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Predictive Efficiency of the Estimators Under Non-Stochastic Restrictions

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ABSTRACT

It consider the simultaneous forecasting of actual and average of study variables in a general linear regression model with a set of non-stochastic restrictions on parameters, we propose some new predictors obtained by combining ridge regression estimators with restricted least squares estimator and reports the performance properties of these predictors on the basis of their risk functions.

Keywords: Ridge regression estimator, restricted least squares, restricted ridge regression estimator, mean value prediction, actual value prediction, prior non-sample information.

INTRODUCTION

Generally predictions from a linear regression are made either for the actual values of the study variable or for the average values at a time, see. Rao and Toutenburg (1995). However situation may arise in which one may be required to consider the predictions of the both actual and the average values simultaneously, see Zellner (1994), M. Tiwari *et al*., (2017).

But if the usual assumption of the full column rank of the design matrix is violated in the linear regression model is violated, the problem of multicollinearity arises. To overcome this, different remedial actions have been proposed. A popular numerical technique to deal with multicollinearity is ridge regression proposed by Hoerl and Kennard (1970a,b). Consequently, several authors have suggested different estimators, see., Sarkar (1992, 1998), Kaciranlar *et al*., (1999). If there is extra non-sample information available, performance of conventional estimators should improve remarkably, see Srivastava and Srivastava (1984).

We attempt in this paper to study the performance properties of predictors based on these improved estimators by using the composite target function proposed to expose the estimator obtained by considering the prior non-sample information, for prediction purpose and analyze their performance properties on the basis of the predictive risks involved.

The organization of the paper is as follows, section 2 deals with model specification and presents target function for the prediction of actual and average values of the study variables. In section 3 by assuming the availability of prior non-sample information, various predictors are introduced. In section 4 properties of these predictors presented in the form of theorems. In section 5 a comparative study of these predictors is carried out while the criteria for their comparison is the risk function. In section 6 proof of the results presented in section 4 are derived.

2. MODEL SPECIFICATIONS AND ESTIMATORS

Let us postulate the following linear regression model

$$Y = X\beta + \sigma\omega \tag{2.1}$$

Where Y is a (n ×1) vector of n-observations on study variables, X is a (n×p) full column rank matrix of known elements, β is a (p×1) vector of regression coefficients, σ being unknown scalar and ω is a (n×1) vector of disturbances with mean zero, variance unity and measures of skewness and kurtosis are γ_1 and (γ_2+3) respectively.

When the situation demands prediction of both average and actual values together, we may define the following target function

$$T(Y) = \alpha Y + (1-\alpha)E[Y] = T \tag{2.2}$$

And use $\hat{T} = X\beta$ for predicting it, where $0 \le \alpha \le 1$ is a non stochastic scalar specifying the weightage to be assigned to the prediction of actual and average values of the study variable. We assume that along with model (2.1), the following exact linear restrictions are available

$$q = Q\beta \tag{2.3}$$

where q is a (J×1) vector of known elements and Q is a (J×p) matrix of known elements with full row rank.

The ordinary least squares estimator of β which do not obey the restrictions given in (2.3) is given by

$$b_o = (X'X)^{-1}X'Y \tag{2.4}$$

Following the method of ridge regression proposed by Hoerl and Kennard (1970a, b) the ridge regression estimator

$$b(k) = W^{-1}X'Y \tag{2.5}$$

where

$$W = (X'X + kI) \tag{2.6}$$

and k is non-negative scalar characterizing the estimator.

