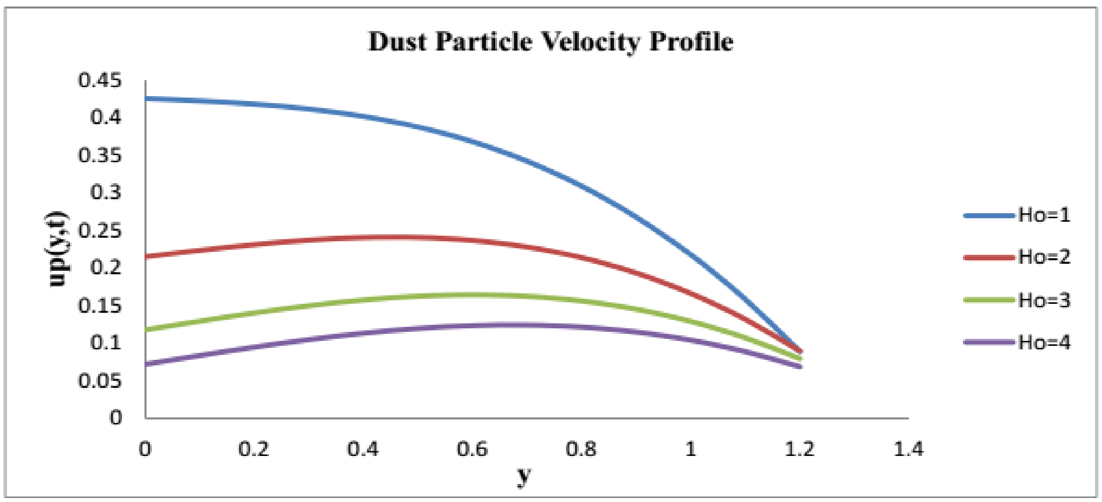

Figure (B)

From figures it is observed that the velocity of both fluid and particle phase starting with same initial value decreases sharply and terminate at some point for every value of magnetic field with respect to layer distance. The distance increases velocity decreases the result is clear that its increasing value reduces the velocity due to increasing pull, However, particle velocity has less initial value in comparison to fluid velocity due to more attraction on particle phase in comparison to fluid phase but both follow the same nature in shape and the pattern of change. Also the terminating point lies around (=1.2 to 1.3) for both cases and this may be applicable various industrial problems from the derived relation.

REFERENCES

1. Bestman A R and Adjepong S K. Unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating fluid. *Astrophysics Space Sci*. 143: 217–224 (1988). Cogley A C L, Vinvent W C and Gilees S E. Differential approximation for radiative transfer in a non-grey gas near equilibrium. *Am. Inst. Aeronaut. Astronaut*. 551 (1968).
2. El-Hakiem M A. MHD oscillatory flow on free convection-radiation through a porous medium with constant suction velocity. *J. Magnet. Magnet. Mater*. 220: 271–276. Han K S, Sung H J and Chung M K 1991 Analysis of heat transfer in a pipe carrying two-phase gas-particle suspension. *Int. J. Heat Mass Transfer* 34(1) pp.69–78 (2000).
3. Kulshretha P K and Puri P. Wave structure in oscillatory Couette flow of a dustygas. *Acta Mech. Springer Verlag* 46: 127–128 (1981).
4. Makinde O D and Mhone P Y. Heat transfer to MHD oscillatory flow in a channel filled with porous medium. *Rom. J. Phys*. 50(9–10): 931–938 (2005).
5. Mansour M A. Forced convection-radiation interaction heat transfer in boundary layer over a flat plate submerged in a porous medium. *Appl. Mech. Eng*. 3: 405–413 (1997).
6. Mbeledogu I U, Amakiri A R C and Ogulu A. Unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. *Int. J. Heat Mass Transfer* 50: 1668–1674 (2007).
7. Park H J and Baek S W. Non gray thermal radiation effects on the sound wave propagation in gasparticle two-phase medium. *J. Quant. Spectroscopy Radiative Transfer* 73(2–5)(15): 397–407 (2002).
8. Prakash O, Kumar D and Dwivedi Y K. MHD free convection flow of a visco-elastic (Kuvshiniski type) dusty gas through a semi infinite plate moving with velocity decreasing exponentially with time and radiative heat transfer. *AIP Adv* 1: 022132 (2011).