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Certain Transformation of Basic Hypergeometric Series

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ABSTRACT

In this paper we establish certain transformation involving multiple q-series with the help of certain known transformations.

Keywords: Multiple q-series, Hypergeometric Series.

I. INTRODUCTION

Recently Denis and Bhagirathi[1] established certain transformations of triple series in terms of triple series. They used Jackson's[4] transformation of a ${}_2\phi_1$ in terms of an abnormal ${}_2\phi_2$. In this paper we shall use a known transformation of a double q-series of two variables in terms of another double q-series due to Denis[2] besides the above mentioned transformation due to Jacksons to establish certain interesting new transformation of multiple series. In the sequel, we shall deduce certain summations of multiple series, believed to be new. These results may be useful in partition theory.

2. WE SHALL USE THE FOLLOWING TRANSFORMATION TO ESTABLISH OUR RESULTS

$$\phi^{(1)}\left[\alpha;\beta,\mu/\beta;\mu;x,y\right] = \prod\begin{bmatrix} x\beta\alpha/\mu, y\alpha/\beta \\ x, \quad y \end{bmatrix}\phi^{(1)}\left[\mu/\alpha;\mu/\beta,\beta;\mu;\alpha\beta x/\mu, y\alpha/\beta\right] \tag{1}$$

(Denis[2])

$${}_2\phi_1\begin{bmatrix} \lambda,\alpha;x \\ \beta \end{bmatrix}{}_2\phi_1\begin{bmatrix} \lambda,\alpha;x \\ \beta \end{bmatrix} = \frac{[\lambda x]_\infty}{[x]_\infty}\,{}_2\phi_2\begin{bmatrix} \lambda,\beta/\alpha;-\alpha x \\ \lambda x,\beta;q^1 \end{bmatrix} \qquad \text{(Jackson}^4\text{)} \tag{2}$$

where

$$[\alpha]_\infty = \prod_{r=0}^{\infty}\left(1-\alpha q^{\gamma}\right);$$

$$\left[\alpha q^m\right]_\infty = \frac{[\alpha]_\infty}{[\alpha]_m} \tag{3}$$

and

$$[\alpha]_{m+n} = [\alpha]_m\left[\alpha q^m\right]_n. \tag{4}$$

3. HERE WE ESTABLISHED THE FOLLOWING TRANSFORMATION

$$\frac{[a]_{m+n+p}[b]_{m+p}[c/b]_n x^m y^n z^p}{[c]_{m+n+p}[\alpha]_p[q]_m[q]_n[q]_p} \tag{1}$$

$$= \frac{[bx]_\infty[ay]_\infty}{[x]_\infty[y]_\infty}\sum_{m,n,p}\frac{[a]_{n+p}[b]_{m+n+p}[c/a]_m\left[abxq^p/-c\right]_n}{[bx]_{m+n+p}[c]_{m+n+p}[ay]_{n+p}}$$

$$\times\frac{(-ax)^m\left(-cy/b\right)^m z^p q^{(n+p)m} q^{(m+n)(m+n+1)/2}}{[q]_m[q]_n[q]_p}$$

$$\sum_{m,n,p}\frac{[\alpha]_{m+n+p}[\beta]_{m+n}[\gamma/\beta]_p x^m y^n z^p}{[\gamma]_{m+n+p}[q]_m[q]_n[q]_p} \tag{2}$$

$$= \frac{[\gamma\beta]_\infty[z\alpha]_\infty}{[y]_\infty[z]_\infty}\sum_{m,n,p}\frac{[\alpha]_{m+p}[\beta]_{m+n+p}[\gamma/\alpha]_n\left[y\beta\alpha q^m/\gamma\right]_p}{[y\beta]_{m+n+p}[\gamma]_{m+n+p}[z\alpha]_{m+p}}\times$$

$$\times\frac{x^m\left(-y\alpha\right)^n\left(-z\gamma/\beta\right)^p q^{(m+p)n} q^{(n+p)(n+p+1)/2}}{[q]_m[q]_n[q]_p},$$

$$\sum_{m,n,p}\frac{[a]_{m+n+p}[b]_{n+p}[c/b]_m x^m y^n z^p}{[c]_{m+n+p}[\alpha]_p[q]_m[q]_n[q]_p} \tag{3}$$

