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On Operational Properties of Rotational Fuzzy Set Model

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ABSTRACT

Fuzzy set theory is a tool to study the rational subjectivities and in this paper, alpha cut representation for a new rotational fuzzy set model is introduced with the geometrical illustration. Also, various operations on the new rotational fuzzy set model has been developed along with the theoretical illustrations. Suitable applications are discussed for the operations defined over the new rotational fuzzy set model.

Keywords: Angular fuzzy set model, rotational fuzzy set model, translational fuzzy set model.

1. INTRODUCTION

Impreciseness while taking decisions are studied by fuzzy set models[9-11]. Fuzzy set is widely used to evaluate and categorize the subjective opinion of the decision makers and Experts. Fuzzy set is an extension of classical set, which provides a tool to grade the qualitative statements in order to reduce the inherent impreciseness within the boundaries. Fuzzy set model has been widely subdivided into two types; Translational and Rotational Fuzzy Set Model[3,5]. Fuzzy numbers such as triangular, trapezoidal and pentagonal fuzzy numbers[17] are commonly used translational fuzzy sets. Translational fuzzy models study the linguistic impreciseness in linear pattern[5]. Rotational fuzzy set models study the linguistic impreciseness in terms of angles[3-5]. T. Pathinathan and K. Ponnivalavan introduced a new type of translational fuzzy number called pentagonal fuzzy number[17] along with various properties and arithmetic operations. Later on, they developed the generalized properties of pentagonal fuzzy numbers with certain theoretical concepts[18]. T. Pathinathan and E. Mike Dison in the year 2017 gave a theoretical explanation for pentagonal fuzzy numbers[16]. Also, they have studied the similarity measures and defuzzification of two pentagonal fuzzy numbers with suitable illustrations[15]. Later on, J. Jesintha Rosline and E. Mike Dison introduced symmetric pentagonal fuzzy number with suitable geometrical illustrations[8].

Angular fuzzy set model[4] was introduced by Fabian C. Hadipriono and Keming Sun in the year 1990 to study the rotational characteristics. The linguistic values in angular fuzzy set model are represented by angles[4]. Angular fuzzy set model captures the structural impreciseness in an engineering mechanism[2,5,6-7,12]. Angular fuzzy set model replaces the other rotational models with its simple, handy interpretation over real structural phenomenon. In the year 2012, Ali Reza Afshari employed angular fuzzy set models to analyze the reason behind project delays[1].

In the early 2018, T. Pathinathan and E. Mike Dison introduced a new type of rotational fuzzy set model[14], where they developed the one-one correspondence among the trigonometric sine function and unit circle. The linguistic values are considered as angles in the newly introduced rotational fuzzy set model. The new rotational fuzzy set model discusses the trans-rotational property between trigonometric sine function into a unit circle, which tries to handle the impreciseness within a single function. Also, illustrations have been made to verify the newly introduced rotational fuzzy set model.

Through this paper, we have extended the theoretical aspects of the newly introduced rotational fuzzy set model. We have introduced the basic fuzzy set theoretic operations such as fuzzy union, fuzzy intersection and fuzzy weighted mean and discussed the alpha-cut representation of rotational fuzzy set model with proper graphical illustration. Rating space for the newly introduced rotational fuzzy set model has been defined with the help of suitable geometrical illustration.

Throughout this paper, we adopt $\underset{\sim}{A}$ notation for representing the fuzzy set, whereas in our previous papers we have used $\sim$ above the letter A. The paper is organized as follows. Section two provides few basic definitions and concepts on rotational fuzzy model with illustration. Section three provides the theoretical background such as rating space and alpha-cut representation for the new rotational fuzzy set model. Section four discusses the fuzzy set theoretic operations on new rotational fuzzy set model with suitable graphical illustrative examples and followed by conclusion in section five.

2. PRELIMINARIES

In this section, we provide some basic notations and definitions related to rotational models. Translational and rotational fuzzy set models have been discussed with the graphical illustration. Along with the models, we have described various notations relating with the new rotational fuzzy set model.

2.1 Fuzzy Set

Let X be a non-empty set and $\underset{\sim}{A}$ is said to be a fuzzy set defined by the function,

$$\underset{\sim}{A} = \quad x, \mu_{\underset{\sim}{A}}(x) \ \mid x \in X$$

where x is an element in the set X which is referred as universe of discourse and $\mu_{\underset{\sim}{A}}(x)$ is the membership value associated with each element of the set X which assigns the continuum value between 0 and 1.

