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Fixed Point Theorems in Multiplicative Cone b-Metric Spaces

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ABSTRACT

In this paper we prove a general fixed point theorem that generalizes various results present in Multiplicative cone b-metric space. Also discuss the unique common fixed point of two pairs of weak commutative mappings on this space.

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INTRODUCTION

Fixed point theory is one of the most fruitful and effective tools in mathematics which has enormous applications within as well as outside the mathematics. It is well known that R_+ is not complete according to the usual metric. To overcome this problem, in 2008, Bashirov *et al*.[1] introduced the notion of multiplicative metric spaces and studied the concept of multiplicative calculus and proved the elementary theorem of multiplicative calculus. In Özavşar and Çevikel[8] investigate multiplicative metric spaces by remarking its topological properties. The idea of b-metric presented by Bakkhtin[6] as a generalized form of metric. Czerwik[12] provided the some fixed point theorems in this space. Hussain and Shah[10] introduced cone b-metric spaces. Zada, Shah and Saifullah[3] introduced the concept of multiplicative cone b-metric and proved some theorems in this space. In this paper, we prove a general fixed point theorem that generalizes various results present in this space. We also obtain unique fixed point of weak commutative mappings on the complete multiplicative cone b- metric space.

PRELIMINARIES

Definition 2.1[1]. Let X be a nonempty set. Multiplicative metric is a mapping $d : X \times X \rightarrow R^+$ satisfying the following conditions:
(i) $d(x, y) \geq 1$ for all $x,y \in X$ and $d(x,y) = 1$ if and only if $x = y$,
(ii) $d(x, y) = d(y, x)$ for all $x,y \in X$,
(iii) $d(x, y) \leq d(x, z) . d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality). Then (X,d) is called a cone multiplicative metric space.

Definition 2.2 [3]. Let $X \neq \phi$ and let $s \geq 1$ be a given real number. A function $d: X \times X \rightarrow E$ is called a multiplicative cone b-metric space, provided that , for all $x,y,z \in X$,

(1) $d(x,y) \geq 1$ for all $x,y \in X$
(2) $d(x,y) = 1$ if and only if $x=y$
(3) $d(x,y)=d(y,x)$ for all $x,y \in X$
(4) $d(x,y) \leq [d(x,z). d(z,y)]^s$ for all $x,y,z \in X$.

Example 2.3[3]. Let $d(x,y) = a^{(\sum_{i=1}^{\infty} |x_i - y_i|^p)^{\frac{1}{p}}}$ for all $x,y \in X$ $p>1$. Then d(x,y) is multiplicative cone b-metric space but not multiplicative cone metric space.

Definition 2.4. Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. if for every multiplicative open ball $B_e(x)$, there exists a natural number N such that $n \geq N$ implies $x_n \in B_e(x)$, then the sequence $\{x_n\}$ is said to be multiplicative convergent to x .

Theorem 2.5[3]. Let (X,d) be a complete multiplicative cone b- metric space with power $s \geq 1$. Suppose the mapping $T:X \rightarrow X$ satisfies the following contractive condition,
$d(Ta,Tb) \leq d(a, b)^{\lambda}$ for all $a,b \in X$,where $0 \leq \lambda < 1$is a constant. Then T has a unique fixed point in X.

Theorem 2.6[3]. Let (X,d) be a complete multiplicative cone b- metric space with power $s \geq 1$. Suppose the mapping $T:X \rightarrow X$ satisfies the following Kannan contractive condition,
$d(Ta,Tb) \leq (d(Ta,a).d(Tb, b))^{\lambda}$ for all $a,b \in X$, where $0 \leq \lambda < ½$ is a constant. Then T has a unique fixed point in X.

Theorem 2.7[3]. Let (X,d) be a complete multiplicative cone b- metric space with power $s \geq 1$. Suppose the mapping $T:X \rightarrow X$ satisfies the following contractive condition,
$d(Ta,Tb) \leq (d(Ta,b). d(a,T b))^{\lambda}$ for all $a,b \in X$, where $0 \leq \lambda < ½$ is a constant. Then T has a unique fixed point in X.

