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Unsteady Flow of an Electrically Conducting Viscous Fluid through a Rectangular Channel

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ABSTRACT

The present article deals with the flow of an electrically conducting viscid fluid in the presence of uniform and transverse magnetic field in a straight tube whose cross-section is a rectangle and considering the flow field under the effect of time varying barometric gradient. The expression for velocity has been obtained by operational methods and the detailed discussion has been made by taking the barometric gradient constant.

Keywords: MHD flow, Rivlin-Ericksen Fluid, Magnetic Field, Pressure Gradient, Viscosity, Magnetic Induction.

1. INTRODUCTION

In the recent years, the motion of Rivlin-Erickson fluid under uniform, exponential pressure gradient, when the fluid is contained between two parallel plates have been investigated by many researchers. Umavathi *et al.* discussed the oscillatory flow of unsteady fluid and observed the heat transfer in porous medium. Pavlov and Teresor gave a note on the stability of flow of conducting fluid in between the parallel planes under oblique magnetic field. Unsteady fluid flow in rectangular channel with heat transfer has been studied by Gireesha *et al.* Makinde studied heat transfer in a channel comprising of porous medium. The motion of viscous elastic fluid under the action of uniform or periodic body force for a finite time has been investigated by Pal and Sen Gupta. Bagchi has considered a similar problem through two parallel planes with transient pressure gradient.

In the present article, an effort is made by considering the flow of viscid conducting liquid through a rectangular channel under some barometric gradient in the presence of uniform magnetic field and then a discussion has been made by taking constant barometric gradient. Effects due to induced magnetic field and perturbation of the field has been neglected.

2. MATHEMATICAL FORMULATION

Here, we have assumed the following notations:

ρ	=	Liquid's density
u	=	Liquid's velocity
η_0	=	Viscosity coefficient
v	=	Kinematic coefficient of viscosity
B_0	=	Magnetic induction
σ	=	Electrical conductivity
β & γ	=	Non- dimensional parameter
(x,y,z)	=	Space co-ordinates
t	=	Time variable

Here, we have considered the class of electrically conducting viscous liquid through a rectangular tube in presence of time varying barometric gradient taking the liquid initially at rest. The walls of the rectangular tube are the planes y = $\pm$b, z = $\pm$c has also considered. The equation of motion is given by

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma {B_0}^2 u}{\rho} \quad (1)$$

Where $v = n_0/\rho$, u(x,y,z) is the velocity of liquid in the direction of x-axis, v & ρ are the kinematic coefficient of density and viscosity of the liquid respectively.

The following non-dimensional quantities have been introduced :

$x' = x/b, y' = y/b, z' = z/b, p' = p\, b^2/\rho v^2, t' = vt/b^2, u' = ub/v$

The equation (1) becomes

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - M^2 u \quad (2)$$

Where $M^2 = \frac{\sigma {B_0}^2 b^2}{v\rho}$ (3)

Initially, the liquid was at rest. The flow takes place under the effect of the time varying barometric gradient under no slip boundary conditions. It is considered that the flow in the region y $\geq$ 0 and z $\geq$ 0 due to symmetric consideration and thus the boundary conditions are as follows:

$$u(1,z,t) = 0 \qquad 0 \leq z \leq h,\ t \geq 0$$
$$\frac{\partial w}{\partial z} = 0 \qquad \text{at } z = h \quad (4)$$
and
$$u(y,h,t) = 0 \qquad 0 \leq y \leq 1,$$

$\frac{\partial w}{\partial z} = 0$ at t=0, where h = c/b (5)

3. SOLUTION OF PROBLEM

Here, we will define the following transforms:

$\bar{u}(l,z,t) = \int_0^1 u(y,z,t))\sin(q_l y)\,dy,$ (6)

$\bar{u}(y,m,t) = \int_0^h u(y,z,t))\sin(q_m z)\,dz$ (7)

where

$q_l = l\,\pi\,,\quad q_m = m\,\pi$ (8)

Inversion formula's of equations (6) & (7) are as follows:

$u(y,z,t) = 2\sum_{l=0}^{\infty}\bar{u}(l,z,t)\sin(q_l y)$ (9)

$u(y,z,t) = \frac{2}{h}\sum_{m=0}^{\infty}\bar{u}(y,m,t)\sin(q_m z)$ (10)

Applying finite sine transform to the equation (2) and using boundary conditions (4) & (5), we obtain

$$\frac{\partial u}{\partial t} = \frac{(-1)^{l+m}F(t)}{q_l q_m} - (q_l^2 + q_m^2)u - M^2 u \quad (11)$$