The restricted least squares estimator obtained by incorporating the restrictions (2.3) is given by

$$b_R = b_o - (X'X)^{-1}Q'\left[Q(X'X)^{-1}Q'\right]^{-1}(Qb_o - q) \tag{2.7}$$

Utilizing (2.4) and (2.5) in (2.7), we obtain restricted ridge regression estimator

$$b_R(k) = b(k) - (X'X)^{-1}Q'\left[Q(X'X)^{-1}Q'\right]^{-1}(Qb(k) - q) \tag{2.8}$$

3. PREDICTIONS WITHIN THE SAMPLE

Employing (2.2), (2.4), (2.5), (2.7) and (2.8) we get the following predictors for the values of the study variables within the sample.

$$\hat{T}_o = Xb_o \tag{3.1}$$

$$\hat{T}_R = Xb_R \tag{3.2}$$

$$\hat{T}(k) = Xb(k) \tag{3.3}$$

$$\hat{T}_R(k) = Xb_R(k) \tag{3.4}$$

4. PROPERTIES OF THE PREDICTORS

Predictor defined in (3.1) is an unbiased with mean squared error matrix and risk function respectively given by

$$Bias(T_o) = E\,[\hat{T}_o - T] = 0 \tag{4.1}$$

$$V(\hat{T}_0) = \sigma^2[\alpha^2 I_n(1-2\alpha)X'(XX)^{-1}X'] \tag{4.2}$$

$$Risk(\hat{T}_0) = \sigma^2[n\alpha^2(1-2\alpha)p] \tag{4.3}$$

Predictor defined in (3.2) is also an unbiased with mean squared error matrix and risk function respectively given by

$$Bias(T_R) = E\,[\hat{T}_R - T] = 0 \tag{4.4}$$

$$V(\hat{T}_R) = V(\hat{T}_0) - \sigma^2 X[(XX)^{-1}A^* - \alpha(A^*(XX)^{-1} + (XX)^{-1}A^{*'})] \tag{4.5}$$

Where

$$A^* = (XX)^{-1}Q'[Q(X'X)Q']^{-1}Q$$

$$Risk(\hat{T}_R) = Risk(\hat{T}_0) - \sigma^2\left[(1-2\alpha)\left\{\frac{trQ'Q}{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j}\right\}\right] \tag{4.6}$$

Where λ_i's , $i = 1, 2, \ldots, p$ are eigen values of the matrix $(X'X)$ and τ_j's, j =1, 2, . . ., J are eigen values of the matrix $[Q(X'X)^{-1}Q']$

Theorem 4.1 For normally distributed disturbances, the bias vector, mean squared error matrix and risk function of the predictor $\hat{T}(k)$ defined in (3.3) up to order $o(\sigma^2)$ of approximations are respectively given by

$$B(\hat{T}(k)) = -kXW^{-1}\beta \tag{4.7}$$

$$M\left[\hat{T}(k)\right] = k^2 XW^{-1}\beta\beta' W^{-1}X' + \sigma^2[\alpha^2 I_n - 2\alpha XW^{-1}X' + XW^{-1}X'XW^{-1}X'] \tag{4.8}$$

$$\text{Risk}(\hat{T}(k)) = Risk(\hat{T}_0) + k^2\beta' W^{-1}X'XW^{-1}\beta + \sigma^2 k\left[\frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i} - \frac{k}{\sum_{i=1}^{p}\lambda_i^2}\right]^2 \tag{4.9}$$

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Theorem 4.2 For normally distributed disturbances, the bias vector, mean squared error matrix and risk function of the predictor $\hat{T}_R(k)$ defined in (3.4) up to order $o(\sigma^2)$ of approximations are respectively given by

$$B(\hat{T}_R(k)) = -kXW^{-1}\beta \tag{4.10}$$

$$M[T_R(k)] = M[T(k)] - [k^2\{XA^*W^{-1}\beta\beta' W^{-1}X' + XAW^{-1}\beta\beta' W^{-1}A^{*\prime} X'\} + \sigma^2\{XA^*W^{-1}X'XW^{-1}X' + XA\,W^{-1}X'XW^{-1}A^*X'\} \tag{4.11}$$

$$Risk\left[\hat{T}_R(k)\right] = Risk\left[\hat{T}(k)\right] - k^2\left[\beta W^{-1}X'XA^*W^{-1}\beta + \sigma^2\left\{(1-2\alpha)\sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j+k}\right)^2 - 2\alpha k\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+k)^2}\right\}\right] \tag{4.12}$$

