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Effects of Viscous Dissipation on Natural Convection Flow Over a Sphere with Temperature Dependent Thermal Conductivity

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ABSTRACT

The effect of viscous dissipation on natural convection flow over an isothermal sphere in presence of temperature dependent thermal conductivity has been studied. In this paper, the thermal conductivity is assumed as a linear function of temperature. A set of non linear partial differential equations along with the boundary conditions are reduced to ordinary differential equations with appropriate corresponding conditions, which are solved numerically applying finite difference method together with Keller-box scheme. The obtained numerical results are presented for the velocity profiles, temperature distributions as well as skin friction coefficients and heat transfer coefficients.

Keywords: Natural convection, viscous dissipation, thermal conductivity.

INTRODUCTION

Natural convection takes place while the density difference occurred due to the temperature variations in the fluid. Natural convection has a great deal of attention to the researchers because of its presence both in nature and engineering applications. In addition the problem of natural convection flow over sphere has much interest to the scientists and researchers for their extensive applications. In engineering applications convection is commonly visualized in the formulation of microstructures during the cooling of molten metals and fluid flows around shrouded heat dissipation fins, solar ponds, petroleum reservoir, nuclear energy, fire engineering etc. A very common industrial application of natural convection is free air cooling without aid of fans. On the other hand, thermal conductivity is a measure of the ability of heat transfer. Considering, the importance of viscous dissipation and thermal conductivity a lot of research works have been accomplished by many researchers. For examples, Alam and Alim[1] investigated the viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. The effect of viscous dissipation on natural convection flow along a sphere with heat generation studied by Akter *et al.*[2]. Miraj *et al*.[3] discussed the conjugate effects of radiation and viscous dissipation on natural convection flow over a sphere with pressure work.

In the aforementioned studies, the thermal conductivity is mentioned as a constant quantity. This physical property may change with the change of temperature. Molla *et al*.[4] analyzed the effect of thermal conductivity on natural convection flow for isothermal sphere. The effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and Joule heating have been examined by Islam *et al.*[5]. Nasrin and Alim[6] have investigated the effects of viscous dissipation and temperature dependent thermal conductivity on magneto hydrodynamic (MHD) free convection flow with conduction and joule heating along a vertical flat plate. Gitima[7] presented analysis of the effect of variable viscosity and thermal conductivity in micro polar fluid for a porous channel in presence of magnetic field.

To the best of our knowledge effect of viscous dissipation on natural convection flow over a sphere in presence of temperature dependent thermal conductivity has not been studied yet. So, the present work demonstrates this issue. The non-dimensional transformed boundary layer equations which govern the flow are solved numerically by using finite difference method together with killer-box method. Numerical calculations were carried out for different values of the non dimensional quantities and then presented in figures.

NOMENCLATURE

a	Radius of the sphere
C_f	Skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
Gr	Grashof number
g	Acceleration due to gravity
T_w	Temperature at the surface
u, v	Dimensionless velocity components along x, y direction
X, Y	Axis in the direction along and normal to the surface respectively
ψ	Stream function
τ_w	Shearing stress
ξ	Dimensionless coordinate along the surface

k_f	Thermal conductivity of the fluid	η	Dimensionless coordinate normal to the surface
N	Viscous dissipation parameter	ρ	Density of the fluid
Nu	Local Nusselt number	μ	Viscosity of the fluid
Pr	Prandtl number	ν	Kinematic viscosity of the fluid
q_w	Heat flux at the surface	θ	Dimensionless temperature function
r	Radial distance from the symmetric axis to the surface	β	coefficient of thermal expansion
T	Temperature of the fluid in the boundary layer	β_0	Strength of magnetic field.
T_∞	Temperature of the ambient fluid	γ	Thermal conductivity variation parameter

FORMULATION OF THE PROBLEM

We consider a steady two-dimensional natural convection boundary layer flow of an electrically conducting and viscous incompressible fluid over a sphere of radius *a*. The surface temperature of the sphere is assumed as T_w and T_∞ being the ambient temperature of the fluid. When $T_w > T_\infty$ an upward flow is established along the surface due to free convection and the flow is downward for $T_w < T_\infty$. The mathematical model for the assumed physical problem is prescribed by the following conservation equation of mass, momentum and energy.