$$=\frac{[xab/c]_\infty}{[x]_\infty}\frac{[zac/b^2]_\infty}{[z]_\infty}\sum_{m,n,p}\frac{[a]_{n+p}[b]_{m+n}[c/a]_m[c/a]_p}{[c]_{m+n+p}[xab/c]_n[zac/b^2]_p}\times$$

$$\times\frac{(xab/c)^m y^n(-zc/b)^p}{[q]_m[q]_n[q]_p}$$

$$\sum_{m,n,p,k}\frac{[\alpha_1]_{m+n+p+K}[\beta_1]_{m+n}[\beta_2]_p[\gamma_2/\beta_2]_k x^m y^n z^p t^k}{[\gamma_1]_{m+n}[\gamma_2]_{p+k}[q]_m[q]_n[q]_p[q]_k} \tag{4}$$

$$=\frac{[z\alpha_1]_\infty[t\alpha_1]_\infty}{[z]_\infty[t]_\infty}\sum_{m,n,p,k}\frac{[\alpha_1]_{m+n+p+k}[\beta_1]_{m+n}[\gamma_2/\beta_2]_p[\beta_2]_k}{[z\alpha_1]_{m+n+p}[t\alpha_1]_{m+n+k}}\times$$

$$\times\frac{x^m y^n(-z\beta_2)^p(-t\gamma_2/\beta_2q^p)^k q^{(k+p)(k+p+1)/2}}{[\gamma_2]_{p+k}[\gamma_1]_{m+n}[q]_m[q]_n[q]_p[q]_k}$$

$$\sum_{m,n,p,k}\frac{[a]_{m+n+p+k}[b]_{m+n+p}[a'/b]_k u^m v^n w^p x^k}{[a']_{m+n+p+k}[q]_m[q]_n[q]_p[q]_k} \tag{5}$$

$$=\frac{[wa]_\infty}{[w]_\infty}\frac{[xa]_\infty}{[x]_\infty}\sum_{m,n,p,k}\frac{[a]_{m+n+p+k}[b]_{m+n+k}[a'/b]_p}{[xa]_{m+n+k}[wa]_{m+n+p+k}}\times$$

$$\times\frac{u^m v^n(-wb)^p(-a'x/b)^k q^{(p+k)(p+k+1)/2}q^{(m+n+k)p}}{[a']_{m+n+p+k}[q]_m[q]_n[q]_p[q]_k}$$

$$\sum_{m,n,p,k}\frac{[a]_{m+n+p+k}[b]_{m+p+k}[a'/b]_n x^m y^n z^p t^k}{[a']_{m+n+p+k}[q]_m[q]_n[q]_p[q]_k} \tag{6}$$

$$=\frac{[xa]_\infty}{[x]_\infty}\frac{[ya]_\infty}{[y]_\infty}\sum_{m,n,p,k}\frac{[a]_{m+n+p+k}[b]_{n+p+k}[a'/b]_m}{[a']_{m+n+p+k}[xa]_{m+n+p+k}[ya]_{n+p+k}}\times$$

$$\times\frac{[xab/a'](-xb)^m(-a'y/b)^n z^p t^k q^{(m+n)(m+n+1)/2}q^{(p+k)n}q^{(n+p+k)n}}{[q]_m[q]_n[q]_p[q]_k}$$

Proof of (3.1)-(3.6)

Here we present a brief out line of the proof of (3.1)-(3.6).

To prove (3.1) we replace the left side of (3.1) by

$$\sum_p \frac{[a]_p[b]_p z^p}{[c]_p[d]_p[q]_p}\phi^{(1)}\left[aq^p;bq^p,c/b;cq^p;x,y\right]$$