Where

$$u = \int_0^1\int_0^h u(y,z,t)\sin(q_l y)\sin(q_m z)\,dy\,dz \quad (12)$$

and

$\frac{\partial \rho}{\partial z}$ = - F(t) where F(t) is an arbitrary function of time

Further we apply Laplace transform in transformed initial condition u=0 at t=0, we get

$$\overline{su} = \frac{(-1)^{l+m}\bar{F}(s)}{q_l q_m} - (q_l^2 + q_m^2)\bar{u} - M^2 u \quad (13)$$

Where $\bar{u}$ and $\bar{F}(s)$ are the Laplace transforms of respective quantities.

On calculating equation (13) and take the help of Inverse Laplace formula we get the analytical expression of the velocity of liquid which is as follows:

$$u = \frac{4}{h}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{l+m}}{q_l q_m}\left[\int_0^1 F(t-\eta)\left\{\frac{e^{s_1\eta}}{(s_1-s_2)(s_1-s_3)} + \frac{e^{s_2\eta}}{(s_2-s_3)(s_2-s_1)} + \frac{e^{s_3\eta}}{(s_3-s_1)(s_3-s_2)}\right\}d\eta\right]\sin(q_l y)\sin(q_m z) \quad (14)$$

where s_1, s_2 and s_3 are the roots of the cubic in obtained by equating the denominator of the integrand of the Laplace Inverse transforms to zero.

4. RESULT AND DISCUSSION

On considering the constant pressure gradient as -1, equation (14) takes the form:

$$u = \frac{4}{h}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{l+m}}{q_l q_m}\left[1 + \frac{1}{(s_1-s_2)(s_2-s_3)(s_3-s_1)}\left\{s_2 s_3(s_2-s_3)e^{s_1 t} + s_3 s_1(s_3-s_1)e^{s_2 t}\right.\right.$$

$$+s_1 s_2 (s_1 - s_2) e^{s_3 t} \}] \frac{\sin(q_l y)\sin(q_m z)}{(q_l^2 + q_m^2)} \quad (15)$$

After a bit calculation equation (15) reduced in the form

$$u(y,z,t) = \frac{1}{2}(h^2 - z^2) - \frac{2}{h}\sum_{m=0}^{\infty} \frac{(-1)^m \sin(q_l y)\sin(q_m z)}{q_m^3 \sinh q_m} + \frac{4}{h}\sum_{m=0}^{\infty} \frac{(-1)^{l+m}}{q_l q_m (s_1 - s_2)(s_2 - s_3)(s_3 - s_1)} [(s_2 - s_3) s_2 s_3 e^{s_1 t} + (s_3 - s_1) s_3 s_1 e^{s_2 t} + (s_1 - s_2) s_1 s_2 e^{s_3 t}] \frac{\sin(q_l y)\sin(q_m z)}{(q_l^2 + q_m^2)} \quad (16)$$

It is noteworthy that the equation (16) gives the velocity of the liquid through the tube under the influence of unit constant pressure gradient. It is also to be noted that if we take M=0 in equation (11) we get a result which is equivalent with those obtained by Gupta and Gupta (1976).

4. REFERENCES

1. Bagchi, K.C. *Indian Journal of Mechanics and Mathematics* IV, (No. 2); p 87 (1965).
2. Gireesha B J, Roopa G S and Bagewadi C S. Unsteady flow and heat transfer of a dusty fluid through a rectangular channel. Hindawi Publishing Corporation, Mathematical Problems in Engineering, Article ID 898720, 17 pages (2010).
3. Gupta, P.K. and Gupta, S.C. *ZAMP*, 27(1); p 119 (1976).
4. Makinde, O. D. and Mhone P Y. Heat transfer to MHD oscillatory flow in a channel filled with porous medium. *Rom. J. Phys.* 50(9–10): 931–938 , Bucharest (2005).
5. Pal, S.K. and Sen Gupta, P.R. *Indian Journal of theoretical Physics*, 34 (No.4); p 183 (1986).
6. Palov, K.B. and Tersor, Y.A. *Applied Math., Mech., Lenings*. 24; P. 1079 (Trans.) (1960).
7. Umavathi J C, Chamkha A J, Mateen A and Al-Mudhaf A. Unsteady oscillatory flow and heat transfer in a horizontal composite porous medium. *Nonlinear Anal.: Model. Control* 14(3): 397–415 (2009).