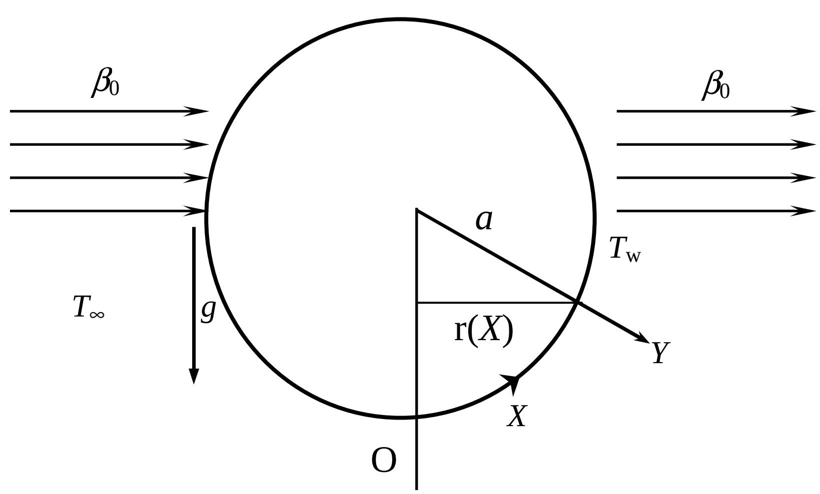

Fig. 1: Physical model and coordinate system

Under these considerations the governing equations are

$$\frac{\partial}{\partial X}(r\,U)+\frac{\partial}{\partial Y}(r\,V)=0 \qquad (1)$$

$$U\,\frac{\partial U}{\partial X}+V\,\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2}+g\,\beta\,(T\text{-}T_\infty)\sin\left(\frac{X}{a}\right) \qquad (2)$$

$$U\,\frac{\partial T}{\partial X}+V\,\frac{\partial T}{\partial Y}=\frac{1}{\rho\,C_p}\frac{\partial}{\partial Y}\left(k_f\,\frac{\partial T}{\partial Y}\right)+\frac{\upsilon}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2 \qquad (3)$$

The boundary conditions for the governing equations are

$$\left.\begin{array}{l} U=V=0,\quad T=T_w\quad on\quad Y=0\\ U\to 0, T\to T_\infty\quad at\quad Y\to\infty \end{array}\right\} \qquad (4)$$

$$r(X)=a\sin\left(\frac{X}{a}\right) \qquad (5)$$

where $r = r(X)$

Where a is the radius of sphere, r is the radial distance from the symmetrical axis to the surface of the sphere, $k\,(T)$ is the thermal conductivity of the fluid depending on the fluid temperature T. Here we will consider the form of the temperature dependent thermal conductivity which is proposed by Charraudeau (1975), as

$$k_f = k_\infty\left(1+\gamma^*(T-T_\infty)\right)$$

where k_∞ is the thermal conductivity of the ambient fluid and γ^* is defined as $\gamma^*=\frac{1}{k_f}\left(\frac{\partial k}{\partial T}\right)_f$. Equation (3) can be reduced into the following form

$$U\,\frac{\partial T}{\partial X}+V\,\frac{\partial T}{\partial Y}=\frac{1}{\rho C_p}\left(\frac{\partial k}{\partial Y}\frac{\partial T}{\partial Y}+k\,\frac{\partial^2 T}{\partial Y^2}\right)+\frac{\upsilon}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2$$

$$=\frac{1}{\rho C_p}\left(\frac{\partial k}{\partial Y}\frac{\partial T}{\partial Y}\right)+\frac{1}{\rho C_p}\left(k\,\frac{\partial^2 T}{\partial Y^2}\right)+\frac{\upsilon}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2 \qquad (6)$$

The above equations are non-dimensionalised using the following substitutions:

$$\xi=\frac{\mathrm{X}}{\mathrm{a}},\eta=\mathrm{Gr}^{\frac{1}{4}}\frac{\mathrm{Y}}{\mathrm{a}},\mathrm{u}=\frac{\mathrm{a}}{\nu}\mathrm{Gr}^{\frac{-1}{2}}\,\mathrm{U},\upsilon=\frac{\mathrm{a}}{\nu}\mathrm{Gr}^{\frac{-1}{4}}V,\theta=\frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_w}-\mathrm{T}_\infty},\theta_w=\frac{\mathrm{T_w}}{\mathrm{T}_\infty} \qquad (7)$$