Theorem 2.8[3]**.** Let (X,d) be a complete multiplicative cone b- metric space with power $s \geq 1$. Suppose the mapping $T:X \to X$ satisfies the following contractive condition,

$d(Ta,Tb) \leq d(a,Ta)^p . d(b,Tb)^q . d(a,b)^r$ for all $a,b \in X$, where p,q,r are non- negative real numbers and satisfy $p+(q+r)^s < 1$. Then T has a unique fixed point in X.

MAIN RESULTS

Theorem 3.1. Let (X,d) be a complete multiplicative cone b-metric space with $s \geq 1$. Suppose the mapping $f: X \to X$ holds the following condition

$$d(fx,fy) \leq d(x,y)^{\lambda_1} d(x,fx)^{\lambda_2} d(y,fy)^{\lambda_3} d(x,fy)^{\lambda_4} d(y,fx)^{\lambda_5} \qquad (*)$$

For all $x,y \in X$, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$ and $\lambda_1 + \lambda_2 + \lambda_3 + s(\lambda_4 + \lambda_5) < 1$

Then f has a unique fixed point in X.

Proof. Let $x_0 \in X$, define $f(x_0) = x_1$, $f(x_1) = x_2 \ldots\ldots$, $f(x_{n-1}) = x_n \ldots\ldots$

From (*), we have

$d(x_n,x_{n+1}) = d(f(x_{n-1}),f(x_n)) \leq d(x_{n-1},x_n)^{\lambda_1} d(x_{n-1},fx_{n-1})^{\lambda_2} d(x_n,fx_n)^{\lambda_3} d(x_{n-1},fx_n)^{\lambda_4} d(x_n,fx_{n-1})^{\lambda_5}$

$= d(x_{n-1},x_n)^{\lambda_1} d(x_{n-1},x_n)^{\lambda_2} d(x_n,x_{n+1})^{\lambda_3} d(x_{n-1},x_{n+1})^{\lambda_4} d(x_n,x_n)^{\lambda_5}$

$\leq d(x_{n-1},x_n)^{\lambda_1} d(x_{n-1},x_n)^{\lambda_2} d(x_n,x_{n+1})^{\lambda_3} [d(x_{n-1},x_n).d(x_n.x_{n+1})]^{s\lambda_4}$

$$= d(x_{n-1},x_n)^{\lambda_1 + \lambda_2 + s\lambda_4} d(x_n,x_{n+1})^{\lambda_3 + s\lambda_4} \qquad (**)$$

$d(x_{n+1},x_n) = d(f(x_n),f(x_{n-1})) \leq d(x_n,x_{n-1})^{\lambda_1} d(x_n,fx_n)^{\lambda_2} d(x_{n-1},fx_{n-1})^{\lambda_3} d(x_n,fx_{n-1})^{\lambda_4} d(x_{n-1},fx_n)^{\lambda_5}$

$= d(x_n,x_{n-1})^{\lambda_1} d(x_n,x_{n+1})^{\lambda_2} d(x_{n-1},x_n)^{\lambda_3} d(x_n,x_n)^{\lambda_4} d(x_{n-1},x_{n+1})^{\lambda_5}$

$\leq d(x_n,x_{n-1})^{\lambda_1} d(x_n,x_{n+1})^{\lambda_2} d(x_{n-1},x_n)^{\lambda_3} [d(x_{n-1},x_n).d(x_n.x_{n+1})]^{s\lambda_5}$

$$= d(x_{n-1},x_n)^{\lambda_1 + \lambda_3 + s\lambda_5} d(x_n,x_{n+1})^{\lambda_2 + s\lambda_5} \qquad (***)$$

From (**) and (***), we get

$d(x_{n+1},x_n) \leq d(x_n,x_{n-1})^h$, where $h = \dfrac{2\lambda_1 + \lambda_2 + \lambda_3 + s(\lambda_4 + \lambda_5)}{2 - (\lambda_2 + \lambda_3 + s(\lambda_4 + \lambda_5))}$