5. A COMPARISON

On comparing the risk associated with the estimator $\hat{T}_o$ and $\hat{T}_R$ defined in (4.3) and (4.6) respectively we see that

$$\text{Risk}(\hat{T}_o) - \text{Risk}(\hat{T}_R) = \sigma^2(1-2\alpha)\left\{\frac{trQ'Q}{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j}\right\} \tag{5.1}$$

For average value prediction i.e., $\alpha = 0$ we see that the predictor $\hat{T}_R$ have smaller risk than the predictor $\hat{T}_o$ and for the actual value prediction $\alpha = 1$, we find that the predictor $\hat{T}_o$ have smaller risk than $\hat{T}_R$. Again we observe that the predictor $\hat{T}_R$ dominates the predictor $\hat{T}_o$ with respect to risk criterion iff, α satisfies

$$0 < \alpha < \frac{1}{2}\frac{\sum_{i=1}^{p}\lambda_i \sum_{j=1}^{J}\tau_j}{trQ'Q} \tag{5.2}$$

On comparing the risk associated with the estimators $\hat{T}_o$ and $\hat{T}(k)$ defined in (4.3) and (4.9), we see that

$$Risk(\hat{T}_o) - Risk(\hat{T}_k) = -k^2\beta' W^{-1}X'XW^{-1}\beta + \sigma^2 k\left[\frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i} - \frac{k}{\sum_{i=1}^{p}\lambda_i^2}\right] \tag{5.3}$$

If we consider the first term of the above expression, we observe that the risk of considering the predictor $\hat{T}(k)$ is more than that of considering the predictor $\hat{T}_o$.

If we consider the whole expression (5.3) we see that find that the predictor $\hat{T}(k)$ dominates the predictor $\hat{T}_o$ with respect to risk if we choose *k* satisfies

$$0 < k < \frac{(\alpha+2)}{\sum_{i=1}^{p}\lambda_i}\left[\frac{1}{\sum_{i=1}^{p}\lambda_i} + \frac{\beta'(X'X)^{-1}\beta}{\sigma^2}\right] \tag{5.4}$$

On comparing the risk associated with the predictors $\hat{T}_R$ and $\hat{T}(k)$ defined in (4.6) and (4.9) we observe that

$$Risk(\hat{T}(k)) - Risk(\hat{T}_R) = \beta'(XX)^{-1}\beta + \sigma^2\left[(1-2\alpha)\frac{trQ'Q}{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j} - k\left\{\frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i} - \frac{k}{\sum_{i=1}^{p}\lambda_i^2}\right\}\right] \tag{5.5}$$

From the above expression, we observe that for zero power of σ, predictor $\hat{T}_R$ has smaller risk than $\hat{T}(k)$. Again, if the whole expression is considered, we observe $\hat{T}_R$ dominates $\hat{T}(k)$ with respect to risk criterion, if

$$0 < k < \frac{(1-2\alpha)}{(\alpha+2)} \frac{trQ'Q}{\sum_{j=1}^{J} \tau_j} \qquad (5.6)$$

On comparing the risk associated with the predictors $\hat{T}_R$ and $\hat{T}_R(k)$ defined in (4.6) and (4.12) we observe that

$$Risk(\hat{T}_R) - Risk(\hat{T}_R(k)) = -k^2 \beta' W^{-1} X' X W^{-1} \beta + \sigma^2 \left[k \left\{ \frac{\alpha+2}{\sum_{i=1}^{p} \lambda_i} - \frac{k}{\sum_{i=1}^{p} \lambda_i^2} \right\} - (1-2\alpha) \frac{trQ'Q}{\sum_{i-1}^{p} \lambda_i \sum_{j=1}^{J} \tau_j} \right] \qquad (5.7)$$

