

# MHD Unsteady Flow with Heat and Mass Transfer through a Non -Homogeneous Porous Medium in Rotating System

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## ABSTRACT

In the present paper we shall discuss unsteady flow, with heat and mass transfer, in an incompressible, electrically conducting and viscous fluid through a time dependent porous medium past an infinite porous vertical plate with constant suction/injection in the presence of a uniform magnetic field applied perpendicular to the flow region. It is considered that the plate is subjected to a constant suction/injection velocity normal to the plate the flow is through a non-homogeneous porous medium. The effects of various parameters on primary velocity, secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocities, rate of heat and mass transfer have been discussed with the help of tables.

**Keywords:** MHD, unsteady flow, non-homogeneous porous medium, heat and mass, velocity, magnetic field, porous vertical plate.

## 1. INTRODUCTION

Free convection problem have attracted a considerable amount of interest because of its importance in atmospheric and oceanic circulations, nuclear reactors, power transformers etc. several authors viz. Sturat (1954), Greenspan (1969), Jana and Dutta (1977), Sinha and Gupta (1980), Gupta *et al.* (1983), Purohit and Sharma (1986), Palec and Dagenet (1987), Singh (1994) have discussed rotating flows, Seth and Banerjee (1996) have studied combined free and forced convection flow of a viscous fluid in rotating channel in the presence of a uniform transverse magnetic field applied parallel to the axis of rotation. Gebhart (1973), Debnath (1973), Acheson and Hide (1973), Reynolds (1975 a, 1975b), Khare (1977), Srinivasan and Kandaswami (1984), Kumar and Mala (1992) Varshney and Johri (1993),

Sharma (1995), Varshney and Varshney (1996) etc. have discussed flow in rotating system in presence magnetic field. Singh *et al.* (2001) have studied free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. Dhiman (2000) have studied a uniform rotation and uniform magnetic field in thermohaline convection. Recently, Kumar *et al.* (2001) have presented a study of the hydrodynamic lubrication of a micropolar fluid between two rotating rollers. More recently, Singh *et al.* (2002) have studied hydromagnetic oscillatory flow of a viscous fluid past a vertical plate in a rotating system. Johri (2003) was investigated approximate solution of the miscible fluid flow through porous media using collocation method.

In the present chapter we shall discuss unsteady flow, with heat and mass transfer, in an incompressible, electrically conducting, viscous fluid through a time dependent porous medium past an infinite porous vertical plate with constant suction/injection in the presence of an uniform magnetic field applied perpendicular to the flow region. It is considered that the plate is subjected to a constant suction/injection velocity normal to the plate and the flow is through a non-homogeneous porous medium. The effects of various parameters on primary velocity, secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocity, temperature field and concentration field have been discussed with the help of figures while the effects of important parameters on in skin-friction due to primary and secondary velocities, rate of heat and mass transfer have been discussed with the help of tables. There are four figure showing effects of the important parameters on primary and secondary velocities. There are six tables showing the effects of various parameters on skin-friction due to primary velocity, secondary velocity, rate of heat transfer and rate of mass transfer.

## 2. FORMULATION OF THE PROBLEM

We consider an unsteady heat and mass transfer flow of an incompressible, electrically conducting, viscous liquid flowing through porous medium, which depends on time such that  $k(t) = k_0(1 + \epsilon e^{int})$  past an infinite, vertical, porous plate with constant heat source in the presence of transverse uniform magnetic field. Further we consider a Cartesian coordinate system choosing x-axis and y-axis in the plane of the porous plate and z-axis normal to the plate with velocity components u,v, w in these directions respectively. Both the liquid and the plate are considered in a state of rigid body rotation about z-axis with uniform angular velocity  $\Omega$ . We also assume that the uniform magnetic field  $\vec{B}_0 = \mu_e \vec{H}$ , where  $\vec{H} = (0, 0, H_0)$  is applied in the z-direction and the magnetic Reynolds number is small. The constant heat source Q is assumed at  $z = 0$ . We take the heat source of absorption type  $Q = Q_0(T - T_\infty)$ . The suction velocity at the plate is  $w = -w_0$  where  $w_0$  is a positive real number and negative sign indicates that the suction is towards the plate. In this analysis buoyancy force, Hall Effect,

effect due to perturbation of the field, induced magnetic field and polarization effect are ignored. Initially, at  $t \leq 0$  the plate and the fluid are at the same temperature  $T_\infty$  and species concentration is uniformly distributed in the flow region such that it is everywhere  $C_\infty$ . When  $t > 0$  the temperature of the plate is raised to  $T_w(1 + \epsilon e^{\text{int}})$  and the concentration level is raised to  $C_w(1 + \epsilon e^{\text{int}})$ . For formulation of mathematical equations the following assumptions have been made:

- (i) The physical properties of the fluid are constant excluding density in the buoyancy force term in the momentum equation.
- (ii) The density is a linear function of temperature and species concentration given by  $\rho = \rho_0 \left[ 1 - \beta_0(T - T_\infty) + \beta(C - C_\infty) \right]$  so that Boussinesq's approximation is taken into account.
- (iii) Following, Gebhart & Pera (1971), the species concentration is very low so that the Soret and Dofour effects are negligible.
- (iv) The induced magnetic field and the heat due to viscous dissipation are negligible.
- (v) The plate is infinite in length so that the physical quantities involved in the governing equations depend on  $z$  and  $t$  only.
- (vi) The magnetic field is not strong enough to cause Joule heating so that the term due to electrical dissipation is neglected in energy equation.

