Journal of Computer and Mathematical Sciences, *Vol.7(12), 710-715 December 2016* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

A Study on Unsteady Dusty Viscous Fluid Flow through a Circular Pipe under the Influence of an Uniform Transverse Magnetic Field

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(Received on: December 5, 2016)

ABSTRACT

In the present paper, the unsteady flow of a viscid dusty fluid through a long circular pipe in the presence of an uniform transverse magnetic field under time varying presence gradient. The expressions for the velocity of the fluid and dust particules have been investigated later on. The results are also determined for weak magnetic field.

Keywords: Unsteady Flow, Viscous Fluid, Circular Pipe, Transverse Magnetic Field, Pressure Gradient.

1. INTRODUCTION

Over the years, many researchers have investigated the unstable flow of a dust-filled viscid fluid in the effect of constant transverse magnetic field. The effect of magnetic field in fully-developed, steady and laminar flow in a pipe has been investigated by Malekzadeh *et al*.[5] Chand *et al.*[2] studied hydro-magnetic periodic flow within a permeable medium under pipe of cicular cross-section with condition of slip flow. Wall porosity plays an indispensable role on many flows which stabilizes the fluid flow and lowers the temperature within the flow field. A comparative study of Rao and Ramamurthy[6] investigated the flow of a dusty viscid fluid in a pipe of cicular cross-section under the influence of an exponential barometric gradient. Das[4] has discussed the problems of flow of viscous conducting liquid in a pipe of cicular cross-section under the effect of oblique magnetic field. A note on dusty gas flow in a streamline boundary layer over a dull body is given by Asmolav[1].

The present article considers the unsteady flow of a dusty viscous fluid under a long pipe of cicular cross-section in the presence of uniform oblique magnetic field and barometric gradient which varies over time and has been investigated. The effect due to induced magnetic field and perturbation of field has been ignored.

2. MATHEMATICAL FORMULATION

Here, we have assumed the following notations:

ρ	=	Fluid's density
u	=	Fluid particle's velocity
v	=	Dust particle's velocity
ν	=	Kinematic co-efficient of the viscosity
B_0	=	Magnetic induction
σ	=	Electrical Conductivity
N_0	=	Dust particle's constant number density
M	=	Dust particle's mass
K	=	Stoke resistance coefficient
t	=	Time variable
(r,θ,z)	=	Cylindrical co-ordinate

Here, we have considered the flow of Rivlin-Ericken dusty viscous fluid through a long circular cylinder of radius 'a' whose axis is along z-axis so that the equation of cylinder is given by r = a. The constant magnetic field B_0 is applied perpendicular to z-axis. Here we have neglected induced magnetic field.

We observe that the equations of motion relevant to the problem are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN_0(v-u)}{\rho} - \frac{\sigma B_0^2 u}{\rho} \tag{1}$$

and

$$m\frac{\partial v}{\partial t} = (u - nv) \tag{2}$$

Initial and boundary conditions are as follows :

$$u = 0,\ v = 0\ \forall\, r, t \le 0 \tag{3}$$
$$u = 0,\ v = o\ \ r = a, t > 0$$
$$u = \alpha,\ v = \beta\ \ \text{at}\, r = 0$$

where α, β are finite quantities

Let us suppose

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = L_0\left(1 + \lambda e^{i\omega t}\right) \tag{4}$$

If we take the following non-dimensional quantities as :

$$u' = \frac{u}{U},\ v' = \frac{v}{U},\ z' = \frac{Z}{b}, r' = \frac{r}{b},\ t' = \frac{tu}{b},$$
$$\tau' = \frac{\tau u}{b},\ p' = \frac{p}{\rho u^2}, W = \frac{wb}{U},\ R = \frac{Ub}{v} \tag{5}$$