2.2 Translational Fuzzy Set Models

Translational fuzzy set models are employed to represent the linguistic impreciseness, where the fuzzy predicates are partitioned into two or more subcategories[5]. Linguistic partition is one of the effective method to study the vagueness among the subcategories. Each linguistic partition is characterized by a fuzzy membership function. Fuzzy numbers are the widely used fuzzy membership function to represent the impreciseness among the partition and their boundaries. Triangular, Trapezoidal and Pentagonal fuzzy numbers are the commonly used fuzzy membership functions to represent the uncertainty among the linguistic partitions.

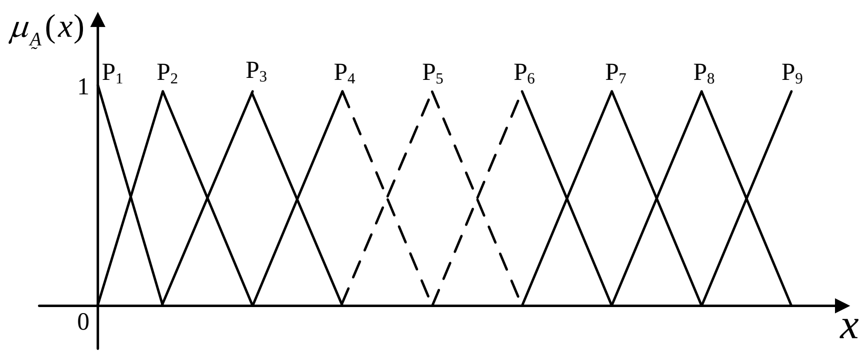


Figure 1: Translational Fuzzy Set Model

In the above figure 1, $P = \ P_1, P_2, P_3 ... P_8$ be the partition and triangular fuzzy membership function is employed to represent the linguistic ambiguities among the categories. For instance, 'age' is the linguistic variable which is further classified into two major categories such as 'young' and 'old' along with the partition of subcategories which ranges from 'very young' to 'very old'. The above figure clearly shows the partitions among the subcategories with the support of triangular fuzzy membership function widely referred as triangular fuzzy number. The domain set has the real values ranging from 0 to 100, where '0' represents the initial value partition P_1 labelling the subcategory "very young" and '100' represents the final value partition P_8 labelling the subcategory "very old" respectively. Each partition is characterized by triangular fuzzy membership function with fuzzy membership value ranging from 0 to 1. In the above figure, the triangular membership functions are taken to be normal, i.e., the core value of each of the triangular partition is 1. The linguistic variables characterized by such functions along with finite partitions are widely referred as translational fuzzy set model.

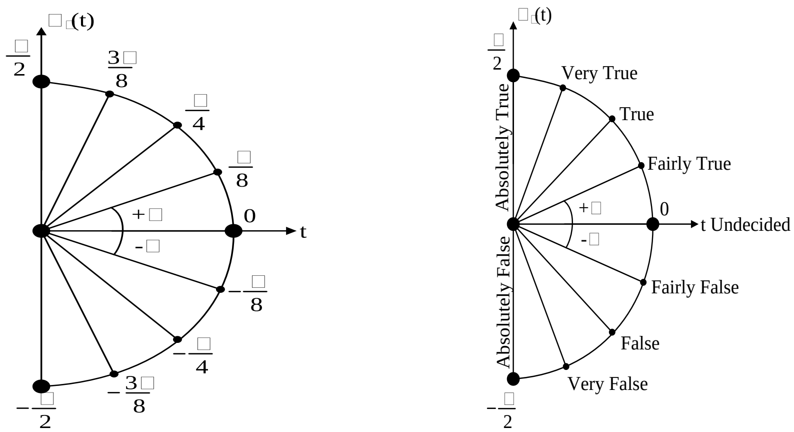
2.3 Angular Fuzzy Set Model

Angular fuzzy set model is introduced to study the rotational behavior and uncertainties observed in the structural mechanics. Fabian C. Hadipriono introduced the angular fuzzy set model in the year 1990 [4]. Later on, the angular fuzzy set models have been employed to study various real life problems involving structural mechanics related uncertainties. Instead of characterizing fuzzy predicates with the triangular, trapezoidal and pentagonal membership function, Hadipriono employed angles to represent the linguistic partitions. In angular fuzzy set model, linguistic partitions have been made on the half circle with an angular representation that takes membership gradation among the linguistic categories between $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Each truth value partition has been represented by an angles. The following table provides the truth value partition made by Hadipriono in the year 1990 [4];

