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Analytical Solution of Surge Hydrodynamic Coefficients by an Offshore-structure Model in a Channel of Finite Width

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ABSTRACT

This paper presents the analytical solutions for the surge radiation problem by a floating single hollow cylindrical structure in a channel of finite width. We used the channel multipoles approach in order to find the solutions including wall conditions. On the basis of channel multipoles, separation of variables methods, we derived the surge radiated velocity potentials in the identified subdomains. Moreover, by using appropriate matching conditions between virtual and physical boundaries of the subdomains, we deduce a system of linear equations to find the unknown coefficients. From the expressions of radiated velocity potentials, we obtain the expressions of hydrodynamic coefficients, namely, added mass and damping coefficients due to surge oscillation of hollow cylindrical structure.

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Keywords: Surge motion, Channel multipoles, Hydrodynamic coefficients, wavenumber, virtual boundary.

1. INTRODUCTION

Many researchers have approached theoretically to develop the problem of radiation of water wave by using different floating structure. Our present investigation is also related to the radiation problem of water wave in channel by a vertical hollow cylinder under the assumptions of linearized water wave theory. Abramowitz and Stegun[1] gave the different recurrence relations and values of the mathematical functions. Bharatkumar *et al*.[2] used a Green's function approach together with the method of images to compute first-order forces and pressures on pairs of circular solid cylinders in a channel. Bhatta and Rahman[3] discussed interactions between single bodies with different cross-sections and waves at uniform depth. Bhattacharjee and Soares[4] analyzed the diffraction of water waves by a floating structure near a wall with step-type bottom topography. Buffer and Thomas[5] used multipole method to compute reflection and transmission coefficients for array of cylinders in a channel. Hassan and Bora[6,7] calculated hydrodynamic coefficients, i.e., added mass and damping coefficient and exciting forces for the two coaxial vertical geometrical shapes in the form of a pair of coaxial vertical cylinders. A useful method developed by Kashiwagi[8] who was constructed three-dimensional Green's function which automatically satisfied the channel wall boundary condition. He also computed mean second-order drift forces on four truncated cylinders arranged in a square and compared his results with experimental data. Linton[9], Linton and Evans[10] and Mclver and Bennett[11] gave how to used multipole method to find the solution of such problems. The use of multipoles also enabled the existence of the phenomenon of trapped modes near bodies in channels, previously undiscovered in the water wave context, to be proved. The same approach was used by Neelamani *et al*.[12] who compared results for a particular two-cylinder geometry with those from experiments. Thomas[13] used the method of images to solve scattering and radiation problems for a single vertical solid circular cylinder in a channel. Thorne[14] derived multipoles in two and three dimensions which allow the straightforward solution of many problems. Mean drift loads on arrays of two and four cylinders extending throughout the water depth were also considered by Williams and Vazquez[15] using a multipole method. Wu *et al*.[16, 17] discussed the hydrodynamic coefficients and wave exciting force for a buoy over a convex body whose radius was larger or smaller than that of the buoy. Yeung and Sphaier[18] have been used the method of images to solve scattering and radiation problems for a single vertical solid circular cylinder in a channel, in some cases extending throughout the entire fluid depth.

Our present investigation is also deals with the radiation of water waves by a vertical hollow cylinder which is placed at centre between the channel walls. In our present paper, we divide the complicated fluid domain into two parts as exterior and interior subdomains and the expression of radiated velocity potentials for each defined domain is being obtained by the method of channel multipole (Linton and Evans[10]) and separation of variables approach.