$h < 1$. Similarly, $d(x_n,x_{n-1}) \leq d(x_{n-1},x_{n-2})^h$

$d(x_{n+1},x_n) \leq d(x_{n-1},x_{n-2})^{h^2} \ldots\ldots$

continue like this , we have

$d(x_n,x_m) \leq d^s(x_n,x_{n-1}).\, d^{s^2}(x_{n-1},x_{n-2}).\, d^{s^3}(x_{n-2},x_{n-3}) \ldots\ldots\, d^{s^{n-m}}(x_{m-1},x_m)$

$\leq d^{sh^{n-1}}(x_1,x_0).\, d^{s^2h^{n-2}}(x_1,x_0) \ldots\ldots\ldots\ldots\ldots\, d^{s^{n-m}h^m}(x_1,x_0)$

$\leq d(x_1,x_0)^{\frac{(sh)^m}{1-sh}}$. this implies $d(x_n,x_m) \to 1$ as $n,m \to \infty$.

Hence$\{x_n\}$ is a Cauchy sequence. By the multiplicative completeness of X, there is $z \in X$ such that $x_n \to z$ as $n \to \infty$.

Now we prove that z is the fixed point of f . We have

$d(f(z),z) \leq [d(f(z),x_{n+1})\, d(x_{n+1},z)]^s \leq [d(f(z), f(x_n))]^s.[\, d(x_{n+1},z)]^s$

$\leq [d(z,x_n)^{\lambda_1}\, d(z,fz)^{\lambda_2}\, d(x_n,fx_n)^{\lambda_3}\, d(z,fx_n)^{\lambda_4}\, d(x_n,fz)^{\lambda_5}]^s . [d(x_{n+1},z)]^s$

$\leq d(f(z),z)^{s(\lambda_2+\lambda_3+\lambda_4+\lambda_5)}$.

Gives f(z)=z i.e., z is a fixed point of f.

Uniqueness. Suppose z,w ($z \neq w$) be two fixed point of f, then from (*), we have

$d(z,w)=d(f(z),f(w)) \leq d(z,w)^{\lambda_1}\, d(z,fz)^{\lambda_2}\, d(w,fw)^{\lambda_3}\, d(z,fw)^{\lambda_4}\, d(w,fz)^{\lambda_5}$

$\leq d(z,w)^{\lambda_1+\lambda_4+\lambda_5}$

This implies d(z,w)=1 i.e., z=w

Hence f has a unique fixed point.

Coollary 3.2. Putting $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$gives 2.5

Coollary 3.3. Putting$\lambda_1 = \lambda_4 = \lambda_5 = 0$and $\lambda_2 = \lambda_3$gives 2.6

Coollary 3.4. Putting $\lambda_1 = \lambda_2 = \lambda_3 = 0$and $\lambda_4 = \lambda_5$gives 2.7

Coollary 3.5. Putting $\lambda_4 = \lambda_5 = 0$gives 2.8

Definition 3.6. Let A and S be self-mappings on multiplicative cone b-metric space (X,d). then A and S are said to be weakly compatible if they commute at their coincident point; that is, Ax=Sx for some xin X implies ASx=SAx.

Theorem 3.7. Let (X,d) be a complete multiplicative cone b-metric space with $s \geq 1$. Suppose the mappings A,B,S and T are four self mappings of X satisfying the following conditions:

1 T(X) $\subseteq$ A(X) and S(X) $\subseteq$ B(X)

2 Suppose there exists $\lambda \in [0,1)$ such that

$d(Sx,Ty) \leq [\max\{d(Ax,By),d(Ax,Sx),d(By,Ty),d(Ax,Ty),d(By,Sx)\}]^{\lambda}$

3 the pairs (S,A) and (T,B) are weakly compatible.

4One of S,T,A and B is continuous,

Then A,B,S and T have a unique common fixed point.