Thus (5) becomes $r(\xi)=a\sin\xi$

Employing the above transformations into the equations (1) to (3), we have

$$\frac{\partial}{\partial\xi}(\mathrm{ru})+\frac{\partial}{\partial\eta}(\mathrm{r}\,\upsilon)=0 \qquad (8)$$

$$u\frac{\partial u}{\partial\xi}+\upsilon\frac{\partial u}{\partial\eta}=\frac{\partial^2 u}{\partial\eta^2}+\theta\sin\xi \qquad (9)$$

$$u\frac{\partial\theta}{\partial\xi}+\upsilon\frac{\partial\theta}{\partial\eta}=\frac{1}{\mathrm{Pr}}(1+\gamma\theta)\frac{\partial^2\theta}{\partial\eta^2}+\frac{1}{\mathrm{Pr}}\gamma\left(\frac{\partial\theta}{\partial\eta}\right)^2+\mathrm{N}\left(\frac{\partial u}{\partial\eta}\right)^2$$

since $\frac{\partial \mathrm{T}_\infty}{\partial\xi}=\frac{\partial T_\infty}{\partial\eta}=0$ and $\upsilon\rho=\mu$ (10)

and the boundary conditions (4) becomes

$$\left.\begin{array}{l} u=v=0,\quad \theta=1\quad at\ \xi=0,\ for\ any\ \eta\\ u=v=0,\quad \theta=1\quad at\ \eta=0, \xi>0\\ u\to 0,\quad \theta\to 0\ \ as\ \eta\to\infty,\ \xi>0 \end{array}\right\} \qquad (11)$$

Here, $Gr=g\beta(\mathrm{T_w}-\mathrm{T}_\infty)\mathrm{a}^3/\nu^2$ is the Grashof number and θ is the non dimensional temperature function,

$N=\frac{\upsilon^2\,Gr}{\rho\,a^2 C_p(T_w-T_\infty)}$ is the viscous dissipation parameter, $\gamma=\gamma^*(T_w-T_\infty)$ is the non-dimensional thermal conductivity variation parameter and $Pr = \frac{\mu C_P}{k}$ is the Prandtl number. To solve equations (9) and (10) subject to the boundary conditions (11), we assume the following variables *u* and *v* where $\psi = \xi\, r(\xi)\, f(\xi,\eta)$ is a non-dimensional stream function which is related to the velocity components in the usual way as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial\eta} \text{ and } v = -\frac{1}{r}\frac{\partial\psi}{\partial\xi} \qquad (12)$$

Putting the above value in equation (9) and (10), we have

$$\frac{\partial^3 f}{\partial\eta^3} + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right) f\frac{\partial^2 f}{\partial\eta^2} - \left(\frac{\partial f}{\partial\eta}\right)^2 + \theta\frac{\sin\xi}{\xi} = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right) \qquad (13)$$

$$\frac{1}{\Pr}(1+\gamma\theta)\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\Pr}\gamma\left(\frac{\partial\theta}{\partial\eta}\right)^2 + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right) f\frac{\partial\theta}{\partial\eta} + N\xi^2\left(\frac{\partial^2 f}{\partial\eta^2}\right) = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right)^2 \qquad (14)$$

Where $\theta = \theta(\eta,\xi)$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &f = \frac{\partial f}{\partial\eta} = 0,\quad \theta = 1 \text{ at } \eta = 0 \quad \text{for any } \eta \\ &f = \frac{\partial f}{\partial\eta} = 0,\quad \theta = 1 \text{ at } \eta = 0,\quad \xi > 0 \\ &\frac{\partial f}{\partial\eta} \to 0,\ \theta \to 0 \text{ as } \eta \to \infty,\quad \xi > 0 \end{aligned}\right\} \qquad (15)$$

It can be seen that near the lower stagnation point of the sphere i.e. $\xi \approx 0$, Eqs. (13) and (14) reduces to the following ordinary differential equations:

$$f''' + 2f\,f'' - (f')^2 + \theta = 0 \qquad (16)$$

$$\frac{1}{\Pr}(1+\gamma\theta)\theta'' + \frac{1}{\Pr}\gamma(\theta')^2 + 2f\theta' = 0 \qquad (17)$$

Corresponding boundary conditions are

$$\left.\begin{aligned} &f(0) = f'(0) = 0, \theta(0) = 1 \\ &f' \to 0, \theta \to 0 \text{ as } \eta \to \infty \end{aligned}\right\} \qquad (18)$$

Where primes denote differentiation of the function with respect to η. In practical applications, the physical quantities of principal interest are the heat transfer and the skin- friction coefficient, which can be written in non- dimensional form as

$$Nu = \frac{aGr^{-1/4}}{k_f(T_w - T_\infty)}q_w \text{ and } C_f = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w \qquad (19)$$