Under above stated restrictions the equations of motion and energy are:

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} + g\beta_0(T - T_\infty) + g\beta(C - C_\infty) - \frac{\nu}{k_0(1 + \epsilon e^{\text{int}})}u - \frac{\sigma}{\rho} \mu_e^2 H_0^2 u \quad (1)$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} - 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\nu}{k_0(1 + \epsilon e^{\text{int}})}v - \frac{\sigma}{\rho} \mu_e^2 H_0^2 v \quad (2)$$

$$\frac{\partial T}{\partial t} - w_0 \frac{\partial T}{\partial z} = \frac{K}{\mu C_p} \frac{\partial^2 T}{\partial z^2} - \frac{Q_0(T - T_\infty)}{\rho C_p} \quad (3)$$

$$\frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} \quad (4)$$

Where  $g$  is the acceleration due to gravity,  $\beta_0$  is the volumetric coefficient of thermal expansion,  $\sigma$  is electrical conductivity of the liquid,  $\rho$  is the density of the liquid,  $\mu_e$  is the magnetic permeability,  $H_0$  is the constant magnetic field,  $k_0$  is the constant permeability of the

medium,  $\mu$  is the coefficient of viscosity,  $K$  is the thermal conductivity,  $C_p$  is the specific heat at constant pressure,  $T$  is the temperature,  $T_w$  is the plate temperature and  $T_\infty$  is the temperature far away from the plate,  $C$  is the concentration of species,  $C_\infty$  is the concentration of species far away from the plate,  $C_w$  is the concentration of species at the plate and the other symbols have their usual meaning.

The boundary conditions relevant to the problem are :

$$u = U_0 \left(1 + \epsilon e^{\text{int}}\right) v = 0, \quad T = T_w \left(1 + \epsilon e^{\text{int}}\right), \\ C = C_w \left(1 + \epsilon e^{\text{int}}\right) \text{ as } z = 0$$

$$u = U(t) \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \\ C \rightarrow C_\infty, \quad \text{as } z \rightarrow \infty \tag{5}$$

We introduce the following non-dimensional quantities:

$$z^* = \frac{w_0 z}{\nu}, \quad t^* = \frac{w_0^2 t}{\nu}, \quad u^* = \frac{u}{U_0}, \quad n^* = \frac{\nu n}{w_0^2}, \quad v^* = \frac{v}{U_0},$$

$$k_0^* = \frac{w_0^2 k_0}{\nu}, \quad C^* = \frac{C - C_\infty}{C_w - C_\infty} \quad \text{and} \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}$$

Using the above stated non-dimensional quantities, the equations (1), (2), (3) and (4) after ignoring the stars over them, reduce to:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} - 2Ev = \frac{\partial^2 u}{\partial z^2} + G_r T + G_m C - \left[ M^2 + \frac{1}{k_0 (1 + \epsilon e^{\text{int}})} \right] u \tag{6}$$

$$\frac{\partial v}{\partial t} - \frac{\partial v}{\partial z} - 2Eu = \frac{\partial^2 v}{\partial z^2} - \left[ M^2 + \frac{1}{k_0 (1 + \epsilon e^{\text{int}})} \right] v \tag{7}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{\rho r} \frac{\partial^2 T}{\partial z^2} - \alpha_0 T \tag{8}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} \tag{9}$$

where  $P_r = \frac{\mu C_p}{K}$  (Prandtl number),  $S_c = \frac{\nu}{D}$  (Schmidt number)

$$G_r = \frac{g \beta_0 \nu (T_w - T_\infty)}{U_0 w_0^2} \text{ (Grashof number)}$$

$$G_m = \frac{g\beta\nu(C_w - C_\infty)}{U_0 w_0^2} \text{ (modified Grashof number),}$$

$$E = \frac{\Omega\zeta}{w_0^2} \text{ (Rotation parameter),}$$

$$M^2 = \frac{\sigma\mu_e^2 H_0^2 \nu}{\rho w_0^2} \text{ (Magnetic parameter)}$$

$$\alpha_0 = \frac{Q_0 \nu^2}{K w_0^2} \text{ (Heat source parameter)}$$

Using  $q = u + iv$  in (6) and (7), we obtain

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + \left[ M^2 + \frac{1}{k_0(1 + \epsilon e^{\text{int}})} + 2iE \right] q = \frac{\partial^2 u}{\partial z^2} + G_r T + G_m C \tag{10}$$

The equation (8) can be written in the following form :

$$\frac{\partial^2 T}{\partial z^2} + P_r \frac{\partial T}{\partial z} - P_r \frac{\partial T}{\partial t} - \alpha_0 T = 0 \tag{11}$$

$$\frac{\partial^2 C}{\partial z^2} + S_c \frac{\partial C}{\partial z} - S_c \frac{\partial C}{\partial t} = 0 \tag{12}$$