If we consider first term of the above expression, we see that the risk of the predictor $\hat{T}_R(k)$ is greater than the risk $\hat{T}_R$. If we consider the second term i.e., term containing σ^2, we find that $\hat{T}_R(k)$ dominates $\hat{T}_R$ with respect to risk criterion, so long as

$$k < \frac{(1-2\alpha)}{(\alpha+2)} \frac{trQ'Q}{\sum_{j=1}^{J} \tau_j} \qquad (5.8)$$

On comparing the expression (4.9) and (4.12), the we observe that up to the unit power of k and $0 \le \alpha < 0.5$, the predictive risk associated with $\hat{T}_R(k)$ is less than the predictive risk associated with $\hat{T}(k)$, if the characterizing scalar k satisfies the constraint

$$0 < k \le \left[\left(\frac{1-2}{2\alpha} \right) \sum_{j=1}^{p} \left(\frac{\lambda_j}{(\lambda_j + k)} \right)^2 \right] \left[\sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} \right]^{-1} \qquad (5.9)$$

and just reverse holds, if $0.5 < \alpha \le 1$.

For $\alpha = 0.5$, the predictive risk associated with $\hat{T}_R(k)$ is smaller than the predictive risk associated with $\hat{T}(k)$, if the lower bound of *k* is

$$k \ge \sigma^2 \left[\beta' W^{-1} X' X A^* W^{-1} \beta \right] \left[\sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} \right] \qquad (5.10)$$

When the aim is to predict the average value of the study variable $(\alpha = 0)$

$$Risk_{AV}\left[\hat{T}(k)\right] = k^2\beta' W^{-1}X'XW^{-1}\beta \; + \; \sigma^2\sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j + k}\right)^2 \tag{5.11}$$

$$Risk_{AV}\left[\hat{T}_R(k)\right] = k^2\beta' W^{-1}X'XAW^{-1}\beta \tag{5.12}$$

On comparing the expression (5.11) and (5.12), we observe that for the average value prediction the predictor $\hat{T}_R(k)$ is better than that of $\hat{T}(k)$ for a fixed value of *k*.

Case II: When the aim is to predict the actual values of the study variable $(\alpha = 1)$

$$Risk_{AC}\left[\hat{T}(k)\right] = k^2\beta' W^{-1}X'XW^{-1}\beta + \sigma^2\left\{n - 2\alpha\sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j + k}\right) + \sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j + k}\right)^2\right\} \tag{5.13}$$

$$Risk_{AC}\left[\hat{T}_R(k)\right] = k^2\beta' W^{-1}X'XAW^{-1}\beta + n\sigma^2 \tag{5.14}$$

On comparing the expressions (5.13) and (5.14), the author finds that for actual value prediction, $\hat{T}_R(k)$ have smaller risk than $\hat{T}(k)$, if the lower bound of *k* is

$$k \geq \sigma\left[\beta' W^{-1}X'XA^* W^{-1}\beta\right]^{-1/2}\left[\left\{2\sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j + k}\right) - \left(\frac{\lambda_j}{\lambda_j + k}\right)^2\right\}\right]^{1/2} \tag{5.15}$$

6. DERIVATIONS OF THE THEOREMS

Proof of the theorem 4.1

By utilizing (2.1), (2.4) and (2.5) we obtain

$$b(k) = [I - kW^{-1}]b_0 \tag{6.1}$$

Utilizing (2.2) and (3.2) in (6.1), we have

$$\hat{T}(k) - T = \phi_0^* + \sigma\phi_1^* \tag{6.2}$$

Where

$$\phi_0^* = -kXW^{-1}\beta \tag{6.3}$$

$$\phi_1^* = \sigma[XW^{-1}X' - \alpha I_n]\omega \tag{6.4}$$

$$E[\phi_0^*] = -kXW^{-1}\beta \qquad (6.5)$$

$$E[\phi_1^*] = 0 \qquad (6.6)$$

Using (6.5), (6.6) in (6.2) we obtain the result (4.7) of the theorem 1
Now

$$[(\hat{T}(k)-T)((\hat{T}(k)-T)' = \phi_0^*\phi_0^{*'} + \sigma[\phi_0^*\phi_1^{*'} + \phi_1^*\phi_0^{*'}] + \sigma^2[\phi_1^*\phi_1^{*'}] \qquad (6.7)$$