Where $\tau_w = \mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ and

$q_w = -k_f\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ are the shearing stress and heat flux, respectively. Using the new variables (7), we have the simplified form of the heat transfer and the skin- friction coefficient as

$$\therefore Nu = -\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} \text{ and}$$

$$\therefore C_f = \xi\left(\frac{\partial^2 f}{\partial\eta^2}\right)_{\eta=0} \qquad (20)$$

RESULTS AND DISCUSSION

In this analysis, we have investigated the effect viscous dissipation on natural convection flow over a sphere in presence of temperature dependent thermal conductivity. The solution of the moderated governing equations (16) and (17) are computed for the different values of viscous dissipation parameter *N* (=0.10, 5.0, 10.0, 15.0), thermal conductivity parameter γ (= 0.10, 050, 1.00, 2.00) and Prandtl number Pr (= 0.1.00, 1.44, 1.99, 2.20). In order to illustrate the physical behavior of the velocity, temperature, skin friction and rate of heat transfer, the numerical results are plotted in the figures two to seven and appropriate discussion is given in the following section.

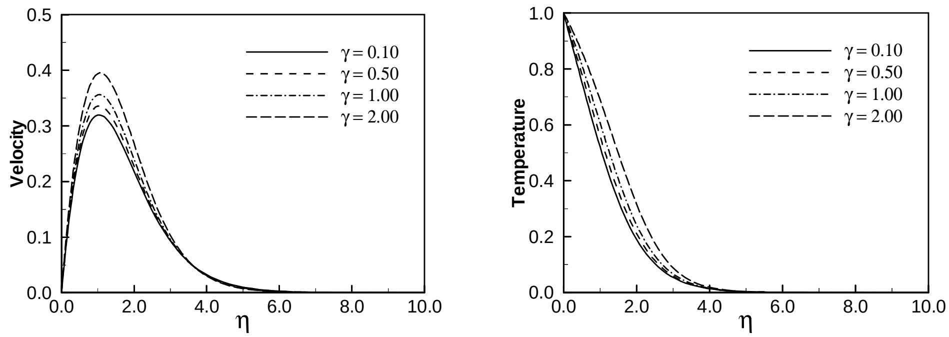

Fig. 2. (a) Variation of velocity and (b) variation of temperature against η for varying of γ with Pr = 1.0 and N = 0.10.

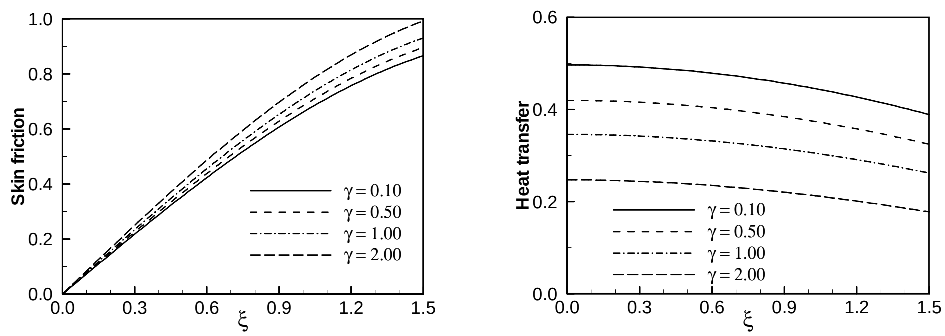

Fig.3. (a) Variation of skin friction and (b) variation of surface temperature against ξ for varying of γ with Pr = 1.0 and N = 0.10.

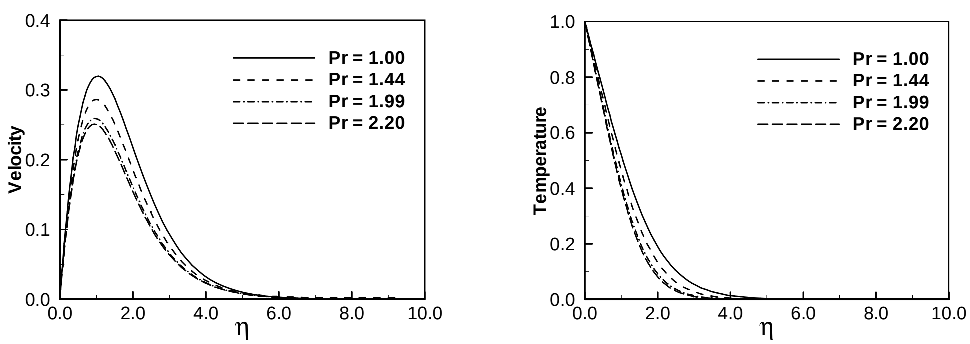

Fig. 4. (a) Variation of velocity and (b) variation of temperature against η for varying of Pr with γ = 0.10 and N = 0.10