The boundary conditions (5) are transformed to :

$$\begin{aligned} q &= 1 + \epsilon e^{\text{int}}, & T &= 1 + \epsilon L_1 e^{\text{int}}, & C &= 1 + \epsilon L_2 e^{\text{int}}, & \text{at } z &= 0 \\ q &\rightarrow 0, & T &\rightarrow 0, & C &\rightarrow 0, & \text{as } z &\rightarrow \infty \end{aligned} \tag{13}$$

$$\text{where } L_1 = \frac{T_w}{T_w - T_\infty} \quad \text{and} \quad L_2 = \frac{C_w}{C_w - C_\infty}$$

### 3. SOLUTION OF THE PROBLEM

In order to solve the equations (10), (11) and (12), we assume the velocity, temperature and concentration in the neighbourhood of the plate as follows:

$$q(z, t) = q_0 + \epsilon q_1(z) e^{\text{int}} \tag{14}$$

$$T(z, t) = T_0 + \epsilon T_1(z) e^{\text{int}} \tag{15}$$

$$\text{and } C(z, t) = C_0 + \epsilon C_1(z) e^{\text{int}} \tag{16}$$

Using equation (1), (2) and (3) in equations (10), (11) and (12), we obtain following equations:

$$q_0''(z) + q_0'(z) - (M_1 + 2iE)q_0(z) = -G_r T_0(z) - G_m C_0(z) \tag{17}$$

$$q_0''(z) + q_0'(z) - [M_1 + (2E + n)]q_1(z) = -G_r T_1(z) - G_m C_1(z) - \frac{1}{k_0} q_0(z) \quad (18)$$

$$T_0''(z) + P_r T_0'(z) - \alpha_0 T_0(z) = 0 \quad (19)$$

$$T_1''(z) + P_r T_1'(z) - (inP_r + \alpha_0) T_1(z) = 0 \quad (20)$$

$$C_0''(z) + S_c C_0'(z) = 0 \quad (21)$$

$$C_1''(z) + S_c C_1'(z) in S_c C_1(z) = 0 \quad (22)$$

Using (14), (15) and (16) in (13) the boundary conditions are reduced to:

$$\begin{aligned} q_0 = 1, \quad q_1 = 1, \quad T_0 = 1, \quad T_1 = L_1, \quad C_0 = 1, \quad C_1 = L_2 \quad \text{at } z = 0 \\ q_0 = 0, \quad q_1 = 0, \quad T_0 = 0, \quad T_1 = 0, \quad C_0 = 0, \quad \text{at } z \rightarrow \infty \end{aligned} \quad (23)$$

The solution of equations (17) to (22) under the boundary conditions (23) are:

$$T_0(z) = e^{-H_2 z} \quad (24)$$

$$T_1(z) = L_1 e^{-H_4 z} \quad (25)$$

$$C_0(z) = e^{-S_c z} \quad (26)$$

$$C_1(z) = L_2 e^{-R_4 z} \quad (27)$$

$$q_0(z) = e^{-H_6 z} + D_1 (e^{-H_2 z} - e^{-H_6 z}) + R_5 (e^{-S_c z} - e^{-H_6 z}) \quad (28)$$

$$\begin{aligned} \text{and } q_1(z) = D_2 e^{-H_2 z} + D_3 e^{-H_4 z} + D_4 e^{-H_6 z} + R_6 e^{-R_4 z} + R_7 e^{S_c z} + R_8 e^{-H_6 z} \\ + (1 - D_2 - D_3 - D_4 - R_6 - R_7 - R_8) e^{-H_8 z} \end{aligned} \quad (29)$$

Substituting the values of  $q_0(z)$  and  $q_1(z)$  in (14), the values of  $T_0(z)$  and  $T_1(z)$  in the equation (15) and the values of  $C_0(z)$  and  $C_1(z)$  in the equation (16) we obtain.

$$\begin{aligned} q(z, t) = e^{-H_6 z} + D_1 (e^{-H_2 z} - e^{-H_6 z}) + R_5 (e^{-S_c z} - e^{-H_6 z}) e^{-H_6 z} \\ + R_6 e^{-R_4 z} + R_7 e^{S_c z} + R_8 e^{-H_6 z} + (1 - D_2 - D_3 - D_4 - R_6 - R_7 - R_8) e^{-H_8 z} \end{aligned} \quad (30)$$

$$T(z, t) = e^{-H_2 z} + \in [L_1 e^{-H_4 z}] e^{\text{int}} \quad (31)$$

$$C(z, t) = e^{-S_c z} + \in [L_2 e^{-R_4 z}] e^{\text{int}} \quad (32)$$

From (30), the steady part of the primary velocity ( $u_0$ ) and the steady part of the secondary velocity ( $v_0$ ) are:

$$\begin{aligned} u_0(z) = F_5 e^{-H_2 z} + P_2 e^{-S_c z} - e^{-A_2 z} (P_2 \cos B_2 z + Q_2 \sin B_2 z) \\ + e^{-A_2 z} (\cos B_2 z - F_5 \cos B_2 z - F_6 \sin B_2 z) \end{aligned} \quad (33)$$

$$\text{and } v_0(z) = F_6 e^{-H_2 z} + Q_2 e^{-S_c z} - e^{-A_2 z} (Q_2 \cos B_2 z + P_2 \sin B_2 z)$$