2. MATHEMATICAL FORMULATION

Let us assume that the fluid is inviscid, homogeneous, incompressible and the motion is irrotational and small in amplitude. This allows us to consider linear water wave theory in the fluid of finite depth h_1. We consider an infinitely long channel of uniform depth h_1 has parallel walls at a distance $2d$ apart. The bottom of the channel is horizontal and impermeable and the draft of the cylinder is at e_1. A right-handed Cartesian coordinate system $Oxyz$ is defined with the origin O at the undisturbed free surface and the $z-$axis coincide with the axis of the cylinder and pointing upwards, the $x-$axis is located in the longitudinal plane of symmetry of the channel and while the $y-$axis is perpendicular to the channel walls. We divide the whole domain into two subdomains: an exterior domain defined by $r \geq R,\ \ 0 < \theta \leq 2\pi,\ \ -h_1 \leq z \leq 0,$ and the ineterior domain defined by $r \geq R,\ \ 0 < \theta \leq 2\pi,\ \ -h_1 \leq z \leq 0$ as shown in Fig. 1. The velocity potential can be written as $\Phi(x, y, z, t) = Re[\phi(x, y, z)e^{-i\,\omega\,t}]$, where Re represents the real part of the complex quantity in bracket and $\phi(x, y, z)$ is the spatial part of the total velocity potential satisfied the following Laplace's equation:

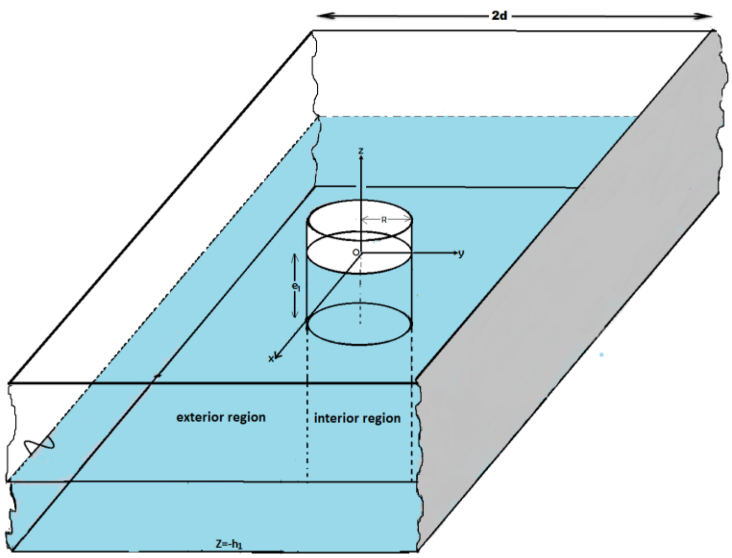

Fig. 1: Schematic diagram of the model

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0. \tag{1}$$

We use the following depth function corresponding to the bottom condition at $z = h_1$ by using separation of variables method

$$Z_m(z) = N_m^{-1/2}\ \cos\lambda_m(z + h_1), \tag{2}$$

where $N_m\ = \frac{1}{2}\left(1 + \frac{\sin 2\,\lambda_m\ h_1}{2\,\lambda_{m\,h_1}}\right)$.

The eigenvalue λ_m can be determined from the dispersion relation

$$\omega^2 = g\,k\tanh(k\,h_1)\,; \qquad \lambda_0\ = -i\,k;\ \text{for}\ \ m = 0, \tag{3}$$
$$\omega^2 = -g\,\lambda_m\tan(\lambda_m\,h_1); \quad \text{for} \qquad m = 1, 2,3, \ldots,$$

where k denotes the wavenumber of the fluid domain and g is the acceleration due to gravity. On the basis of separation of variables method, use equation (2) in equation (1), we get the Helmholtz's equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - \lambda_m^2\phi\ = 0. \tag{4}$$

To convert into polar co-ordinates, we used the following relation:

$$x = r\cos\theta,\ \ y = r\sin\theta. \tag{5}$$

3. BOUNDARY-VALUE PROBLEM

In this section, we setup the governing equation of the surge motion and the boundary conditions

3.1. Governing equation and boundary conditions

Suppose hollow cylindrical structure is forced to oscillate in surge motion only with unit amplitude and draft e_1 in the channel of finite depth. So corresponding to small surge motion the radiated velocity potentials ϕ_{rad} satisfy the following governing equation and boundary conditions:

$$\nabla^2\phi_{rad} = 0, \qquad \text{in the respective domain} \tag{6}$$

$$\frac{\partial\phi_{rad}}{\partial z} - \frac{\omega^2}{g}\phi_{rad} = 0, \qquad z = 0 \tag{7}$$

$$\frac{\partial\phi_{rad}}{\partial z} = 0, \qquad z = -h_1 \;, \tag{8}$$

Body boundary condition in surge motion

$$\frac{\partial\phi_{rad}}{\partial r} = -i\omega\cos\theta, \qquad r = R,\; -e_1 < z < 0 \;, \tag{9}$$