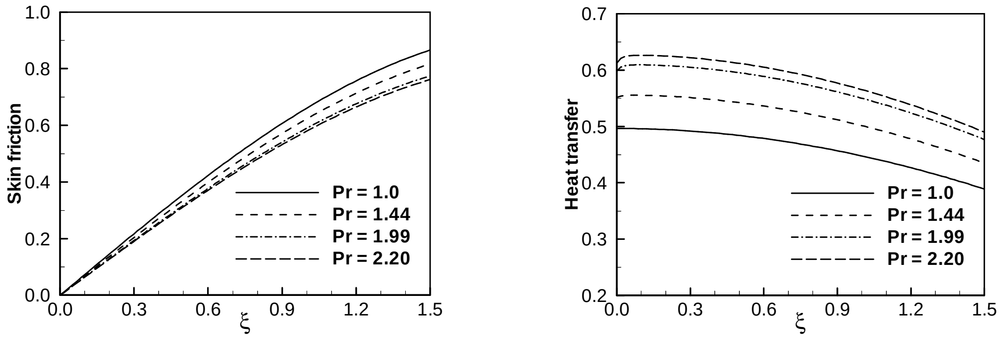

Fig.5. (a) Variation of skin friction and
(b) variation of surface temperature against η for varying of *Pr* with γ = 0.10 and *N* = 0.10

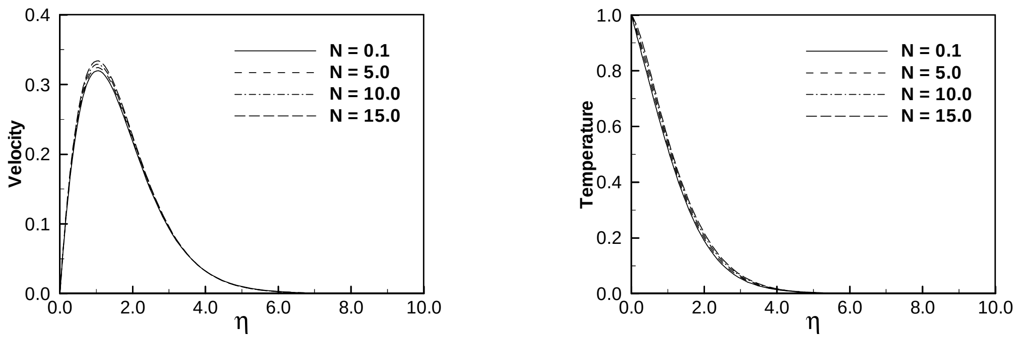

Fig. 6. (a) Variation of velocity and
(b) variation of temperature against η for varying of *N* with γ = 0.10 and *Pr* = 1.0

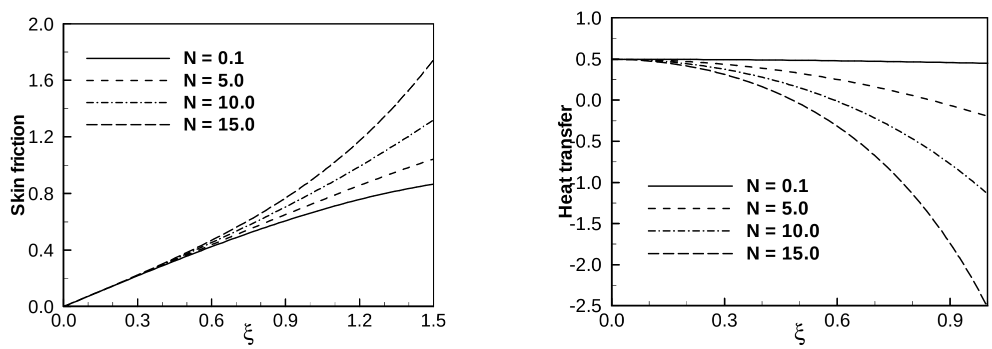

Fig.7. (a) Variation of skin friction and
(b) variation of surface temperature against ξ for varying of *N* with *Pr* = 1.0, and γ = 0.10