$$-e^{-A_2 z} (\sin B_2 z - F_6 \cos B_2 z - F_5 \sin B_2 z) \tag{34}$$

From (30), the unsteady part i.e. time dependent part of the primary velocity ( $u_1$ ) and time dependent parts of the secondary velocity ( $v_1$ ) are:

$$\begin{aligned} u_1(z) = & (F_9 \cos B_1 z + F_{10} \sin B_1 z) e^{-A_1 z} (F_{10} \cos B_2 z + F_{12} \sin B_2 z) e^{-A_1 z} \\ & + (F_{13} \cos B_3 z - F_{14} \sin B_3 z) e^{-A_1 z} + F_7 e^{-H_2 z} \\ & + e^{-P_1 z} (P_3 \cos Q_1 z + Q_3 \sin Q_1 z) + P_4 e^{-S_C z} \\ & + (P_5 \cos B_2 z + Q_5 \sin B_2 z) e^{-A_2 z} \end{aligned} \tag{35}$$

$$\begin{aligned} v_1(z) = & (F_{10} \cos B_1 z + F_9 \sin B_1 z) e^{-A_1 z} + (F_{12} \cos B_2 z + F_{11} \sin B_2 z) e^{-A_2 z} \\ & - (F_{14} \cos B_3 z + F_{13} \sin B_3 z) e^{-A_1 z} + F_8 e^{-H_2 z} \\ & + e^{-P_1 z} (Q_3 \cos Q_1 z - P_3 \sin Q_1 z) + Q_4 e^{-S_C z} \\ & + (Q_5 \cos B_2 z - P_5 \sin B_2 z) e^{-A_2 z} \end{aligned} \tag{36}$$

Therefore substituting these values of  $u_0(z)$ ,  $v_0(z)$ ,  $u_1(z)$  and  $v_1(z)$  the primary velocity  $u(z, t)$  and secondary velocity  $v(z, t)$  can be written as

$$u(z, t) = u_0(z) + \epsilon (u_1 \cos nt - v_1 \sin nt) \tag{37}$$

$$v(z, t) = v_0(z) + \epsilon (v_1 \cos nt - u_1 \sin nt) \tag{38}$$

Hence, from (31) and (32), the primary and secondary velocities at  $nt = \frac{\pi}{2}$  are:

$$u\left(z, \frac{\pi}{2n}\right) = u_0(z) - \epsilon v_1(z) \tag{39}$$

$$v\left(z, \frac{\pi}{2n}\right) = v_0(z) - \epsilon u_1(z) \tag{40}$$

$$H_2 = \frac{P_r + \sqrt{P_r^2 + 4\alpha_0}}{2}, \quad M_1 = M^2 + \frac{1}{k_0},$$

$$H_4 = A_1 + iB_1 = \frac{1}{2} \left[ P_r + \sqrt{P_r^2 + 4\alpha_0 + i4nP_r} \right],$$

$$H_6 = A_2 + iB_2 = \frac{1}{2} \left[ 1 + \sqrt{P_r^2 + 4M_1 + i8E} \right],$$

$$H_8 = A_3 + iB_3 = \frac{1}{2} \left[ 1 + \sqrt{P_r^2 + 4M_1 + i2(4E + n)} \right],$$

$$R_4 = P_1 + iQ_1 = \frac{1}{2} \left[ S_C + \sqrt{S_C^2 + i4nS_C} \right],$$

$$R_5 = P_2 + iQ_2 = \frac{-G_m}{S_C^2 - S_C - M_1 - 2iE},$$

$$R_6 = P_3 + iQ_3 = \frac{-G_m L_2}{R_4^2 - R_4 - M_1 - i(2E + n)},$$

$$R_7 = P_4 + iQ_4 = \frac{-R_5}{k_0 [S_C^2 - S_C - M_1 - (2E + n)]},$$

$$R_8 = P_5 + iQ_5 = \frac{-R_5}{k_0 [F_4 - i(2E + n)]},$$

$$D_1 = F_5 + iF_6 = \frac{-G_r}{[F_1 - i2E]},$$

$$D_2 = F_7 + iF_8 = \frac{-D_1}{[F_2 - ik_0(2E + n)]},$$

$$D_3 = F_9 + iF_{10} = \frac{-G_r L_1}{[F_3 - i(2E + n)]},$$

$$D_4 = F_{11} + iF_{12} = \frac{-(1 - D_1)}{k_0 [F_4 - i(2E + n)]},$$

$$A_1 = \frac{P_r}{2} + \frac{1}{2\sqrt{2}} \left[ \sqrt{(P_r^2 + 4\alpha_0)^2 + 16n^2 P_r^2} + (P_r^2 + 4\alpha_0) \right]^{1/2},$$

$$B_1 = \frac{1}{2\sqrt{2}} \left[ \sqrt{(P_r^2 + 4\alpha_0)^2 + 16n^2 P_r^2} - (P_r^2 + 4\alpha_0) \right]^{1/2},$$



$$A_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \sqrt{(1+4M_1)^2 + 64E^2} + (1+4M_1) \right]^{1/2},$$

$$B_2 = \frac{1}{2\sqrt{2}} \left[ \sqrt{(1+4M_1)^2 + 64E^2} - (1+4M_1) \right]^{1/2},$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[ \sqrt{(1+4M_1)^2 + 16(2E+n)^2} + (1+4M_1) \right]^{1/2},$$

