Journal of Computer and Mathematical Sciences, *Vol.9(5), 384-390 May 2018* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Fixed Point Results in Menger Space with E.A. Property

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(Received on: April 5, 2018)

ABSTRACT

In this paper we have proved some fixed point results in Menger Space with E.A. property for single and multivalued maps under hybrid contractive conditions.

Keywords: Menger Space, Property E.A., JSR Mapping, Probabilistic Metric Space.

1. INTRODUCTION

There are several generalizations of metric space out of which one generalization is Menger Space it was introduced in 1942 by Menger[5]. Menger exploited distribution functions instead of non-negative real numbers as values of the metric, the notion of probabilistic metric space correspond to situation when exactly the distance between the two points is not known but probabilities of this distance are known. A probabilistic generalization of metric space appears to be interest in the investigation of physical quantities and physiological threshold. It is also a fundamental importance in probabilistic functional analysis. Schweizer and Sklar[9] studied this concept and then the important development of Menger Space theory was due to Sehgal[11] and Bharucha-Reid[1,2]. The fixed point theory in PM- spaces which was developed by Schweizer and Sklar[9,10]. Simultaneously Sehgal[11] initiated the study of contraction mapping theorems in PM-spaces. Followed by, several contraction mapping theorems for commuting mappings have been proved in PM-spaces; see for instance Ciric *et al*.[3], Pant and Kumar[8].

The study of fixed points for non-compatible mappings can be extended to the class of non-expansive or Lipschitz mapping pairs, even without necessity of continuity.

The notions of improving commutatively of mapping have been extended to PM-space by various mathematician such as, Singh and Pant[13] extended the notion of weak communality (introduced by Sessa[12] in Metric Spaces), Mishra[6] extended the notion of compatibility (introduced by Jungck[4] in metric space) and Ciric and Milovanovic Arandjelovic[3] extended the notion of point wise weak commutatively (introduce by Pant[8] in metric space) to PM-space. These mathematicians have also proved some common fixed point theorem for contraction mapping by applying them in PM (Probabilistic Metric) space.

In This present paper we have proved some fixed point results in Menger Spaces with E.A. property.

Before going to our main results we need the following definitions, theorems, preliminaries and basic results.

2. PRELIMINARIES

Definition 2.1: A mapping Δ: [0, 1] * [0,1] $\rightarrow$[0,1] is called t- norm if the following conditions are satisfied

(1) Δ (a,1) = a for all a ϵ [0,1], Δ (0,0) = 0,

(2) Δ (a,b) = Δ (b,a)

(3) Δ (c,d) $\leq$ Δ (a,b) for c $\geq$ a, d $\geq$ b, and

(4) Δ (Δ (c, d), c) = Δ (a,Δ (b, c)) for all a, b, c ϵ [0,1]

Example: (2.1) Δ (a,b) = ab, **(2)** Δ (a,b) = min(a, b) **(3)** Δ (a,b) = max (a+b-1;0)

Definition 2.2: A Menger probabilistic space is a triplet (X, F, Δ) where (X, F) is a PM-space and Δ is a t- norm with the following condition

$F_{p,r}$ **(t+s)** $\geq$ Δ ($F_{p,r}$ **(t)**,$F_{p,r}$ **(s)**) for all p, q, r ϵ X and t, s $\geq$ 0.

The above inequality is called Menger's triangle inequality.

Definition 2.3:A sequence $\{x_n\}$in (X, F,Δ) is said to be convergent to a point x ϵ X if for every ε >0 and λ >0, there exists an integer N=N (ε ,λ) such that $F_{x_n,X}(\varepsilon)\rightarrow 1-\lambda\ \forall\ n \geq N(\varepsilon, \lambda)$.

Definition 2.4: A sequence $\{x_n\}$ in (X,F,Δ) is said to be Cauchy sequence if for every ε > 0 and λ >0 , there exists an integer N=N (ε ,λ) such that $F_{x_n, x_m}(\varepsilon)\rightarrow 1-\lambda\ \forall$ n, m $\geq$ N (ε ,λ).

Definition 2.5: A Menger Space (X, F, Δ) with the continuous t- norm is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 2.6: let A Menger Space (X, F, Δ) .Two mapping f, g : X $\rightarrow$ X are said to be compatible if and only if $F_{fgx_n, gfx_n}(t)\rightarrow 1$ for all t >0 whenever $\{x_n\}$ in X such that fx_n, gx_n $\rightarrow$ z for some z ϵ X·

Definition 2.7: Two self-mapping f and g of a probabilistic metric space (X,F,Δ) are said to be point wise R- weakly commuting if given x ϵ X› there exists R > 0 such that

$F_{fgx, gfx}(t) \geq F_{fx, gx}(t/R)$ for t >0.