Proof. Since SX $\subseteq$ BX, consider the point $x_0 \in X$, there exists $x_1 \in X$ such that $Sx_0 = Bx_1 = y_0$;

$\exists\ x_2 \in X$ such that $Tx_1 = Ax_2 = y_1$;………;

$\exists\ x_{2n+1} \in X$ such that $Sx_{2n} = Bx_{2n+1} = y_{2n}$;

$\exists\ x_{2n+2} \in X$ such that $T x_{2n+1} = Ax_{2n+2} = y_{2n+1}$;………

Now we can define a sequence $\{ y_n \}$ in X, we have

$d(y_{2n} ,y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1})$

$\leq [\max\{d(A x_{2n},B x_{2n+1}),d(A x_{2n},S x_{2n}),d(B x_{2n+1},T x_{2n+1}),d(A x_{2n},T x_{2n+1}),d(B x_{2n+1}, S x_{2n})\}]$ λ

$= [\max\{d(y_{2n-1}, y_{2n}),d(y_{2n-1}, y_{2n}),d(y_{2n}, y_{2n+1}),d(y_{2n-1}, y_{2n+1}),d(y_{2n}, y_{2n})\}]^{\lambda}$

$\leq [\max\{d(y_{2n-1}, y_{2n}),d(y_{2n}, y_{2n+1}),d(y_{2n-1}, y_{2n+1}),1\}]^{\lambda}$

$\leq [\max\{d(y_{2n-1}, y_{2n}),d(y_{2n}, y_{2n+1}),d(y_{2n-1}, y_{2n})^{s} . d(y_{2n}, y_{2n+1})^{s} , 1\}]^{\lambda}$

$\leq d(y_{2n-1}, y_{2n})^{s\lambda} . d(y_{2n}, y_{2n+1})^{s\lambda}$

This implies that $d(y_{2n} ,y_{2n+1}) \leq d(y_{2n-1}, y_{2n})]^{s\lambda/1-s\lambda}$.

Let $\frac{s\lambda}{1-s\lambda} = h$, then

$d(y_{2n},y_{2n+1})\leq d^{h}(y_{2n-1},y_{2n})$. (1)

We also obtain $d(y_{2n+1}, y_{2n+2}) \leq d^{h}(y_{2n}, y_{2n+1})$ (2)

From (1) and (2), we know $d(y_n, y_{n+1})\leq d^{h}(y_{n-1}, y_n)\leq\ldots\ldots\ldots\leq d^{h^n}(y_1,y_0)$, $\forall n\geq 2$. Let $m,n \in N$ such that $m\geq n$, then we get

$d(y_m,y_n)\leq d(y_m,y_{m-1})^{s}.d(y_{m-1}, y_n)^{s}\leq d^{s}(y_m,y_{m-1}).\ d^{s^2}(y_{m-1},y_{m-2}).d^{s^2}(y_{m-2},y_{m-3})$

$\leq\ldots\ldots\ldots\ldots\leq d^{\frac{(sh)^n}{1-sh}}(y_1,y_0)$.

This implies that $d(y_m,y_n)\to 1$ as $m,n \to\infty$. Hence $\{y_n\}$ is a multiplicative Cauchy sequence. By the completeness of X, there exists $z\in X$ such that $y_n\to z$ as $n\to\infty$.

Moreover , $\{Sx_{2n}\} = \{Bx_{2n+1}\} = \{y_{2n}\}$

And $\{Tx_{2n+1}\} =\{Ax_{2n+2}\} = \{y_{2n+1}\}$ are subsequence of $\{y_n\}$, so we obtain

$\lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Bx_{2n+1}= \lim_{n\to\infty} Tx_{2n+1}=\lim_{n\to\infty} Ax_{2n+2}=z$.

Case 1. Suppose that A is continuous, then $\lim_{n\to\infty} ASx_{2n}=\lim_{n\to\infty} A^2x_{2n} = Az$. Since A and S are weak commutative mappings, then

$d(ASx_{2n}, Sax_{2n})\leq d(Sx_{2n},Ax_{2n})$.