The effect of thermal conductivity variation parameter, γ on the velocity and temperature profiles with Prandlt number *Pr* =1.0 and viscous dissipation parameter *N*=0.10 are shown in figures 2(a) and 2(b), respectively. In Figs. 2(a) and 2(b), it is seen that both the velocity and temperature increases as the thermal conductivity variation parameter γ increase. From figure 2(a), the maximum values of the velocity are 0.31971, 0.33611, 0.55647 and 0.39532 for γ = 0.10, 0.50, 1.0, 2.0, respectively at η=1.05539. It is found that the velocity increases by 23.65% for the variation of γ from 0.10 to 2.0. Moreover, in Fig. 2(b) the temperature increases with increasing γ along η direction up to the maximum value and gradually decreases to zero. Fig. 3(a) and Fig. 3(b) illustrate the effect of thermal conductivity variation parameter γ on the skin friction and heat transfer co-efficient against ξ with *Pr* =1.0 and *N* = 0.10. Figs. 3(a)-3(b) depicts that the values of the skin-friction coefficient C_f increases and the Nusselt number *Nu* decreases for increasing values of γ from 0.10 to 2.0. The values of skin-friction coefficient C_f increase by 14.6 % and the Nusselt number *Nu* decrease by 50.204% , respectively while the value of γ changes from 0.10 to 2.0.

Figs. 4(a) and 4(b) deal with the effect of *Pr* on the velocity and temperature against η associate with thermal conductivity variation parameter γ = 0.10 and viscous dissipation parameter *N* = 0.10. Fig.4 (a) shows that the increasing values of Prandlt number *Pr* leads to the decrease in the velocity profiles. The maximum values of the velocity are 0.31971, 0.28640, 0.25901 and 0.25097 for *Pr* = 1.0, 1.44, 1.99 and 2.20, respectively, which occur at η = 1.05539 for the first maximum value and η = 0.99806 for second, η = 0.94233 for third and last maximum value. It is calculated that the velocity decreases by 21.501 % as *Pr* extent from 1.0 to 2.20. Again from Fig 4(b), it is found that the temperature profile decreases with the increasing value of Prandlt number, *Pr*. Figs. 5(a) and 5(b) display the numerical results for the effect of *Pr* on the skin friction coefficient and rate of heat transfer against ξ with γ = 0.10 and *N* = 0.10. From Fig.5(a), we observed that the skin friction coefficient decrease monotonically for the higher value of *Pr*. On the other hand, in Fig.5(b), the rate of heat transfer increases monotonically for selected values of *Pr* along ξ direction. It can be conclude that the skin friction coefficient decrease by 11.998 % and local Nusselt number increase by 23.384 % for different value of *Pr* from 1.0 to 2.20.

Figs. 6(a)-(b) represent the effect of viscous dissipation parameter, *N* on the velocity and temperature profiles with *Pr* = 1.0 and γ = 0.10. We observed in Fig.6(a) that the velocity goes significantly upward with the increasing viscous dissipation parameter *N*. The maximum values of the velocity are 0.33971, 0.32428, 0.32903 and 0.33385 for *N* =0.1, 5.0, 10.0 and15.0 respectively. Counting these peak values of the velocity, we have seen that the velocity rises by 4.422 % as *N* increases from 0.1 to 15.0. On the other hand in Fig. 6(b) exhibits the temperature profile increases for the greater *N*. Figs.7(a)-(b) demonstrate the effect of viscous dissipation parameter *N* on the local skin friction coefficient against ξ with *Pr* =1.0 and γ = 0.10. It is observed from Fig. 7(a) that the increasing value of the viscous dissipation parameter *N*, causes the greater skin friction coefficient on the surface of the sphere. On contemporary of Fig. 7(b) depicts the heat transfer rate, which is gradually decreased for higher values of *N*.

CONCLUSION

A numerical study has been carried out to study the effects of viscous dissipation on natural convection flow over a sphere in presence of temperature dependent thermal conductivity. Numerical solutions are presented for the fluid flow and heat transfer characteristics, and their dependence on the pertinent parameters is discussed. Some of the important conclusions of the present study are:

- The velocity and the temperature of the fluid increase with the increase in thermal conductivity parameter and viscous dissipation parameter.
- The increase in the thermal conductivity parameter and viscous dissipation parameter leads to increase the local skin friction coefficient C_f and a decrease the local heat transfer rate Nu.
- An increase in the values of *Pr* leads to decrease the velocity and the temperature profiles as well as the local skin friction coefficient but the local rate of heat transfer increases.

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