And T and R-weak commutative of type (A_g) if $F_{ffx, gfx}(t) \geq F_{fx,gx}(t/R)$

Definition 2.8: Let $\{x_n\}$ be a sequence in PM space (X, F, Δ). The self map S and T are called non – compatible if there exist at least one sequence $\{x_n\}$ in X such that $\lim_n Tx_n = \lim_n Sx_n$= p for some p in X but $\lim_n F_{STxn, TSxn,}(t) \neq 1$ or $\lim_n F_{STxn,TSxn,}(t)$ does not exists.

By using definitions mentioned above and simple mathematical analysis recently in 2012, Shukla.M., Dubey.R., and Patel. S.K. Established the following theorems.

Theorem 2.1: Let s and t be non – compatible self mapping of a complete Menger Space (X, F, Δ), where t is continuous and Δ (t, t) ≥t for all t in [0, 1] such that

(2.1.1) $\overline{T(x)} \subset S(X)$ where $\overline{T(x)}$ is closure of the range of T

(2.1.2) $F_{Tx,Ty} \geq \min\left\{\left(F_{Sx,Sy}(t), F_{Sx,Sy}(t)\right), \frac{F_{Sx,Sy}(t)F_{Sx,Sy}(t)}{F_{Sx,Sy}(t)F_{Sx,Sy}(t)}, \frac{F_{Sx,Sy}(t)F_{Sx,Sy}(t)}{F_{Sx,Sy}(t)F_{Sx,Sy}(t)}\right\}$

If T and S be weak compatible of type A, then S and T have a unique common fixed point.

They also make a modification in the condition of theorem 1 with R-weak commutative of type (A_g), we get discontinuity at common fixed point. Let S and T satisfying following conditions:

(2.1.3) $\lim_{n\to\infty} T\,Tx_n = T_p$ and $\lim_{n\to\infty} S\,Tx_n = S_p$

Whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} T\,x_n = \lim_{n\to\infty} S\,x_n = p$ for some p in X

Theorem 2.2: Let S and T be non – compatible self mapping of a Menger Space (X, F, Δ) satisfying 2.1.2 (theorem -2.1) and the condition 2.1.3. If T and S were R- weak commutative of type (A_g), then S and T have a unique common fixed point and fixed point is affixed point discontinuity

In support examples are also provided.

3: FIXED POINT RESULTS IN MENGER SPACE WITH E.A. PROPERTY

The aim of this portion is to define to strength and to emphasize the role of property E.A. in the Existence of common fixed points and prove our main result for hybrid pair of single valued and multi valued map under hybrid contractive condition in Menger Space.

Definition 3.1: Let s: X→X and T: X→CB(X) be mapping in Menger Space (X, F, Δ) then,

(1) s is said to be T weakly commuting at x ϵ X if ssx ϵ Tsx
(2) s and T are weakly compatible if they commute at their coincidence points, i.e. If sTx = Tsx whenever sx ϵ Tx
(3) s and T are (IT) commuting at x ϵ X if sTx is sub set whenever sx ϵ T

Definition 3.2: Let (X, F, Δ) be a Menger Space. If u ϵ X, ε >0, λ ϵ (0, 1) , then an (ε ,λ) neighborhood of u, called U_u (ε ,λ) is defined as

$U_u(\varepsilon, \lambda) - \{v \in X; F_{u,v}(\varepsilon) > 1 - \lambda\}$.

An (ε ,λ) –topology in X is the topology induced by the family

$\{U_u(\varepsilon,\lambda)\ ;\ u \in X,\ \varepsilon > 0,\ \lambda \in (0,1)\}$ of neighborhood. If t is continuous, then Menger Space (X,F,Δ) is a Hausdroff space in the (ε ,λ)- topology

Definition 3.3: Let (X, F, Δ) be a Menger Space. Maps f, g: X→X are said to satisfy the property (E.A.) if there exist a sequence $\{x_n\}$ in x such that:

$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = z \in X$.

Definition 3.4: Maps f: X →X and T: X →CB(X) are said to satisfy the property (E.A.) if there exist a sequence $\{x_n\}$ in X, some z in X and A in CB(X) such that
$\lim_{n\to\infty} f x_n = z \in A = \lim_{n\to\infty} T x_n$.