Let $n\to\infty$, we get $\lim_{n\to\infty} d(SAx_{2n},Az)\leq d(z,z) = 1$, i.e., $\lim_{n\to\infty} SAx_{2n}= Az$,

$d(SAx_{2n}, Tx_{2n+1})\leq\{\max\{d(A^2x_{2n}, Bx_{2n+1}), d(A^2x_{2n}, SAx_{2n}),d(Bx_{2n+1},Tx_{2n+1}),d(SAx_{2n}, Bx_{2n+1}),d(A^2x_{2n}, Tx_{2n+1})\}\}^{\lambda}$.

Let $n\to\infty$, we get $d(Az,z)\leq\{\max\{d(Az,z), d(Az,Az),d(z,z),d(z,z),d(Az,z),(Az,z)\}\}^{\lambda}$

$=\{\max\{d(Az,z),1\}\}^{\lambda}=d^{\lambda}(Az,z)$.

This implies $d(Az,z) =1$, i.e., $Az= z$,

$d(Sz,Tx_{2n+1})\leq\{\max\{d(Az,Bx_{2n+1}), d(Az,Sz),d(Bx_{2n+1}, Tx_{2n+1}),d(Sz,Bx_{2n+1}), d(Az,Tx_{2n+1})\}\}^{\lambda}$.

Let $n\to\infty$, we have

$d(Sz,z)\leq\{\max\{d(Az,z),d(z,Sz),d(z,z),d(Sz,z),d(z,z)\}\}^{\lambda}$

$=\{\max\{d(Sz,z),1\}\}^{\lambda}= d^{\lambda}(Sz,z)$,which implies $d(Sz,z)=1$, i.e., $Sz =z$,

$z=Sz \in SX \subseteq BX$, so there exists $z^{*}\in X$ such that $z= Bz^{*}$,

$d(z,Tz^{*}) = d(Sz, Tz^{*})$

$\leq\{\max\{d(Az,Bz^{*}),d(Az,Sz),d(Bz^{*},Tz^{*})d(Sz,Bz^{*}),d(Az,Tz^{*})\}\}^{\lambda}$

$=\{\max\{d(z,Tz^{*}),1\}\}^{\lambda}=d^{\lambda}(z,Tz^{*})$,which implies $d(Tz^{*},z) =1$ i.e., $Tz^{*}= z$.

Since B and T are weak commutative , then $d(Bz,Tz) = d(BTz^{*},TBz^{*})\leq d(Bz^{*}, Tz^{*}) = d(z,z) =1$, so $Bz =Tz$, $d(z,Tz) = d(Sz,Tz)$