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial\phi_{rad}}{\partial r} - ik\phi_{rad}\right) = 0\,, \tag{10}$$

and the channel walls condition is

$$\frac{\partial\phi_{rad}}{\partial y} = 0, \qquad y = \pm d \;. \tag{11}$$

Since we have divided fluid domain into two subdomains, one is interior and the other is exterior domain. Therefore, the solution of the boundary value problem to be obtained in these physical domain. Suppose ϕ_{rad}^{ext} and ϕ_{rad}^{int} are the radiated velocity potentials in the exterior and interior domain respectively.

4. CHANNEL MULTIPOLE APPROACH TOWARDS SOLUTIONS

Since we consider only the surge motions and the fluid region is symmetric about the centerline of the channel. Therefore, We need to consider only the symmetric part of the velocity potential. Hence let the multipoles is $\psi_{n,m}^s$ which is symmetric about the centerline of the channel and the multipoles $\psi_{n,m}^s$ will then satisfy Helmholtz's equation, channel wall condition (11) and the radiation condition. The channel multipoles are singular solutions of the Helmholtz equation which has singularity at origin. Therefore we can choose for $m = 0$, the multipole $\psi_{n,m}^s$ is a singular part of $H_n^1(kr)\cos n\theta$ and for $m \geq 1$, the multipole $\psi_{n,m}^s$

is a singular part of $K_n \ \lambda_m r \ \cos n\theta$. Let us consider the integral representation for $H_n^1 \ .$ and $K_n \ .$ as

$$H_n^{(1)}(kr) = -\frac{1}{\pi}\int_{-a+i\infty}^{a-i\infty} e^{ikr\cos\omega} e^{in(\omega-\pi/2)} d\omega, \ \ 0 < a < \pi \tag{12}$$

$$K_n(\lambda_m r) = -\frac{1}{2} i \int_{a-i\infty}^{a+i\infty} e^{-k_m r\cos\omega} e^{in\omega} d\omega \ \ -\frac{1}{2}\pi < a < \frac{1}{2}\pi, \tag{13}$$

where $H_n^1 \ .$ is Hankel function of first kind of order n and $K_n \ .$ is the modified Bessel function of second kind of order n. By using change of variables, let $\omega = -\theta - iu$ and multiply both sides by $i^n \ e^{in\theta}$. The integral (12) becomes as,

$$H_n^{(1)}(kr) i^n e^{in\theta} = -\frac{i}{\pi}\int_{i(a+\theta)-\infty}^{i(b+\theta)+\infty} e^{ikx\cosh u} e^{ky\sinh u} e^{nu} du \tag{14}$$

we can rewrite the equation by restriction on the value of θ as

$$H_n^{(1)}(kr) i^n e^{in\theta} = -\frac{i}{\pi}\int_{-\infty}^{i\pi+\infty} e^{ikx\cosh u} e^{ky\sinh u} e^{nu} du \tag{15}$$

Combined equation (12) with similar expression where n is replaced by $-n$ and considering the cases of even and odd values n separately, it follow as:

$$H_{2n}^{(1)}(kr)\cos 2n\theta = -(-1)^n \frac{i}{\pi}\int_{-\infty}^{\infty+i\pi} e^{ky\sinh u}\cos(kx\cosh u) e^{-2nu} du \tag{16}$$

now we split the integral into four part, namely $(-\infty, 0)$, $\left(0, \frac{i\pi}{2}\right)$, $(\frac{i\pi}{2}, \ i\pi)$ and $(\, i\pi, \ \infty + i\pi)$.