Table 1: Truth Value Partition of Angular Fuzzy Set Model

Truth value partition	Angular Linguistic values
Absolutely True (AT)	$\theta = \pi/2$
Very True (VT)	$\theta = 3\pi/8$
True (TR)	$\theta = \pi/4$
Fairly True (FT)	$\theta = \pi/8$
Undecided (UN)	$\theta = 0$
Fairly False (FF)	$\theta = -\pi/8$
False (FA)	$\theta = -\pi/4$
Very False (VF)	$\theta = -3\pi/8$
Absolutely False (AF)	$\theta = -\pi/2$

The geometrical representation of angular fuzzy set model and truth values in terms of angular fuzzy set model are illustrated in the below figure.



(a) **(b)**

Figure 2: (a) Angular Fuzzy Set Model (b) Angular Fuzzy Set Model for Truth values

3. NEW ROTATIONAL FUZZY SET MODEL

Adjective pairs such as tall and short, fast and slow, young and old, strong and weak etc are partitioned into two or more subcategories by adjoining linguistic hedges with the fuzzy predicate. Different scales are employed to study the partition (gradation) among the adjective pairs. Translational fuzzy set models such as Triangular, Trapezoidal and Pentagonal fuzzy numbers are used to represent the multi-granular gradation among the adjective pairs. The new rotational fuzzy set model accommodates the adjective pairs such as young and old, true and false, bad and good, pleasant and unpleasant etc in the unit circle. Transition between the two adjective pairs has been captured by partitioning the two extreme linguistic terms in a unit circle.

Transition among the linguistic terms such as young and old, good and bad etc in the unit circle is observed by the deviation among the angles in all the four quadrants respectively. For instance the typical illustration in[14] exemplifies the partition among the linguistic terms “young” and “old” in 1st, 2nd, 3rd and 4th quadrant respectively. The transition and partition among the linguistic terms “young” and “old” is graphically shown in the below figure;

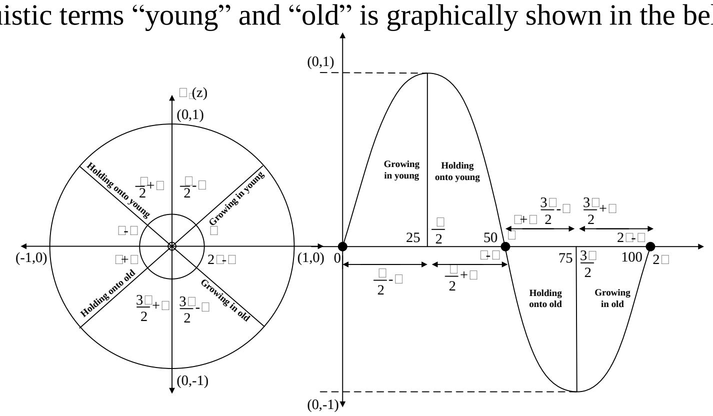


Figure 3: New Rotational Fuzzy Set Model

The partition between the linguistic terms "young" and "old" further classified into four major categories which is shown in the below table;

Table 2: Fuzzy Predicate with Partitions

Fuzzy Predicate	Fuzzy Partition
Growing in young	Absolutely young Very young More or less young Possibly young
Holding onto young	Possibly not young More or less not young Not very young Absolutely not young
Holding onto old	Absolutely not old Not very old More or less not old Possibly not old
Growing in old	Possibly old More or less old Very old Absolutely old

The fuzzy partition of the linguistic category such as, growing in young, holding onto young, holding onto old and growing in old has been geometrically illustrated in the below figure.