Now we transform $\phi^{(1)}$ with the help of (2.1) by replacing α,β and μ by aq^p, bq^p and cq^p respectively, to get

$$\frac{[abx/c]_\infty}{[x]_\infty}\frac{[ay/b]_\infty}{[y]_\infty}\sum_{m,n,p}\frac{[a]_p[b]_{n+p}[c/a]_{m+n}[c/b]_m(abx/c)^m z^p q^{pm}}{[c]_{m+n+p}[d]_p[abx/c]_p[q]_m[q]_n[q]_p}$$

which can be put in the form,

$$\frac{[abx/c]_\infty}{[x]_\infty}\frac{[ay/b]_\infty}{[y]_\infty}\sum_{m,p}\frac{[a]_p[b]_p[c/a]_m[c/b]_m(abx/c)^m z^p q^{pm}}{[c]_{m+p}[d]_p[abx/c]_p[q]_m[q]_p}\times$$

$$\times {}_2\phi_1\left[\begin{matrix} bq^p,\dfrac{cq^m}{a};ya/b \\ cq^{m+p}\end{matrix}\right]$$

now we transform the ${}_2\phi_1$ with the help of (2.2) by replacing λ,α,β and x by bq^p, cq^p/a, cq^{m+p} and ya/b respectively, to get

$$\frac{[abx/c]_\infty}{[x]_\infty}\frac{[ay]_\infty}{[y]_\infty}\sum_{m,n,p}\frac{[a]_{n+p}[b]_{n+p}[c/a]_m[c/b]_m(abx/c)^m}{[ay]_{n+p}[c]_{m+n+p}[abx/c]_p}\times$$

$$\times\frac{(-cy/b)^n z^p q^{(p+n)m}q^{n(n+1)/2}}{[d]_p[q]_m[q]_n[q]_p}$$

Which can be again put in the form,

$$\frac{[abx/c]_\infty}{[x]_\infty}\frac{[ay]_\infty}{[y]_\infty}\sum_{n,p}\frac{[a]_{n+p}[b]_{n+p}(-cy/b)^n z^p q^{n(n+1)/2}[c/b]_m}{[ay]_{n+p}[c]_{n+p}[abx/c]_p[d]_p[q]_m[q]_n[q]_p}\times$$

$$\times {}_2\phi_1\left[\begin{matrix} c/a, c/b; \frac{abx}{c}q^{n+p} \\ cq^{n+p} \end{matrix}\right]$$

Now making use of (2.2) to transform the ${}_2\phi_1$ - series, we get the right side of (3.1) after some simplification.

The proof (3.2) also follows similarly.

The proof of (3.3) follows similarly where we apply (2.1) and (2.2) once with proper choice of the parameters and the argument.

To prove (3.4) we write, the left side as

$$\sum_{m,n}\frac{[\alpha_1]_{m+n}[\beta_1]_{m+n}x^m y^n}{[\gamma_1]_{m+n}[q]_m[q]_n}\phi^{(1)}\left[\begin{matrix}\alpha_1 q^{m+n}, \beta_2, \gamma_2/\beta_2; z, t \\ \gamma_2\end{matrix}\right]$$

Now transforming the "$\phi^{(1)}$" with the help of (2.1) with proper choice of α, β and μ respectively, to get

$$\frac{[z\beta_2\alpha_1/\gamma_2]_\infty}{[z]_\infty}\frac{[t\alpha_1/\beta_1]_\infty}{[t]_\infty}\sum_{m,n,p}\frac{[\gamma_2/\alpha_1 q^{m+n}]_p[\alpha_1]_{m+n}[\beta_1]_{m+n}}{[z\beta_2\alpha_1/\gamma_2]_{m+n}[t\alpha_1/\beta_2]_{m+n}}\times$$

$$\times\frac{x^m y^n\left[\frac{z\beta_2\alpha_1}{\gamma_2}q^{m+n}\right]^p}{[q]_m[q]_n[q]_p}\cdot {}_2\phi_1\left[\begin{matrix}\frac{\gamma_2 q^p}{\alpha_1 q^{m+n}}, \beta_2; t\alpha_1 q^{m+n}\big/\beta_2 \\ \gamma_2 q^p\end{matrix}\right]$$

Now we use the transformation (2.2) twice with proper choice of λ, α, β and the argument x each time to get the right side after some simplification.

The proof (3.5) follows similarly.