$$H_{2n}^{(1)} = (-1)^{n+1}\left[\int_{-\infty}^{0} e^{ky\sinh u}\cos(kx \ \cosh u)\, e^{-2nu} du + \int_0^{\frac{i\pi}{2}} e^{ky\sinh u}\cos(kx \ \cosh u)\, e^{-2nu} du + \int_{\frac{i\pi}{2}}^{i\pi} e^{ky\sinh u}\cos(kx \ \cosh u)\, e^{-2nu} du + \int_{i\pi}^{\infty+i\pi} e^{ky\sinh u}\cos(kx \ \cosh u)\, e^{-2nu} du\right] \tag{17}$$

$$= I_1 + I_2 + I_3 + I_4 \tag{18}$$

Substituting $u = -\beta$, $u = i\left(\frac{\pi}{2} - \alpha\right)$, $u = i(\frac{\pi}{2} + \alpha)$ and $u = (\beta + i\,\pi)$ in I_1, I_2, I_3 and I_4, respectively, then we have

$$I_1 \ = \ -(-1)^n \frac{i}{\pi}\int_0^{\infty} e^{-ky\sinh\beta}\cos(kx\cosh\beta) e^{2n\beta} d\beta \tag{19}$$

$$I_2 \ = \ \frac{1}{\pi}\int_0^{\frac{1}{2}\pi} e^{iky\cos\alpha}\cos(kx\sin\alpha) e^{2n\alpha i} d\alpha \tag{20}$$

$$I_3 = \frac{1}{\pi}\int_0^{\frac{1}{2}\pi} e^{iky\cos\alpha}\cos(kx\sin\alpha)e^{2n\alpha i}d\alpha \tag{21}$$

$$I_4 = -(-1)^n\frac{i}{\pi}\int_0^{\infty} e^{-ky\sinh\beta}\cos(kx\cosh\beta)e^{-2n\beta}d\beta \tag{22}$$

On summation of all $I_1,\ I_2,\ I_3,\ I_4$, we have

$$H_{2n}^{(1)}(kr)\cos 2n\theta = \frac{2}{\pi}\int_0^{\frac{1}{2}\pi} e^{iky\cos\alpha}\cos(kx\sin\alpha)\cos 2n\alpha d\alpha - (-1)^n\frac{2i}{\pi}\int_0^{\infty} e^{-ky\sinh\beta}\cos(kx\cosh\beta)\cosh 2n\beta d\beta \tag{23}$$

Similarly, we can obtain the other integral as

$$H_{(2n+1)}^{(1)}(kr)\cos(2n+1)\theta = \frac{2}{\pi}\int_0^{\frac{1}{2}\pi} e^{iky\cos\alpha}\sin(kx\sin\alpha)\sin(2n+1)\alpha d\alpha + (-1)^{n+1}\frac{2i}{\pi}\times \int_0^{\infty} e^{-ky\sinh\beta}\sin(kx\cosh\beta)\cosh(2n+1)\beta d\beta, \tag{24}$$

Now plugging $\alpha = sin^{-1}z,\quad \beta = cosh^{-1}z$ into equation (23)-(24), then it gives

$$H_{2n}^{(1)}(kr)\cos 2n\theta = -\frac{2i}{\pi}\int_0^{\infty}\frac{e^{-k\zeta y}\cos kxz a_{2n}(z)}{\zeta}dz \tag{25}$$

$$H_{2n+1}^{(1)}(kr)\cos(2n+1)\theta = -\frac{2i}{\pi}\int_0^{\infty}\frac{e^{-k\zeta y}\sin kxz b_{2n+1}(z)}{\zeta}dz \tag{26}$$

where

$$\zeta(z) = \begin{cases} -i(1-z)^{1/2}, & z \le 1 \\ (z^2-1)^{1/2}, & z > 1, \end{cases} \tag{27}$$

$$a_{2n}(z) = \begin{cases} \cos(2n\sin^{-1}z), & z \le 1 \\ (-1)^n\cosh(2n\cosh^{-1}z), & z > 1, \end{cases} \tag{28}$$

$$b_{2n+1}(z) = \begin{cases} \sin[(2n+1)\sin^{-1}z], & z \le 1 \\ (-1)^n\cosh[(2n+1)\cosh^{-1}z], & z > 1. \end{cases} \tag{29}$$