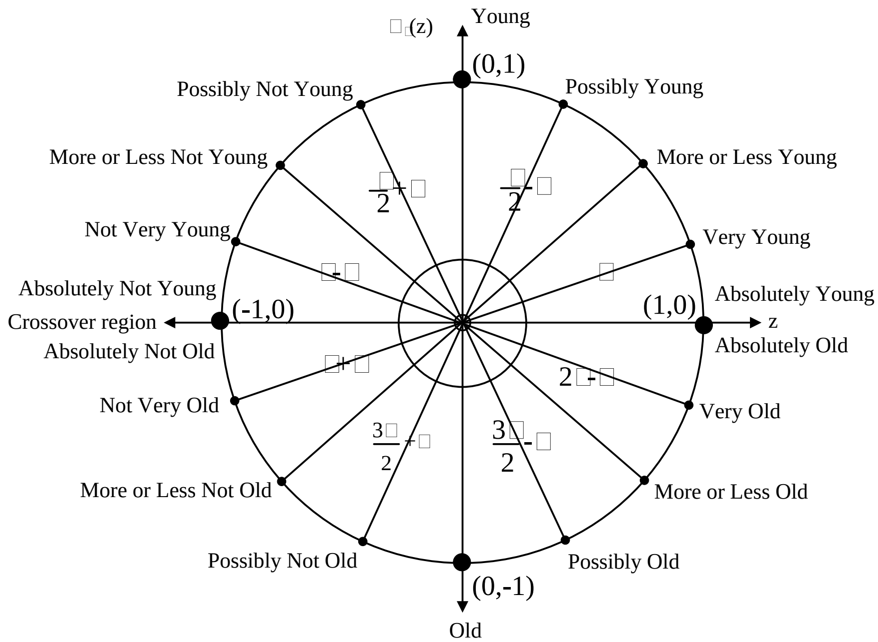


Figure 4: Trans-rotational fuzzy model for truth values

The table discusses the angular partition among the linguistic categories.

Table 3: Angular Partition of Linguistic Categories

Fuzzy Predicate	Angle	Fuzzy Predicate	Angle
Absolutely young	$\theta = 0$	Absolutely not old	$\theta = \pi$
Very young	$\theta = \pi/8$	Not very old	$\theta = 9\pi/8$
More or less young	$\theta = \pi/4$	More or less not old	$\theta = 5\pi/4$
Possibly young	$\theta = 3\pi/8$	Possibly not old	$\theta = 11\pi/8$
Young	$\theta = \pi/2$	Old	$\theta = 3\pi/2$
Possibly not young	$\theta = 5\pi/8$	Possibly old	$\theta = 13\pi/8$
More or less not young	$\theta = 3\pi/4$	More or less old	$\theta = 7\pi/4$
Not very young	$\theta = 7\pi/8$	Very old	$\theta = 15\pi/8$
Absolutely not young	$\theta = \pi$	Absolutely old	$\theta = 2\pi$

3.1 Rotational Fuzzy Membership Function

Membership function of the new rotational fuzzy set model is defined as follows,

$$\underset{\sim}{A} = \left\{ \left(\theta, \phi_{\underset{\sim}{P_i}}(z) \right) \mid z \in S_1 \right\}$$

and $\phi_{\underset{\sim}{P_i}}(z)$ is defined as,

$$\phi_{\underset{\sim}{P_i}}(z) = z \cos\left(\theta^*\right), \; i \in 1, 2, \ldots n$$

where, z represents the fuzzy element in an unit circle S_1 (Universe of Discourse), θ^* represents the angle partitions of the linguistic values and $\phi_{\underset{\sim}{P_i}}(z)$ represents the membership functions of various fuzzy proposition. The range of cosine function varies from 0 to 360^0. Since the universe of discourse is unit circle, the angle partition has been made for the range varies from 0 to 360^0.

4. THEORETICAL CONCEPTS AND PROPERTIES OF NEW ROTATIONAL FUZZY SET MODEL

4.1 Rating Space for New Rotational Fuzzy Set Model

Rating space for the new rotational fuzzy set model consists of continuous functions with parameter r that satisfies the following equations;

$$R\left(r, x\right) = rx$$

with $0 \le x \le 1$ and $-1 \le r \le 1$. Here x is the domain of the rating space and r is a rating value. The following table provides the various rating space values for the typical illustration which we have discussed in the above section.