$$B_3 = \frac{1}{2\sqrt{2}} \left[ \sqrt{(1+4M_1)^2 + 16(2E+n)^2} - (1+4M_1) \right]^{1/2},$$

$$P_1 = \frac{S_c}{2} + \frac{1}{2\sqrt{2}} \left[ S_c \sqrt{S_c^2 + 16n^2} + S_c^2 \right]^{1/2},$$

$$Q_1 = \frac{1}{2\sqrt{2}} \left[ S_c \sqrt{S_c^2 + 16n^2} - S_c^2 \right]^{1/2},$$

$$P_2 = \frac{-G_m d_1}{d_1^2 + 4E^2},$$

$$Q_2 = \frac{-2G_m E}{d_1^2 + 4E^2},$$

$$P_3 = \frac{-G_m L_2 d_2}{d_2^2 + d_3^2},$$

$$Q_3 = \frac{G_m L_2 d_3}{d_2^2 + d_3^2},$$

$$P_4 = \frac{-d_4}{k_0 [d_1^2 + (2E+n)^2]},$$

$$Q_4 = \frac{-d_5}{k_0 [d_1^2 + (2E+n)^2]},$$

$$P_5 = \frac{d_6}{k_0 [a_2^2 + (2E+n-b_2)^2]},$$

$$Q_5 = \frac{d_7}{k_0 [a_2^2 + (2E+n-b_2)^2]},$$

$$\begin{aligned}
 F_1 &= H_2^2 - H_2 - M_1, & F_2 &= k_0(H_2^2 - H_2 - M_1), \\
 F_3 &= (a_1 + ib_1) = H_4^2 - H_4 - M_1, & F_4 &= (a_2 + ib_2) = H_6^2 - H_6 - M_1, \\
 F_5 &= \frac{-G_r F_1}{F_1^2 + 4E^2}, & F_6 &= \frac{-2G_r E}{F_1^2 + 4E^2}, \\
 F_7 &= \frac{-F_2 F_5 + k_0 F_6 (2E + n)}{F_2^2 + k_0^2 (2E + n)^2}, & F_8 &= \frac{-F_2 F_6 - k_0 F_5 (2E + n)}{F_2^2 + k_0^2 (2E + n)^2}, \\
 F_9 &= \frac{-G_r L_1 a_1}{a_1^2 + (2E + n - b_1)^2}, & F_{10} &= \frac{-G_r L_1 (2E + n - b_1)}{a_1^2 + (2E + n - b_1)^2}, \\
 F_{11} &= \frac{a_2 (F_5 - 1) - F_6 (2E + n - b_2)}{k_0 [a_2^2 + (2E + n - b_1)^2]}, \\
 F_{12} &= \frac{a_2 F_6 + (F_5 - 1) (2E + n - b_2)}{k_0 [a_2^2 + (2E + n - b_1)^2]}, \\
 F_{13} &= 1 - F_7 - F_9 - F_{11} - P_3 - P_4 - P_5, \\
 F_{14} &= F_8 + F_{10} + F_{12} + Q_3 + Q_4 + Q_5, \\
 a_1 &= A_1^2 - B_1^2 - A_1 - M_1, & a_2 &= A_2^2 - B_2^2 - A_2 - M_1, \\
 b_1 &= 2A_1 B_1 - B_1, & b_2 &= 2A_2 B_2 - B_2, \\
 d_1 &= S_c^2 - S_c - M_1, & d_2 &= P_1^2 - Q_1^2 - P_1 - M_1, \\
 d_3 &= 2P_1 Q_1 - Q_1 - 2E - n, & d_4 &= P_2 d_1 - Q_2 (2E + n), \\
 d_5 &= Q_2 d_1 - P_2 (2E + n), & d_6 &= P_2 a_1 - Q_2 (2E + n - b_2), \\
 \text{and } d_7 &= Q_2 a_2 - P_2 (2E + n - b_2),
 \end{aligned}$$

#### 4. SKIN FRICTION AND HEAT TRANSFER

The non-dimensional skin-friction at the plate is:

$$\tau = \left( \frac{\partial q}{\partial z} \right)_{z=0} e^{\text{int}} = \left[ \left( \frac{\partial q_0}{\partial z} \right)_{z=0} + i \left( \frac{\partial q_1}{\partial z} \right)_{z=0} \right] e^{\text{int}} = \tau_p + i \tau_s \tag{41}$$