The proof of (3.6) follows similarly where we apply (2.1) once and (2.2) twice with proper choice of the parameters and the argument:

4. SPECIAL CASES

The transformation (3.1)-(3.6) provide q-analogues of important transformation of ordinary hypergeometric functions. It we let $q \to 1$ in (3.1)-(3.6), we get the following transformations respectively.

$$F\begin{bmatrix} a; -, -, b; -, c-b, -; x, y, z \\ c; -, -, -; -, -, d; \end{bmatrix} \tag{1}$$

$$= (1-x)^{-b}(1-y)^{-a} \times$$

$$\times F\begin{bmatrix} b; -, a, -; c-a, abx-c, -; \dfrac{x}{(x-1)}, \dfrac{y}{(x-1)(1-y)}, z \\ c; -, -, -; -, -, -; \end{bmatrix}$$

$$F\begin{bmatrix} \alpha; \beta, -, -; -, -, \gamma-\beta; x, y, z \\ \gamma; -, -, -; -, -, -; \end{bmatrix} \tag{2}$$

$$= (1-y)^{-\beta}(1-z)^{-\alpha} \times$$

$$F\begin{bmatrix} \beta; -, -, \alpha; -, \gamma-\alpha, -; \dfrac{x}{(1-y)(1-z)}, \dfrac{y}{(y-1)}, \dfrac{z}{(z-1)} \\ \gamma; -, -, -; -, -, -; \end{bmatrix}$$

$$F\begin{bmatrix} a; -, b, -; c-b, -, -; x, y, z \\ c; -, -, -; -, -, -; \end{bmatrix} \tag{3}$$

$$= (1-x)^{c-a-b}(1-z)^{2b-a-c} \times$$

$$\times F\begin{bmatrix} -; b, a, -; c-a, -, c-b; x, \dfrac{y}{(1-x)}, \dfrac{z}{(z-1)} \\ c; -, -, -; -, -, -; \end{bmatrix}$$

$$F\begin{bmatrix} \alpha_1; -, -, -, -; \beta_1, -, -, -, -, -; -, -, \beta_2, \gamma_2-\beta_2; x, y, z, t \\ -; -, -, -, -; \gamma_1, -, \gamma_2, -, -, -; -, -, -, -; \end{bmatrix} \tag{4}$$

$$= \left[(1-z)(1-t)\right]^{-\alpha} \times$$

$$F\begin{bmatrix} \alpha_1; -, -, -, -; \beta_1, -, -, -, -, -; -, -, \gamma_2-\beta_2, \beta_2; \times \\ -; -, -, -, -; \gamma_1, -, \gamma_2, -, -, -; -, -, -, -; \end{bmatrix}$$

$$\times \frac{x}{(1-z)(1-t)}, \frac{y}{(1-z)(1-t)}, \frac{z}{(z-1)}, \frac{t}{(t-1)}\Bigg]$$

$$F\left[\begin{matrix} a;\ b, -, -, -; -, -, -, -, -, -; -, -, a'-b; u, v, w, x \\ a'; -, -, -, -; -, -, -, -, -, -; -, -, -, -; \end{matrix}\right] \tag{5}$$

$$= \left[(1-w)(1-x)\right]^{-a} \times$$

$$F\left[\begin{matrix} a;\ -, -, b, -; -, -, -, -, -, -; -, -, -, a'-b; \times \\ a'; -, -, -, -; -, -, -, -, -, -; -, -, -, - ; \end{matrix}\right.$$

$$\times \frac{u}{(1-x)(1-w)}, \frac{v}{(1-x)(1-w)}, \frac{x}{(x-1)(1-w)}, \frac{w}{(w-1)}\Bigg]$$

$$F\left[\begin{matrix} a;\ -, b, -, -; -, -, -, -, -, -; -, a'-b, -, -; x, y, z, t \\ a'; -, -, -, -; -, -, -, -, -, -; -, - , -, -; \end{matrix}\right] \tag{6}$$

$$= \left[(1-x)(1-y)\right]^{-a} \times$$

$$F\left[\begin{matrix} a;\ -, -, b, -; -, -, -, -, -, -; a'-b, xab-a', -, -; \times \\ a'; -, -, -, -; -, -, -, -, -, -; - , - , -, -; \end{matrix}\right.$$

$$\times \frac{x}{(x-1)}, \frac{y}{(x-1)(1-y)}, \frac{z}{(1-x)(1-y)}, \frac{t}{(1-x)(1-y)}\Bigg]$$

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