Table 4: Linguistic Partition with Rating Values

Fuzzy Predicate	Rating value	Fuzzy Predicate	Rating value
Absolutely young	(1,0)	Absolutely not old	(-1,0)
Very young	(0.9238,0.3826)	Not very old	(-0.9238,-0.3826)
More or less young	(0.7071,0.7071)	More or less not old	(-0.7071,-0.7071)
Possibly young	(0.3826,0.9238)	Possibly not old	(-0.3826,-0.9238)
Young	(0,1)	Old	(0,-1)
Possibly not young	(-0.3826,0.9238)	Possibly old	(0.3826,-0.9238)
More or less not young	(-0.7071,0.7071)	More or less old	(0.7071,-0.7071)
Not very young	(-0.9238,0.3826)	Very old	(0.9238,-0.3826)
Absolutely not young	(-1,0)	Absolutely old	(1,0)

4.2 Alpha Cut Representation of Rotational Fuzzy Set Model.

The one – one correspondence between the trigonometric sine function and the unit circle is discussed with the help of illustration in[14]. This section introduces the alpha-cut representation of the new rotational fuzzy set model based on the angular representation. The new rotational fuzzy set model has been partitioned into four major dimensions, whereas in unit circle it has been viewed as four different quadrants. And it has been observed that each quadrant (dimension) has angle variation from $0^0 \le z \le 90^0$, $90^0 \le z \le 180^0$, $180^0 \le z \le 270^0$ and $270^0 \le z \le 360^0$ respectively. For instance, the figure given below contains the representation of $\alpha - cut$ in the sine curve and its correspondence in the unit circle.

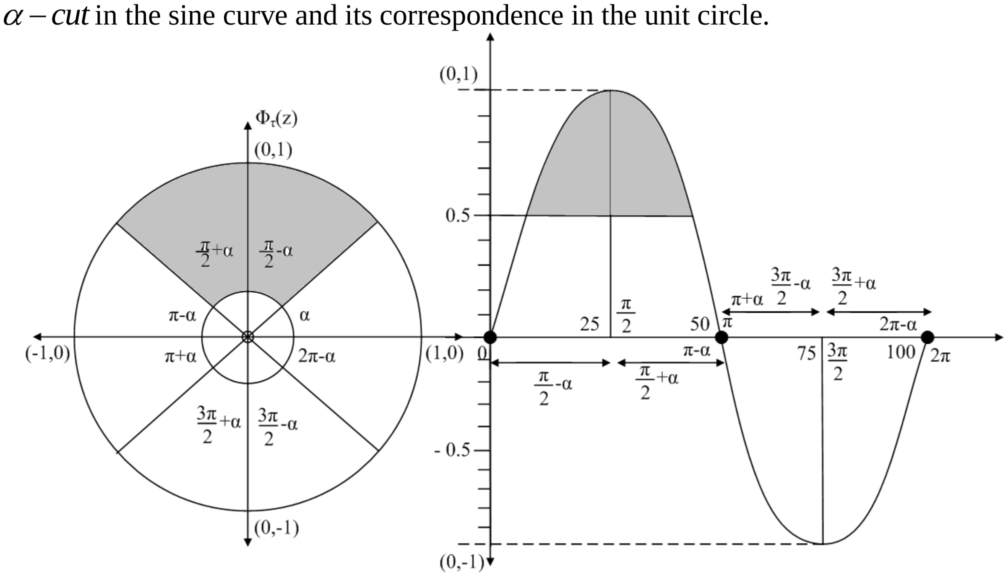


Figure 5: Alpha-cut Representation on Linguistic Variable 'Young'