Hence, primary skin-friction ( $\tau_p$ ) due to primary velocity is:

$$\tau_p = F_{15} + \epsilon (F_{17} \cos nt + F_{18} \sin nt) \tag{42}$$

Also, secondary skin-friction ( $\tau_s$ ) due to secondary velocity is:

$$\tau_s = F_{16} - \epsilon (F_{18} \cos nt - F_{17} \sin nt) \tag{43}$$

where

$$F_{15} = A_2 F_5 - B_2 F_6 - H_2 F_5 - A_2 + P_2 (A_2 - S_c) - B_2 Q_2,$$

$$F_{16} = A_2 F_6 - B_2 F_5 - H_2 F_6 - B_2 + Q_2 (A_2 - S_c) + B_2 P_2,$$

$$F_{17} = A_3 F_{13} - B_2 F_{14} - H_2 F_7 - A_1 F_9 + B_1 F_{11} - A_2 F_{11} + B_2 F_{12},$$

$$F_{18} = B_3 F_{13} - A_3 F_{14} + H_2 F_8 + B_1 F_9 + A_1 F_{10} + B_2 F_{11} + A_2 F_{12},$$

The rate of heat transfer at the plate in terms of Nusselt number ( $N_u$ ) is:

$$N_u = \left( \frac{\partial T}{\partial z} \right)_{z=0} e^{int} = \left[ \left( \frac{\partial T_0}{\partial z} \right)_{z=0} + i \epsilon \left( \frac{\partial T_1}{\partial z} \right)_{z=0} \right] e^{int} \tag{44}$$

Hence, considering that real part only is of significance, the rate of heat transfer is:

$$N_u = -H_2 - \epsilon [A_1 L_1 \cos nt - B_1 L_1 \sin nt] e^{int} \tag{45}$$

The rate of mass transfer at the plate in terms of Sherwood number ( $S_h$ ) is:

$$S_h = \left( \frac{\partial C}{\partial z} \right)_{z=0} e^{int} = \left[ \left( \frac{\partial C_0}{\partial z} \right)_{z=0} + i \epsilon \left( \frac{\partial C_1}{\partial z} \right)_{z=0} \right] e^{int} \tag{46}$$

Hence, considering that real part only is of significance, the rate of mass transfer is:

$$S_h = -S_c - \epsilon [P_1 L_2 \cos nt - Q_1 L_2 \sin nt] e^{int} \tag{47}$$

and  $d_7 = Q_2 a_2 - P_2 (2E + n - b_2)$ ,

**Table - 1**  
**Skin friction due to primary velocity**  
**(Cooling case at n = 5.0, t = 1.0 and  $\epsilon = 0.002$ )**

$P_r$	$S_c$	M	$k_0$	$\alpha_0$	$G_r$	$G_m$	E	$t_p$
0.71	0.66	1.0	20.0	1.0	5.00	8.0	1.0	4.39557
7.00	0.66	1.0	20.0	1.0	5.00	8.0	1.0	3.02178
0.71	0.78	1.0	20.0	1.0	5.00	8.0	1.0	4.19363
0.71	0.66	2.0	20.0	1.0	5.00	8.0	1.0	2.33361
0.71	0.66	1.0	40.0	1.0	5.00	8.0	1.0	4.40933
0.71	0.66	1.0	20.0	2.0	5.00	8.0	1.0	4.14551
0.71	0.66	1.0	20.0	1.0	10.00	8.0	1.0	6.39133
0.71	0.66	1.0	20.0	1.0	5.00	12.0	1.0	6.52598
0.71	0.66	1.0	20.0	1.0	5.00	8.0	2.0	2.48726

**Table – 2**

**Skin friction due to secondary velocity (Cooling case at  $n = 5.0$ ,  $t = 1.0$  and  $\epsilon = 0.002$ )**

$P_r$	$S_c$	$M$	$k_0$	$\alpha_0$	$G_r$	$G_m$	$E$	$\tau_p$
0.71	0.66	1.0	20.0	1.0	5.00	8.0	1.0	-3.4703
7.00	0.66	1.0	20.0	1.0	5.00	8.0	1.0	-2.88124
0.71	0.78	1.0	20.0	1.0	5.00	8.0	1.0	-3.22668
0.71	0.66	2.0	20.0	1.0	5.00	8.0	1.0	-1.42073
0.71	0.66	1.0	40.0	1.0	5.00	8.0	1.0	-3.50291
0.71	0.66	1.0	20.0	2.0	5.00	8.0	1.0	-3.30506
0.71	0.66	1.0	20.0	1.0	10.00	8.0	1.0	-4.11685
0.71	0.66	1.0	20.0	1.0	5.00	12.0	1.0	-4.50417
0.71	0.66	1.0	20.0	1.0	5.00	8.0	2.0	-3.99068

**Table – 3**

**Skin friction due to primary velocity (Heating case at  $n = 5.0$ ,  $t = 1.0$  and  $\epsilon = 0.003$ )**

$P_r$	$S_c$	$M$	$k_0$	$\alpha_0$	$G_r$	$G_m$	$E$	$\tau_p$
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	0.40407
7.00	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	1.77784
0.71	0.78	1.0	20.0	1.0	-5.00	8.0	1.0	0.20212
0.71	0.66	2.0	20.0	1.0	-5.00	8.0	1.0	-0.8868
0.71	0.66	1.0	40.0	1.0	-5.00	8.0	1.0	0.41182
0.71	0.66	1.0	20.0	2.0	-5.00	8.0	1.0	0.65414
0.71	0.66	1.0	20.0	1.0	-10.0	8.0	1.0	-1.5916
0.71	0.66	1.0	20.0	1.0	-5.00	12.0	1.0	2.53447
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	2.0	-0.7074