Definition 3.5: Let f, g, S, G: X→X be mapping in Menger Space. The pair (f, S) and (g, G) are said to be satisfy the common property (E.A.) if there exist two sequence $\{x_n\}$, $\{y_n\}$ in X and some z in X such that ,
$\lim_{n\to\infty} G y_n = \lim_{n\to\infty} S x_n = \lim_{n\to\infty} f x_n = \lim_{n\to\infty} g y_n = z$

Definition 3.6: Let f, g: X→X and S, G: X→CB(X) and be mapping on Menger Space .The maps pair (f, S) and (g, G) are said to satisfy the common property (E.A.) if there exist two sequence $\{x_n\}$, $\{y_n\}$ in X and some z in X, and A, B in CB(X) such that
$\lim_{n\to\infty} S x_n = A$ and $\lim_{n\to\infty} G y_n = B$, $\lim_{n\to\infty} g y_n = z \in A \cap B$.

Definition 3.7: Let (X, F, Δ) be a Menger Space. Let f and g be two self maps of a Menger space. The pair {f, g} is said to be f-JSR mapping iff
$\mu F(fgx_n, gx_n ; p) \geq \mu F(ffx_n, fx_n ; p)$
Where μ = lim sup or lim inf and $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = z$ for some $z \in X$ and for all $\Delta(p, p) > p$

Example 3.1: Let X = [0, 1] with d (x, y) = $|x - y|$ and f, g are two self mapping on X defined by
$fx = \frac{2}{x+2}$, $gx = \frac{1}{x+1}$ for $x \in X$
Now the sequence $\{x_n\}$ in X is defined as $x_n = \frac{1}{n}$, $n \in N$ then we have $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = 1$
$|fgx_n - gx_n| \to \frac{1}{3}$ and $|ffx_n - fx_n| \to \frac{2}{3}$ as $n \to \infty$
Clearly we have, $|fgx_n - gx_n| < |ffx_n - fx_n|$

Thus pair {f, g} is f - JSR mapping. But this pair is neither compatible nor weakly compatible or other non commuting mapping S. Hence pair of JSR mapping is more general then others.

Let f: X →X self map of a Menger Space (X, F, Δ) and S : X → CB (X) be multivalued map. The pair {f, S} is said to be hybrid S-JSR mapping for all
$\Delta(p, p) > p$ if and only if
$\mu F(Sfx_n, fx_n; p) \geq \mu F(SSx_n, x_n; p)$
where, μ = lim sup or lim inf and $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} g x_n = z \in A = \lim_{n\to\infty} S x_n$.

4. MAIN RESULTS

Theorem 4.1: Let (X, F, Δ) be a Menger Space. Let f, g: X→X and
S,G: X →CB(X) such that:

(4.1.1) (f, S) and (g, G) satisfy the common property (EA),
(4.1.2) f (X) and g(X) are closed
(4.1.3) pair (f, S) is S-JSR map and pair (g, G) is G – JSR map,
(4.1.4) $F_{Sx,Gy}(kp) \geq \emptyset\left[min\begin{Bmatrix} F_{fx,gy}(p), F_{fx,Sx}(p), F_{gy,Gy}(p) \\ F_{fx,Gy}(p), F_{Sx,gy}(p), \end{Bmatrix}\right]$
Then f, g, S and G have a common fixed point in X.

Proof: By (4.1.1) there exist two sequence $\{x_n\}$ and $\{y_n\}$ in X and u ϵ X, A, B in CB(X) such that

$\lim_{n\to\infty} Sx_n =$ A and $\lim_{n\to\infty} Gy_n$ = B, And
$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = u \in A \cap B$.
Since f (X) and g (X) are closed, we have u = fv and u = gr for some v, r ϵ X.
Now by (4.1.4) we get

$$F_{Sx_n,Gr}(kp) \geq \emptyset\left[min\begin{Bmatrix} F_{fx_n,gr}(p), F_{fx_n,Sx_n}(p), F_{gr,Gr}(p) \\ F_{fx_n,Gr}(p), F_{Sx_n,gr}(p), \end{Bmatrix}\right]$$

On taking limit n→∞, we obtain

$$F_{AGr}(kp) \geq \emptyset\left[min\begin{Bmatrix} F_{fv,gr}(p), F_{fv,A}(p), F_{gr,Gr}(p) \\ F_{fv,Gr}(p), F_{A,gr}(p), \end{Bmatrix}\right]$$
$$\geq \emptyset\, F_{gr,Gr}(p)$$
$$> F_{gr,Gr}(p)$$

Since, gr = fv ϵ A and $F_{gr,Gr}(p) \geq F_{A,Gr}(p) > F_{gr,Gr}(p)$. Hence gr ϵ Gr

Similarly, $F_{sv,Gy_n}(kp) \geq \emptyset\left[min\begin{Bmatrix} F_{fv,gy_n}(p), F_{fv,sv}(p), F_{gy_n,Gy_n}(p) \\ F_{fv,Gy_n}(p), F_{sv,gy_n}(p), \end{Bmatrix}\right]$

$$F_{sv,B}(kp) \geq \emptyset\left[min\begin{Bmatrix} F_{fv,gr}(p), F_{fv,sv}(p), F_{gr,B}(p) \\ F_{fv,B}(p), F_{sv,gr}(p), \end{Bmatrix}\right]$$
$$\geq \emptyset\, F_{fv,sv}(p)$$
$$> F_{fv,sv}(p)$$

Since fv = gr ϵ B and $F_{fv,sv}(p) \geq F_{B,sv}(p) > \underline{F_{fv}}_{,sv}(p)$, we get fv ϵ sv .