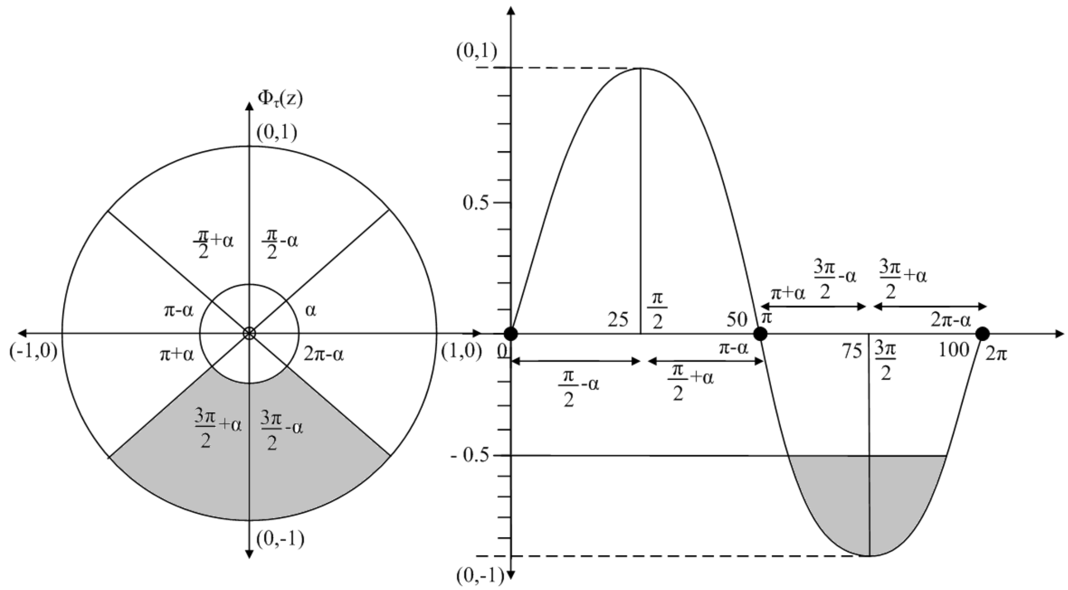


Figure 6: Alpha-cut Representation on Linguistic Variable 'Old'

5. OPERATIONS ON ROTATIONAL FUZZY SET MODEL

5.1 Fuzzy Union

Fuzzy union of the new rotational fuzzy set model aggregates two or more opinions collected over an issue in a certain time and provides the best possible solution. Fuzzy membership function defined in section 3 is employed to study the subjective opinion of the Experts.

Let $\phi_{P_1}(z)$ and $\phi_{P_2}(z)$ be two fuzzy membership function which is defined as follow,

$\phi_{P_1}(z) = z\cos\ \theta_1$ and $\phi_{P_2}(z) = z\cos\ \theta_2$

Then fuzzy union for the new rotational fuzzy set model has been defined by,

$$\phi_{P_1}(z) \cup \phi_{P_2}(z) = z\cos\ \theta_1 \ \cup z\cos\ \theta_2$$

And also the fuzzy union for the new rotational fuzzy set has been calculated by the following formula;

$$\arccos\left[z\cos\ \theta_1\ , z\cos\ \theta_2\ \right].$$

The following figure gives the geometrical illustration of fuzzy union of the new rotational fuzzy set model with an example.

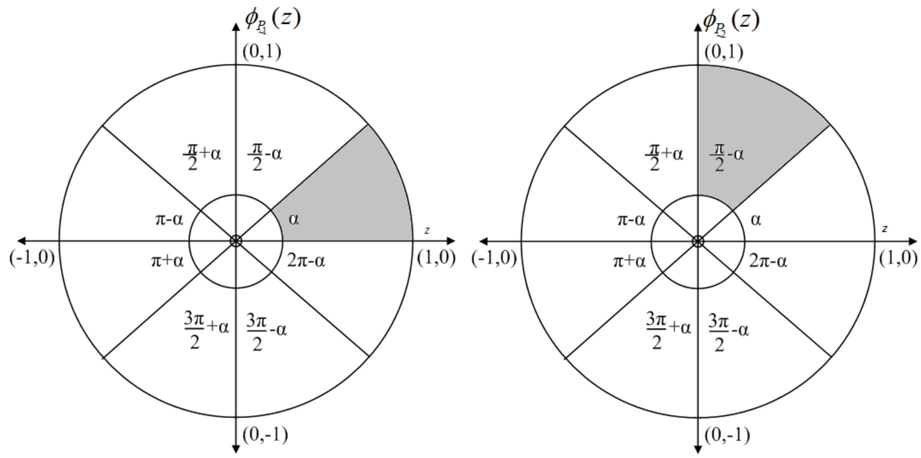


Figure 7: Rotational Fuzzy Set Model

In the above figure, unit circle with gradations from 0^0 to 360^0 acts as the universe of discourse with each quadrant signifies the transition among the linguistic variable. For instance, 'young' is the linguistic variable with transitions from 'growing in young' to 'holding onto old'. Whereas as the membership function in the 1st quadrant characterizes the category 'growing in young', 2nd quadrant characterizes the category 'holding onto young', 3rd quadrant characterizes the category 'growing in old' and 4th quadrant characterizes the category 'holding onto old'. The two unit circle represents the two different opinion about an age of a person. The unit circle (a) represents the angular value of linguistic partition 'more or less young' with the membership grade ranges from $\theta = 0^0$ and $\theta = \pi/4$ and the unit circle (b) represents the angular value of linguistic partition 'possibly young' with the membership grade ranges from $\theta = \pi/4$ and $\theta = \pi/2$. The unit circle (c) represents the fuzzy union of the above two linguistic opinion and the membership grade ranges from $\theta = 0^0$ and $\theta = \pi/2$. Let $\phi_{P_1}(z)$ and $\phi_{P_2}(z)$ be two fuzzy membership function which describes the above situation is defined as, $\phi_{P_1}(z) = z\cos\ \theta_1$ and $\phi_{P_2}(z) = z\cos\ \theta_2$ with fuzzy union is calculated as follows;