**Table – 4**

**Skin friction due to primary velocity (Heating case at  $n = 5.0$ ,  $t = 1.0$  and  $\epsilon = 0.002$ )**

$P_r$	$S_c$	$M$	$k_0$	$\alpha_0$	$G_r$	$G_m$	$E$	$\tau_s$
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	-2.17660
7.00	0.66	1.0	20.0	1.0	-5.00	8.0	1.0	-2.76516
0.71	0.78	1.0	20.0	1.0	-5.00	8.0	1.0	-1.93319
0.71	0.66	2.0	20.0	1.0	-5.00	8.0	1.0	-0.92693
0.71	0.66	1.0	40.0	1.0	-5.00	8.0	1.0	-2.19954
0.71	0.66	1.0	20.0	2.0	-5.00	8.0	1.0	-2.34162
0.71	0.66	1.0	20.0	1.0	-10.0	8.0	1.0	-1.52985
0.71	0.66	1.0	20.0	1.0	-5.00	12.0	1.0	-3.21067
0.71	0.66	1.0	20.0	1.0	-5.00	8.0	2.0	-2.49533

**Table – 5**

**Rate of heat transfer in terms of Nusselt Number ( $\epsilon = 0.005$ )**

S.No.	$P_r$	$n$	$t$	$N_u$
1.	0.71	5.00	1.0	-1.42517
2.	0.025	5.00	1.0	-1.01767
3.	7.00	5.00	1.0	-7.18105
4.	11.4	5.00	1.0	-11.5488
5.	0.71	10.0	1.0	-1.42642
6.	0.71	5.00	2.0	-1.42457

**Table – 6**  
**Rate of mass transfer in terms of Sherwood Number ( $\epsilon = 0.005$ )**

S.No.	$P_r$	$n$	$t$	$N_u$
1.	0.66	5.0	1.0	-0.67355
2.	0.22	5.0	1.0	-0.22733
3.	0.30	5.0	1.0	-0.30869
4.	0.60	5.0	1.0	-0.61283
5.	0.78	5.0	1.0	-0.79493
6.	0.66	10.0	1.0	-0.67654
7	0.66	5.0	2.0	-0.67229

## 5. DISCUSSION AND CONCLUSION

We have observed the effects of Prandtl number parameter ( $P_r$ ), Schmidt number ( $S_c$ ), constant permeability parameter ( $k_0$ ), heat source parameter ( $\alpha_0$ ) magnetic parameter ( $M$ ), Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ) on primary and secondary velocities. These effects are shown in figures. The effects of important parameter on rate of heat transfer, rate of mass transfer and skin-friction due to primary and secondary velocities have also been observed. These effects are computed in table.

Figure-1 shown the effects of Prandtl number ( $P_r$ ) Schmidt number ( $S_c$ ), constant permeability parameter ( $k_0$ ) and heat source parameter ( $\alpha_0$ ) on primary velocity ( $u$ ) at  $M = 1.0$ ,  $G_r = 5.0$ ,  $G_m = 8.0$ ,  $E = 1.0$ ,  $n = 5.0$ ,  $t = 1.0$  and  $\epsilon = 0.005$ . It is observed that primary velocity ( $u$ ) increases as  $z$  increases and after attaining a maximum value near the plate, it decreases rapidly as  $z$  increases. It is also noted that an increase in  $\alpha_0$  and decrease in  $P_r$  results in an increase while an increase in  $S_c$  and a decrease in  $k_0$  result in a decrease in primary velocity.

Figure -2 shows effects of magnetic parameter ( $M$ ), Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), on primary velocity ( $u$ ) at  $P_r = 0.71$ ,  $S_c = 0.66$ ,  $k_0 = 20.0$ ,  $\alpha_0 = 1.0$ ,  $n = 5.0$ ,  $t = 1.0$ , and  $\epsilon = 0.005$ . It is observed that primary velocity ( $u$ ) increases as  $z$  increases and after attaining a maximum value near the plate, it decreases rapidly as  $z$  increases. It is also noted that an increase in  $M$ ,  $G_r$ , or  $G_m$  results in an increase while an increase in  $E$  result in a decrease in primary velocity.

Figure-3 shows effects of Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), permeability parameter ( $k_0$ ) and heat source parameter  $\alpha_0$  on secondary velocity ( $v$ ) at  $M = 1.0$ ,  $G_r = 5.0$ ,  $G_m = 8.0$ ,  $\epsilon = 1.0$ ,  $n = 5.0$ ,  $t = 1.0$ , and  $\epsilon = 0.005$ . It is observed that secondary velocity ( $v$ ) decreases as  $z$  increases and after attaining a maximum value near the plate, it increases rapidly as  $z$  increases. It is also noted that an decrease in  $P_r$  or  $k_0$  results in an decrease and an increase in secondary velocity respectively while an increase in  $\alpha_0$ ,  $k_0$  or  $S_c$  result in an increase in secondary velocity.

Figure -4 shows effects of magnetic parameter ( $M$ ), Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), on secondary velocity ( $v$ ) at  $P_r = 0.71$ ,  $S_c = 0.66$ ,  $k_0 = 20.0$ ,  $\alpha_0 = 1.0$ ,  $n = 5.0$ ,  $t = 1.0$ , and  $\epsilon = 0.005$ . It is observed that secondary velocity ( $v$ ) decreases as  $z$  increases and after attaining a minimum value near the plate it increases

rapidly as  $z$  increases. It is also noted that an increase in  $M$  or  $E$  results in an increase while an increase in  $G_r$  or  $G_m$  result in a decrease in secondary velocity.