Where

$$\phi_0^*\phi_0^{*'} = k^2XW^{-1}\beta\beta'W^{-1}X' \qquad (6.8)$$

$$\phi_0^*\phi_1^{*'} = -kXW^{-1}\beta\omega'[XW^{-1}X' - \alpha I_n)' \qquad (6.9)$$

$$\phi_1^*\phi_1^{*'} = \sigma^2[XW^{-1}X' - \alpha I_n]\omega\omega'[XW^{-1}X' - \alpha I_n]' \qquad (6.10)$$

Now

$$E[\phi_0^*\phi_0^{*'}] = k^2XW^{-1}\beta\beta'W^{-1}X' \qquad (6.11)$$

$$E[\phi_0^*\phi_1^{*'}] = E[\phi_1^*\phi_0^{*'}] = 0 \qquad (6.12)$$

$$E[\phi_1^*\phi_1^{*'}] = \sigma^2[\alpha^2 I_n - 2\alpha XW^{-1}X' + XW^{-1}X'XW^{-1}X'] \qquad (6.13)$$

Utilizing (6.11), (6.12) and (6.13) in (6.7) we obtain result (4.8) of the theorem 1
Again

$$[\hat{T}(k)-T)'(\hat{T}(k)-T)] = \phi_0^{*'}\phi_0^* + \sigma[\phi_0^{*'}\phi_1^* + \phi_1^{*'}\phi_0^*] + \sigma^2[\phi_1^{*'}\phi_1^*] \qquad (6.14)$$

Where

$$\phi_0^{*'}\phi_0^* = \beta'W^{-1}X'XW^{-1}\beta \qquad (6.15)$$

$$\phi_0^{*'}\phi_1^* = -\sigma k\beta'W^{-1}X'\omega'[XW^{-1}X' - \alpha I_n]' \qquad (6.16)$$

$$\phi_1^{*}{}'\phi_1^* = \sigma^2\omega'[XW^{-1}X' - \alpha I]'[XW^{-1}X' - \alpha I]\omega \qquad (6.17)$$

$$E[\phi_0^{*'}\phi_0^*] = \beta'W^{-1}X'XW^{-1}\beta \qquad (6.18)$$

$$E[\phi_0^{*'}\phi_1^*] = E[\phi_1^{*'}\phi_0^*] = 0 \qquad (6.19)$$

$$E[\phi_1^{*}{}'\phi_1^*] = \sigma^2\left[(n-p)\alpha^2 + \sum_{i=1}^{p}\left\{\frac{\alpha k}{\lambda_i + k} - \frac{(1-\alpha)\lambda_i}{\lambda_i + 1}\right\}^2\right] \qquad (6.20)$$

Using (6.18), (6.1) and (6.20) in (6.14) we obtain the result (4.9) of the theorem 1

Proof of the theorem 2

By considering exact linear restrictions as given in (2.3) and using (2.4), (2.5) and (2.7) we obtain

$$b_R(k) = \beta + \sigma X\left[A' - kAW^{-1}\right](X'X)^{-1}\omega \qquad (6.21)$$