Use the method of Thorne[14], in order to satisfy the channel wall condition on $y = \pm d,$ we need to add $H_{2n}\cos 2n\,\theta$, a function of the form

$$-\frac{2i}{\pi}\int_0^{\infty}\frac{A(z)\cosh -k\zeta y\cos kxz a_{2n}(z)}{\zeta}dz \tag{30}$$

On apply channel wall condition, which gives

$$A(z) = e^{-k\zeta y}/\zeta \sinh k\zeta d \tag{31}$$

Multipoles expansions can be written in terms of polar co-ordinates by using the following identities given by Abramowitz and Stegun[1],

$$\cosh k\zeta y \cos kxz = \sum_{p=0}^{\infty} \mu_p J_{2p}(kr) \cos 2p\theta a_{2p}(z), \tag{32}$$

$$\cosh k\zeta y \sin kxz = 2\sum_{p=0}^{\infty} J_{2p+1}(kr) \cos(2p+1)\theta b_{2p+1}(z), \tag{33}$$

$$\sinh k\zeta y \cos kxz = -2i\sum_{p=0}^{\infty} \mu_p J_{2p+1}(kr) \sin(2p+1)\theta a_{2p+1}(z), \tag{34}$$

$$\sinh k\zeta y \sin kxz = -2i\sum_{p=1}^{\infty} J_{2p}(kr) \sin 2p\theta b_{2p}(z), \tag{35}$$

hence the resultant expressions of multipoles for $m = 0$ is given by

$$\psi_{n,0}^{s} = H_n^{(1)}(kr) \cos n\theta + \sum_{p=0}^{\infty} E(p, n; 0) J_p(kr) \cos p\theta, \tag{36}$$

and for $m \geq 1$, the multipoles is given by

$$\psi_{n,m}^{s} = K_n(\lambda_m r) \cos n\theta + \sum_{p=0}^{\infty} E(p, n; m) I_p(\lambda_m r) \cos p\theta. \tag{37}$$

Here $I_n(.)$ is the first kind of modified Bessel function of order n and the parameter $E(.,.;.)$ appeared in equations (36) and (37) are given by

$$E(2p, 2n; m) = \begin{cases} -\dfrac{2i\mu_p}{\pi} \int_0^{\infty} \dfrac{e^{-k\zeta d} a_{2p}(z) a_{2p}(z)}{\zeta \sinh k\zeta d} dz + \dfrac{\mu_p}{kd} \sum_{j=0}^{l} \mu_j z_j^{-1} a_{2p}(z_j) a_{2n}(z_j), & m = 0 \\ \mu_p \int_1^{\infty} \dfrac{e^{-\lambda_m dz} a_{2p}(z) a_{2n}(z)}{\zeta \sinh \lambda_m dz} dz, & m \geq 1 \end{cases} \tag{38}$$

$$E(2p+1, 2n+1; m) = \begin{cases} -\dfrac{4i}{\pi} \int_0^{\infty} \dfrac{e^{-k\zeta d} a_{2p+1}(z) b_{2n+1}(z)}{\zeta \sinh k\zeta d} dz + \dfrac{2}{kd} \sum_{j=0}^{l} \mu_j z_j^{-1} b_{2p+1}(z_j) b_{2n+1}(z_j), & m = 0 \\ 2\int_1^{\infty} \dfrac{e^{-\lambda_m dz} a_{2p+1}(z) a_{2n+1}(z)}{\zeta \sinh \lambda_m dz} dz, & m \geq 1 \end{cases} \tag{39}$$

and

$$E(2p, 2n+1; m) = E(2p+1, 2n; m) = 0 \text{ for all } m$$

where the integral for $m = 0$ is taken to be a principal value of integral for all the singularities which satisfies the Helmholtz's equation and also integrand considered as a function of complex variable z, has simple poles at $k\varsigma d = \pm j\pi i$, $j = 1,2,3, ...$, i.e. at $z = \pm z_j$ we have

$$z_j = (1 - (j\pi/kd)^2)^{1/2},\ j = 0, 1, 2, ..., l$$
$$z_j = i((j\pi/kd)^2 - 1)^{1/2},\ j \geq l + 1$$

where $l\pi < kd < (l+1)\pi$.