$\leq \{\max\{d(Az,Bz),d(Az,Sz),d(Bz,Tz),d(Sz,Bz),d(Az,Tz)\}\}^{\lambda}$
$= \{\max\{d(z,Tz),1\}\}^{\lambda}$
$= d^{\lambda}(z,Tz)$, which implies $d(Tz,z) = 1$, i.e., $Tz=z$.
Case 2. Suppose that B is continuous, we can obtain the same result by the way of Case 1.
Case 3. Suppose that S is continuous, then $\lim_{n\to\infty} SAx_{2n} = \lim_{n\to\infty} S^2x_{2n} = Sz$.
Since A and S are weak commutative, then $d(ASx_{2n}, Sax_{2n}) \leq d(Sx_{2n},Ax_{2n})$.
Let $n\to\infty$, we get
$\lim_{n\to\infty} d(ASx_{2n},Sz) \leq d(z,z) = 1$, $\lim_{n\to\infty} ASx_{2n} = Sz$, $d(S^2x_{2n}, Tx_{2n+1})$
$\leq \{\max \{d(ASx_{2n}, Bx_{2n+1}),d(ASx_{2n}, S^2x_{2n}),d(Bx_{2n+1}, Tx_{2n+1}),d(S^2x_{2n},Bx_{2n+1}), d(ASx_{2n},Tx_{2n+1})\}\}^{\lambda}$.
Let $n\to\infty$, we have
$d(Sz,z) \leq \{\max\{d(Sz,z),d(Sz,Sz),d(z,z),d(Sz,z),d(Sz,z)\}\}^{\lambda}$
$= \{\max\{d(Sz,z),1\}\}^{\lambda} = d^{\lambda}(Sz,z)$, which implies $d(Sz,z)=1$, i.e., $Sz = z$,
$z=Sz \in SX \subseteq BX$, so there exists $z^*\in X$ such that $z= Bz^*$,
$d(S^2 x_{2n},Tz^*) = d(Sz, Tz^*)$
$\leq \{\max\{d(ASx_{2n} ,Bz^*), d(ASx_{2n},S^2x_{2n}),d(Bz^*,Tz^*)d(S^2x_{2n} ,Bz^*),d(ASx_{2n} ,Tz^*)\}\}^{\lambda}$
Let $n\to\infty$, we get
$d(Sz,Tz^*) \leq \{\max\{d(Sz,z),d(Sz,Sz),d(z,Tz^*),d(Sz,z),d(Sz,Tz^*)\}\}^{\lambda}$
$d(z,Tz^*) = \{\max\{d(z,Tz^*),1\}\}^{\lambda} = d^{\lambda}(z,Tz^*)$, which implies $d(z, Tz^*) = 1$ i.e., $Tz^* = z$.
Since T and B are weak commutative, then
$d(Tz,Bz) = d(TBz^*,BTz^*) \leq d(Tz^*,Bz^*) = d(z,z) = 1$, so $Bz = Tz$,
$d(Sx_{2n}, Tz)$
$\leq \{\max\{d(Ax_{2n}, Bz), d(Ax_{2n}, Sx_{2n}),d(Bz,Tz),d(Sx_{2n}, Bz),d(Ax_{2n}, Tz0\}\}^{\lambda}$.
Let $n\to\infty$, we get $d(z,Tz) \leq \{\max\{d(z,Tz), d(z,z),d(Tz,Tz),d(z,Tz),d(z,Tz)\}\}^{\lambda}$
$= \{\max\{d(z,Tz),1\}\}^{\lambda} = d^{\lambda}(z,Tz)$.
This implies $d(z,Tz) = 1$, i.e., $Tz= z$,
$z=Tz \in TX \subseteq AX$, so there exists $z^{**}\in X$ such that $z= Az^{**}$,
$d(Sz^{**},z) = d(Sz^{**}, Tz)$
$\leq \{\max\{d(Az^{**},Bz),d(Az^{**},Sz^{**}),d(Bz,Tz)d(Sz^{**},Bz),d(Az^{**},Tz)\}\}^{\lambda}$
$= \{\max\{d(z,z),d(z,Sz^{**}),d(Bz,Bz)d(Sz^{**},z),d(z,z)\}\}^{\lambda}$
$= \{\max\{d(Sz^{**},z),1\}\}^{\lambda} = d^{\lambda}(Sz^{**},z)$, which implies $d(Sz^{**},z) = 1$ i.e., $Sz^{**} = z$.
Since A and S are weak commutative, then $d(Az,Sz) = d(ASz^{**}, SAz^{**}) \leq d(Sz^{**}, Az^{**}) = d(z,z) = 1$, so $Az=Sz$
We obtain $Sz = Tz = Az = Bz = z$, so z is a common fixed point of S,T,A and B.

Case 4. Suppose that T is continuous, we can obtain the same result by the way of Case 3.
Now, we prove that S, T, A and B have a unique common fixed point. Suppose that $w \in X$ is also a common fixed point of S, T, A and B, then
$D(z,w) = d(Sz,Tw)$
$\leq \{\max \{d(Az, Bw),d(Az, Sz),d(Bw, Bw),d(Sz, Bw),d(Az,Tw)\}\}^{\lambda}$.

$\leq\{\max\ \{d(z,w),1\}\}^{\lambda} = d^{\lambda}(z,w)$.
This implies d(z,w) = 1, i.e., z = w.
This is a contradiction. So S, T, A and B have a unique common fixed point.

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