Now as pair (f, S) is an S – JSR map therefore fp ϵ Sp

And similarly as pair (g, G) is G- JSR maps therefore gu ϵ Gr

$$F_{fx_n,gu}(p) \geq F_{sx_n,Gu}(k) \geq \emptyset\left[min\begin{Bmatrix} F_{fx_n,gu}(p), F_{fx_n,sx_n}(p), F_{gu,Gu}(p) \\ F_{fx_n,Gu}(p), F_{sx_n,gu}(p), \end{Bmatrix}\right]$$

On taking limit n → ∞, we obtain

$$F_{u,gu}(p) \geq \emptyset\left[min\begin{Bmatrix} F_{u,gu}(p), F_{u,A}(p), F_{gu,Gu}(p) \\ F_{u,Gu}(p), F_{A,gu}(p), \end{Bmatrix}\right]$$
$$\geq \emptyset\left[min\begin{Bmatrix} F_{u,gu}(p), F_{u,A}(p), F_{gu,Gu}(p) \\ F_{u,Gu}(p), F_{A,u}(p/2), F_{u,gu}(p/2) \end{Bmatrix}\right]$$

By triangular inequality and as u ϵ $A \cap B$, we obtain

$F_{u,gu}(p) \geq F_{u,gu}(p)$ It follows that gu = u.

Again,

$F_{fu,gxn}(p) \geq F_{su,Gxn}(kp) \geq \emptyset\left[min\left\{\begin{matrix} F_{f\mathrm{u},gx_n}(p), F_{f\mathrm{u},su}(p), F_{gx_n,Gx_n}(p) \\ F_{f\mathrm{u},Gx_n}(p), F_{su,gx_n}(p), \end{matrix}\right\}\right]$

On taking limit n→ ∞, we obtain

$F_{fu,gu}(p) \geq \emptyset\left[min\left\{\begin{matrix} F_{f\mathrm{u},u}(p), F_{f\mathrm{u},su}(p), F_{u,Gu}(p) \\ F_{f\mathrm{u},B}(p), F_{su,u}(p), \end{matrix}\right\}\right]$

$\geq \emptyset\left[min\left\{\begin{matrix} F_{fu,u}(p), F_{fu,su}(p), F_{u,Gu}(p) \\ F_{fu,u}(p/2), F_{u,B}\left(\frac{p}{2}\right), F_{su,u}(p) \end{matrix}\right\}\right]$

By triangular inequality and as u ϵ $A \cap B$, we obtain

$F_{fu,u}(p) \geq F_{fu,u}(p)$ It follows that fu = u.

Hence u = fu ϵ Su and u = gu ϵ Su.

Example 4.1: Let = [1, ∞) with usual metric. Define S: x →X as Sx = $\frac{2+x}{3}$ and T: CB(X) → X as Tx = [1, 2+x]. Consider the sequence {x_n} = {3+$\frac{1}{n}$}. Then all conditions are satisfy of the theorem and hence 3 is the common fixed point.

Theorem 4.2: Let (X, F, Δ) be a Menger Space. Let f, g: X →X and S_i, Gj: X → CB(X) such that:

(4.2.1) (f, S_i) and (g, G_j) satisfy the common property of (E.A.).

(4.2.2) f(X) and g(X) are closed,

(4.2.3) pair (f, S_i) is S_i – JSR and pair (g, G_j) is G_j – JSR Map

(4.2.4) $F_{S_i x, g_j y}(Kp) \geq \emptyset\left[min\left\{\begin{matrix} F_{f\mathrm{x},gy}(p), F_{f\mathrm{x},s_i x}(p), F_{gy,,Gjy}(p) \\ F_{f\mathrm{x},G_j y}(p), F_{s_i x,gy}(p), \end{matrix}\right\}\right]$ then f, g, S_i, G_j have a common fixed point in X.

Proof: same as theorem 4.1 for each sequence S_i and G_j.

5: CONCLUSION

E.A. property play vital role in the existence of common fixed points for hybrid pair of single valued and multivalued maps under hybrid contractive conditions in Menger space. Here JSR mappings more generalized then other non commutative mappings for hybrid pairs.

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