5. RADIATED VELOCITY POTENTIAL

In order to find radiated velocity potential in identified sub-domain, we solved the boundary value problem by using separation of variables and channel multipoles methods as given in the above section on basis of Linton and Evans[8]. Since corresponding to the surge motion only, we have only odd multiples of n exist because of symmetry about the Ox axis and antisymmetric about Oy axis. Therefore, the solutions of the boundary-value problem for exterior and interior domain are given by

$$\phi_{rad}^{ext} = \sum_{m=0}^{\infty} Z_m(z) \sum_{p=0}^{\infty} A_{2p+1,m} \sum_{n=0}^{\infty} \left[U_{2n+1}(\lambda_m r)\delta_{np} + E(2n+1, 2p+1; m) V_{2n+1}(\lambda_m r) \right] \cos(2n+1)\theta, \tag{40}$$

$$\phi_{rad}^{\text{int}} = \sum_{n=0}^{\infty} \left[Z_0(z) B_{2n+1,0}\ \mathrm{J}_{2n+1}(kr) + \sum_{m=1}^{\infty} Z_m(z) B_{2n+1,m}\ \mathrm{I}_{2n+1}(\lambda_m r) \right] \cos(2n+1)\theta, \tag{41}$$

Where $A_{2p+1,m}$ and $B_{2n+1,m}$ are the unknown coefficients which is to be determined by using matching conditions and δ_{np} is the Kronecker delta function. The radial functions $S_n(.)$ and $T_n(.)$ are given by

$$U_n(\lambda_m r) = H_n^{(1)}(kr), \quad m = 0, \tag{42}$$
$$U_n(\lambda_m r) = K_n(\lambda_m r), \quad m = 1, 2, 3, ... \tag{43}$$
$$V_n(\lambda_m r) = H_n^{(2)}(kr), \quad m = 0, \tag{44}$$
$$V_n(\lambda_m r) = I_n(\lambda_m r), \quad m = 1, 2, 3, ... \tag{45}$$

5.1 Matching conditions

We can have the appropriate matching conditions by means of continuity of pressure and that of velocity along the virtual boundaries as depicted in Figure 1. At $r = R$, i.e., along the curved surface of cylinder, extended up to the bottom, we have

$$\phi_{rad}^{ext} = \phi_{rad}^{\text{int}} \qquad -h_1 \leq z \leq -e_1 \tag{46}$$

$$\frac{\partial \phi_{rad}^{ext}}{\partial r} = \begin{cases} \dfrac{\partial \phi_{rad}^{\text{int}}}{\partial r}, & \text{for} \quad -h_1 \leq z \leq e_1 \\ -i\omega \cos\theta, & \text{for} \quad -e_1 \leq z \leq 0 \ . \end{cases} \tag{47}$$

In order to find the unknown coefficients which are present in the expression of the radiated velocity potential, we apply these matching conditions and then yields

$$\sum_{p=0}^{\infty} A_{2p+1,m}\left[U_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V_{2n+1}(\lambda_m R)\right](\lambda_m R) = B_{2n+1,m}V_{2n+1}(\lambda_m R) \tag{48}$$

$$\frac{1}{h_1}\sum_{m=0}^{\infty}\int_{-h_1}^{-h_2} Z_m(z)Z_l(z)dz \sum_{p=0}^{\infty} A_{2p+1,m}\left[U'_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V'_{2n+1}(\lambda_m R)\right] = \frac{1}{h_1}\times \sum_{m=0}^{\infty} A_{2n+1,m}V'_{2n+1}(\lambda_m R)\int_{-h_1}^{-h_2} Z_m(z)Z_l(z)dz. \tag{49}$$

and from other matching conditions gives
and

$$\frac{1}{h_1}\sum_{m=0}^{\infty}\int_{-h_2}^{0} Z_m(z)Z_l(z)dz \sum_{p=0}^{\infty} A_{2p+1,m}\left[U'_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V'_{2n+1}(\lambda_m R)\right] = -\frac{i\omega}{-h_1}\times \int_{-e_1}^{0} Z_l(z)dz. \tag{50}$$