$$\arccos\left[z\cos\ \theta_1\ \cup z\cos\ \varphi_1\ \right] = \arccos\left[1\cos\ \pi/8\ \cup 1\cos\ \pi/8\ \right]$$

$$= \arccos\ 0.8534\ = 31.42^0$$

$$\arccos\left[z\cos\ \theta_2\ \cup z\cos\ \varphi_2\ \right] = \arccos\left[1\cos\ 3\pi/8\ \cup 1\cos\ 3\pi/8\ \right]$$

$$= \arccos\ 0.1463\ = 81.58^0$$

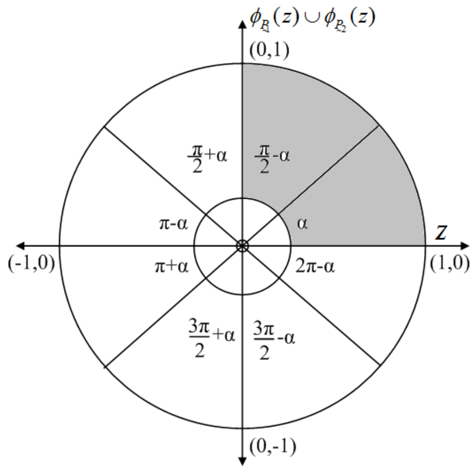


Figure 8: Fuzzy Union of Rotational Fuzzy Set Model

5.2 Fuzzy Intersection

Let $\phi_{P_1}(z)$ and $\phi_{P_2}(z)$ be two fuzzy membership function which is defined as follow,

$$\phi_{P_1}(z) = z\cos\ \theta_1 \ \text{ and } \phi_{P_2}(z) = z\cos\ \theta_2$$

Then fuzzy intersection for the new rotational fuzzy set model has been defined by,

$$\phi_{P_1}(z) \cap \phi_{P_2}(z) = z\cos\ \theta_1\ \cap z\cos\ \theta_2$$

And also the fuzzy union for the new rotational fuzzy set has been calculated by the following formula;

$$\arccos\left[z\cos\ \theta_1\ , z\cos\ \theta_2\ \right].$$

The following figure gives the geometrical illustration of fuzzy union of the new rotational fuzzy set model.

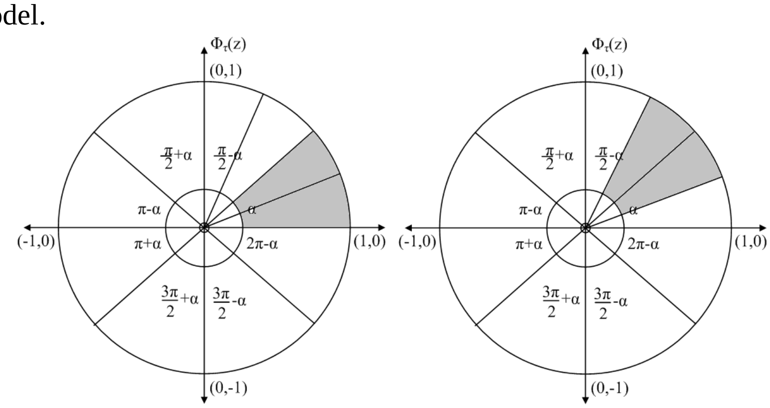


Figure 9: Rotational Fuzzy Set Model

In the above figure, unit circle with gradations from 0^0 to 360^0 acts as the universe of discourse with each quadrant signifies the transition among the linguistic variable. The unit circle (a) represents the angular value of linguistic partition 'more or less young' with the membership grade ranges from $\theta = 0^0$ and $\theta = \pi/4$ and the unit circle (b) represents the angular value of linguistic partition 'possibly young' with the membership grade ranges from $\theta = \pi/4$ and $\theta = \pi/2$. The unit circle (c) represents the fuzzy intersection of the above two linguistic opinion and the membership grade ranges from $\theta = 0^0$ and $\theta = \pi/2$. Let $\phi_{\underset{\sim}{P}_1}(z)$ and $\phi_{\underset{\sim}{P}_2}(z)$ be two fuzzy membership function which describes the above situation is defined as, $\phi_{\underset{\sim}{P}_1}(z) = z\cos\ \theta_1$ and $\phi_{\underset{\sim}{P}_2}(z) = z\cos\ \theta_2$ with fuzzy intersection is calculated as follows;