Where

$$A = [I - (X^{'}X)^{-1}Q^{'}(Q(X^{'}X)^{-1}Q^{'})^{-1}Q] \qquad (6.22)$$

Using (2.2), (2.7) and (6.21) we obtain

$$\hat{T}_R(k) - T = \psi_0 + \sigma\psi_1 \qquad (6.23)$$

Where

$$\psi_0 = -kXAW^{-1}\beta \qquad (6.24)$$

$$\psi_1 = \left[\alpha I - X\left\{A - kAW^{-1}\right\}(X'X)^{-1}X'\right]\omega \qquad (6.25)$$

$$E[\psi_0] = -kXAW^{-1}\beta \qquad (6.26)$$

$$E[\psi_1] = 0 \qquad (6.27)$$

Utilizing (6.26) and (6.27) in (6.23), we obtain the result (4.10) of the theorem 2

$$[(\hat{T}_R(k) - T)\ (\hat{T}_R(k) - T)'] = \psi_0\psi_0' + \sigma[\psi_0\psi_1' + \psi_1\psi_0'] + \sigma^2[\psi_1\psi_1'] \qquad (6.28)$$

where

$$\psi_0\psi_0' = k^2 XAW^{-1}\beta\beta' W^{-1}A' X' \qquad (6.29)$$

$$\psi_0\psi_1' = -kXAW^{-1}\beta\omega'[\alpha I_n - X(X^{'}X)^{-1}\{A - kAW^{-1}\}X'] \qquad (6.30)$$

$$\psi_1\psi_1' = [\alpha I_n - X\{A - kAW^{-1}\}(X^{'}X)^{-1}X']\omega$$

$$\omega'[\alpha I_n - X(X^{'}X)^{-1}\{A - kAW^{-1}\}X'] \qquad (6.31)$$

It can be seen that

$$E[\psi_0\psi_0'] = 0 \qquad (6.32)$$

$$\mathrm{E}[\psi_0\psi_1'] = 0 \qquad (6.33)$$

$$\mathrm{E}[\psi_1\psi_1'] = [\alpha I_n - X\{A - kAW^{-1}\}(X^{'}X)^{-1}X']$$

$$[\alpha I_n - X(X^{'}X)^{-1}\{A - kAW^{-1}\}X'] \qquad (6.34)$$

Utilizing (6.32), (6.33) and (6.34) in (6.28), we obtain the result (4.11) of the theorem 2

Now

$$[(\hat{T}_R(k) - T)'(\hat{T}_R(k) - T)] = \psi_0'\psi_0 + \sigma[\psi_0'\psi_1 + \psi_1'\psi_0] + \sigma^2[\psi_1'\psi_1] \qquad (6.35)$$

Where

$$\psi_0'\psi_0 = k^2\beta' W^{-1}A' X' XAW^{-1}\beta \qquad (6.36)$$

$$\psi_0'\psi_1' = -k\beta W^{-1}A'X\left[\alpha I_n - X\{A - kW^{-1}\}(X'X)^{-1}X'\right]\omega \qquad (6.37)$$

$$\psi_1'\psi_1 = \omega'[\alpha I_n - X(X^{'}X)^{-1}\{A - kW^{-1}A'\}X'][\alpha I_n - X\{A - kAW^{-1}\}(X^{'}X)^{-1}X']\omega \qquad (6.38)$$

Now

$$E[\psi_0'\psi_0] = 0 \qquad (6.39)$$

$$E[\psi_0\psi_1'] = 0 \qquad (6.40)$$

$$E[\psi_1'\psi_1] = \sigma^2\Big[n\alpha^2 + (1-2\alpha)p - k^2\Big\{XA^*W^{-1}\beta\beta'W^{-1}X' + XAW^{-1}\beta\beta'W^{-1}A^*X'\Big\} + \left\{(1-2\alpha)\sum_{j=1}^{p}\left(\frac{\lambda_j}{\lambda_j+k}\right)^2 - 2\alpha k\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+k)^2}\right\}\Big] \qquad (6.41)$$

Utilizing (6.39), (6.40) and (6.41) in (6.35), we obtain the result (4.12) of the theorem 2

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