Equations (49) and (50) can be rewritten in compact form as follows:

$$\sum_{p=0}^{\infty} A_{2p+1,l}\left[U'_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V'_{2n+1}(\lambda_m R)\right] = \frac{1}{h_1}\times \sum_{m=0}^{\infty} B_{2n+1,m}V'_{2n+1}(\lambda_m R)N(\lambda_m,\lambda_l,-h_1,-e_1) - \frac{i\omega}{h_1}P_l. \tag{51}$$

where $N(\lambda_m,\ \lambda_l, -h_1,\ -e_1\ ) = \int_{-h_1}^{-e_1} Z_m(z),\ \ Z_l(z)\ dz,$ and $P_l = \int_{-e_1}^{0}\ Z_l(z)\ dz.$
The equation (51) can be combined with (48) to give a set of system of linear equations

$$\sum_{p=0}^{\infty} A_{2p+1,l}\left[U'_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V'_{2n+1}(\lambda_m R)\right] - \sum_{m=0}^{\infty} P_{2n+1,m}\sum_{p=0}^{\infty} A_{2p+1,m}\times \left[U_{2n+1}(\lambda_m R)\delta_{np} + E(2n+1,2p+1,m)V_{2n+1}(\lambda_m R)\right]N(\lambda_m,\lambda_l,-h_1,-h2) = -\frac{i\omega}{h_1}P_l. \tag{52}$$

where as

$$Q_{nm} = \begin{cases} \dfrac{kJ'_n(kR)}{J_n(kR)}, & m=0 \\ \dfrac{\lambda_m J'_n(\lambda_m R)}{J_n(kR)}, & m>0 \end{cases}$$

The unknown coefficients $A_{2n+1,\ m}$ and $B_{2n+1,\ m}$ can be obtained by solving equations (52) and then equation (48). If we truncated the equations by letting $n = 0,1,2,\dots\dots M$ and $l = 0,1,2,\dots\dots N$, then we reached to the system of linear equations having order of $(M+1)\,(N+1)\ \times\ (N+1)\,(M+1)$.

6. HYDRODYNAMIC COEFFICIENTS

The radiation force F_{r1} can be written as the real part of X_1, where X_1 given by

$$X_1 = -i\,\rho\,\omega R \int_0^{2\pi}\int_{-e_1}^{0} \xi_1 \left[\phi_{rad}^{ext}(R,z) - \phi_{rad}^{int}(R,z)\right] \cos\theta \, dz\, d\theta, \tag{53}$$

where ξ_1 is displacement due to surge motion. This radiation force F_{r1} can be decomposed into components in phase with the acceleration and the velocity of the cylinder in the following way

$$F_{r1} = -\left(\mu_{11}\frac{\partial^2 \chi_1}{\partial t^2} + \lambda_{11}\frac{\partial \chi_1}{\partial t}\right). \tag{54}$$

Hence the added mass (μ_{11}) and damping coefficients (λ_{11}) are given by

$$\mu_{11} + \frac{i}{\omega}\lambda_{11} = -\rho R \sum_{m=0}^{\infty}\left[\sum_{p=0}^{\infty} A_{2p+1,m} E(0,2p+1,m) V_0(\lambda_m R) - B_{1,m} U_1(\lambda_m r)\right] N_m^{-1/2} \frac{\sin\lambda_m h_1 - \sin\lambda_m (h_1 - e_1)}{\lambda_m} \tag{55}$$

7. CONCLUSION

By the approach of the method of channel multipoles and separation of variables techniques, we obtained the analytical solutions for the radiation problem in surge motion of single hollow cylindrical structure in a channel of finite width. We determined the unknown coefficients appearing in the radiated velocity potential expressions by using the appropriate matching conditions along the physical and virtual boundaries between the identified subdomains and then we derived theoretically the hydrodynamics coefficients, i.e., added mass and damping coefficients from the expressions of surge radiated potentials in order to know the influences of channel wall, different drafts etc. on the hydrodynamic coefficients. Our mathematical model can be considered as one kind of wave energy device, i.e., Oscillating water column (OWC). The analytical solutions of the problem may be expected to design the proper device in order to extract maximum energy.

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