$$\arccos\left[z\cos\ \theta_1 \ \cap z\cos\ \varphi_1\ \right] = \arccos\left[1\cos\ \pi/8\ \cap 1\cos\ \pi/8\ \right]$$

$$= \arccos\ 0.8534\ = 31.42^0$$

$$\arccos\left[z\cos\ \theta_2 \ \cap z\cos\ \varphi_2\ \right] = \arccos\left[1\cos\ 3\pi/8\ \cap 1\cos\ 3\pi/8\ \right]$$

$$= \arccos\ 0.1463\ = 81.58^0$$

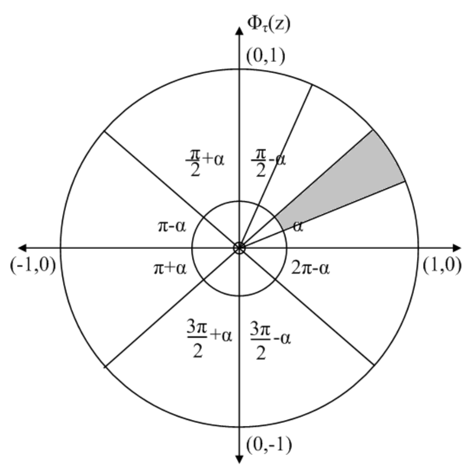


Figure 10: Fuzzy Intersection of Rotational Fuzzy Set Model

5.3 Fuzzy Mean of New Rotational Fuzzy Set Model

Let $\phi_{\underset{\sim}{P}_1}(z)$ and $\phi_{\underset{\sim}{P}_2}(z)$ be two fuzzy membership function which is defined as follow,

$$\phi_{\underset{\sim}{P}_1}(z) = z\cos\ \theta_1 \ \text{ and } \phi_{\underset{\sim}{P}_2}(z) = z\cos\ \theta_2$$

Then fuzzy intersection for the new rotational fuzzy set model has been defined by,

$$\phi_{P_1}(z) \sim \phi_{P_2}(z) = z\cos\ \theta_1\ \sim z\cos\ \theta_2$$

And also the fuzzy union for the new rotational fuzzy set has been calculated by the following formula;

$$\arccos\left[z\cos\ \theta_1\ ,z\cos\ \varphi_1\ \right]+\arccos\left[z\cos\ \theta_1\ ,z\cos\ \varphi_2\ \right].$$

The fuzzy weighted mean value for the example given in the section 5.1 is calculated by using the new rotational fuzzy set model is given as follows;

$\arccos\left[z\cos\ \theta_1\ ,z\cos\ \varphi_1\ \right]+\arccos\left[z\cos\ \theta_1\ ,z\cos\ \varphi_2\ \right]=56.5^0$.

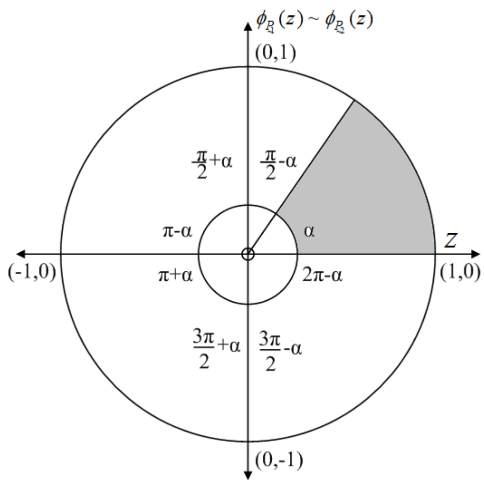


Figure 11: Fuzzy Weighted Mean of Rotational Fuzzy Set Model

6. Conclusion

Through this paper, we have defined various properties related to the new rotational fuzzy set model along with the geometrical illustration. Also we have introduced various arithmetic operations such as fuzzy union, fuzzy intersection and fuzzy weighted mean for the new rotational fuzzy set model. Arithmetic operations has been discussed with the help of graphical illustration.

7. ACKNOWLEDGMENTS

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