The effects of the parameter namely Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), magnetic parameter ( $M$ ), permeability parameter ( $k_o$ ), heat source parameter ( $\alpha_o$ ) Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), at  $n=5.0$ ,  $t=1.0$ , and  $\epsilon = 0.005$ , on skin friction ( $\tau_p$ ) due to primary velocity and skin-friction ( $\tau_s$ ) due to secondary velocity. In the computation of numerical values for skin-friction due to primary velocity and secondary velocity, in the computation of numerical values for skin-friction due to primary velocity and secondary velocity. We have taken two important cases namely cooling case and heating case. These cases are of immense importance in astrophysical problems and industrial technology, where heating and cooling of the plates have economic applications. Therefore the cases of externally cooled plate ( $G_r > 0$ ) and externally heated plate ( $G_r < 0$ ) are studied taking numerical values of various parameter encountered in the equations of the skin-friction. The value of Prandtl number ( $P_r$ ) is chosen as  $P_r = 0.71$  which corresponds to water, which correspond to air. The numerical values of the remaining parameters are chosen arbitrary. These effects are shown in tables (1)-(4). The effects of Prandtl number ( $P_r$ ), frequency parameter ( $n$ ) and time parameter ( $t$ ) on rate of heat transfer [expressed in terms of Nusselt number ( $N_u$ )] and the effects of Schmidt number ( $S_c$ ), frequency parameter ( $n$ ) and time parameter ( $t$ ) on rate of mass transfer [expressed in terms of Sherwood number ( $S_h$ )] are numerically expressed in table-5 and table-6 respectively.

Table-1 represents the skin-friction ( $\tau_p$ ) to show the effects of Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), magnetic parameter ( $M$ ), permeability parameter ( $k_o$ ), heat source parameter ( $\alpha_o$ ) Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), at  $n = 5.0$ ,  $t = 1.0$ , and  $\epsilon = 0.005$  for cooling case. It is observed an increase in  $P_r$ ,  $S_c$ ,  $M$ ,  $\alpha_o$  or  $E$  decreases skin-friction due to primary velocity while an increase in  $k_o$ ,  $G_r$  or  $G_m$  increases skin-friction due to primary velocity in cooling case.

Table-2 represents the skin-friction ( $\tau_s$ ) due to secondary velocity to show the effects of Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), magnetic parameter ( $M$ ), permeability parameter ( $k_o$ ), heat source parameter ( $\alpha_o$ ) Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), at  $n = 5.0$ ,  $t = 1.0$ , and  $\epsilon = 0.005$  for cooling case. It is observed increase in  $P_r$ ,  $S_c$ ,  $M$  or  $\alpha_o$  increases skin-friction due to secondary velocity while an increase in  $k_o$ ,  $G_r$ ,  $G_m$  or  $E$  decreases skin-friction due to secondary velocity in cooling case.

Table-3 represents the skin-friction ( $\tau_p$ ) due to primary velocity to show the effects of Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), magnetic parameter ( $M$ ), permeability parameter ( $k_o$ ), heat source parameter ( $\alpha_o$ ) Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and rotation parameter ( $E$ ), at  $n=5.0$ ,  $t=1.0$ , and  $\epsilon = 0.005$  for heating case. It is observed an increase in  $P_r$ ,  $k_o$ ,  $\alpha_o$  or  $G_m$  increases skin-friction due to primary velocity while an increase in  $S_c$ ,  $M$ ,  $G_r$ , or  $E$  decreases skin-friction due to primary velocity in heating case. For  $G_r$ , we are considering magnitude only due to heating case.

Table-4 represents the skin-friction ( $\tau_s$ ) due to secondary velocity to show the effects of Prandtl number ( $P_r$ ), Schmidt number ( $S_c$ ), magnetic parameter ( $M$ ), permeability parameter ( $k_o$ ), heat source parameter ( $\alpha_o$ ) Grahsop number ( $G_r$ ), modified Grahsop number ( $G_m$ ) and rotation parameter ( $E$ ), at  $n=5.0$ ,  $t=1.0$ , and  $\epsilon = 0.005$  for heating case. It is observed an increase in  $S_c$ ,  $M$  or  $G_r$  increases skin-friction due to secondary velocity while an increase in  $P_r$ ,  $k_o$ ,  $\alpha_o$   $G_m$  or  $E$  decreases skin-friction due to secondary velocity in heating case. For  $G_r$ , we are considering magnitude only due to heating case.

The effects of  $P_r$ ,  $n$  and  $t$  on the rate of heat transfer, expressed in terms of Nusselt number at  $\epsilon = 0.005$  are numerically represented in table-5. It is observed that a decrease in  $P_r$  increases the rate of heat transfer and vice-versa. It is also observed that the effects of increase in  $n$  or  $t$  are opposite to each others.

Table-6 shows the effects of  $S_c$ ,  $n$  and  $t$  on the rate of mass transfer, expressed in terms of Sherwood number at  $\epsilon = 0.005$ . It is observed that a decrease in  $S_c$  increases the rate of mass transfer. It is also observed that the effects of increase in  $n$  or  $t$  mass transfer are reciprocal to each other.

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