After dropping the prime, the relations (1), (2) and (4) reduces in the form

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{R}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + M_0(v - u) - Mu \quad (6)$$

$$\frac{\partial v}{\partial t} = u - v \quad (7)$$

$$\frac{\partial P}{\partial z} = L_0\left(1 + \lambda e^{i\omega t}\right) \quad (8)$$

where,

$M_0 = \frac{nN_0}{\rho}$ (Mass concentration parameter of dust particle)

and relaxation parameter τ is defined as:

$$\tau = \frac{MU}{Kb} \quad (9)$$

wherein

$$M = \frac{\sigma B_0^2}{\rho}\frac{b}{U}$$

using the relation (5) into (3), then we we obtain a new type of boundary conditions:

$u = 0, v = 0$ at $r = b/a\ (c_1)$ $t > 0$ (10)

$u = \alpha, v = \beta$ at $r = 0$

3. SOLUTION OF THE PROBLEM

In view of equation (8), we can take

$$u = L_0[u_1(r) + \lambda u_2(r) e^{i\omega t}] \quad (11)$$

The boundary conditions for u_1 and u_2 are given by

$u_1(c_1) = 0$ $u_2(c_1) = 0$ $t > 0$ (12)

Eliminating v from equation (6) and (7), we get

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{\partial P}{\partial z}\right) + \frac{1}{R}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \frac{M_0+1+Mr}{\tau}\frac{\partial u}{\partial t}$$
$$+ \frac{1}{\tau}\left\{-\frac{\partial P}{\partial z} + \frac{1}{R}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - Mu\right\} \quad (13)$$

Substituting the values of $\left(-\frac{\partial P}{\partial z}\right)$ and u from equations (8) and (11) into equation (13), we obtain

$$\frac{1}{\tau}\left[\frac{1}{R}\left(u''_1 + \frac{1}{r}u_1\right) - Mu + 1\right] + \frac{\lambda e^{i\omega\tau}}{R\tau}\left[(1 + i\omega\tau)\left(u''_2 + \frac{1}{r}u_2\right)\right]$$
$$= \left[i\omega(1 + i\omega\tau)\left(1 + \frac{M_0}{(1+i\omega\tau)} + \frac{M}{i\omega}\right)u_2 + R(1 + i\omega\tau)\right] \quad (14)$$

Comparing the terms containing $e^{i\omega\tau}$ and not containing $e^{i\omega\tau}$ to zero, we get

$$\left(u''_1 + \frac{1}{r}u_1\right) - MRu_1 = -R \quad (15)$$

and

$$u''_2 + \frac{1}{r}u_2 - R\left(i\omega + \frac{i\omega M_0}{(1+i\omega\tau)} + M\right)u_2 = -R \quad (16)$$

If we take $r\sqrt{RM} = z$, then equation (15) reduces in the form

$$z^2 s'' + zs' - z^2 s = 0 \quad (17)$$

$$zs' - z^2 s = 0 \quad (18)$$

where $s = u_1 - \frac{1}{M}$ (19)

Equation (19) in a form of Bessel differential equation of zeroth order and its solution can be written as

$$s = AJ_0(iz) \quad = AJ_0\left(ir\sqrt{MR}\right) \tag{20}$$

Where A is constant.

By virtue of equation (20), equation (19) becomes

$$u_1 = \frac{1}{M} + AJ_0\left(ir\sqrt{MR}\right) \tag{21}$$

Using boundary condition (10) into equation (21) we get

$$u_1 = \frac{1}{M}\left[1 - \frac{J_0\left(ir\sqrt{MR}\right)}{J_0\left(ic_1\sqrt{MR}\right)}\right] \tag{22}$$

Equation (16) can be written as

$$r^2u_2'' + ru_2' \text{-} R(c+id)r^2u_2 = -Rv^2 \tag{23}$$

Where $c = M + \frac{M_0\tau\omega^2}{1+\omega^2\tau^2}$

$$d = \omega\left[1 + \frac{M_0}{1+\omega^2\tau^2}\right]$$

If we take $r\sqrt{R(c + id)} = Y$, then equation (23) reduces in the form

$$Y^2Q'' + YQ' - Y^2Q = 0 \tag{24}$$

where in $Q = u_2 - \frac{1}{c+id}$ (25)

Hence solution of equation (24) becomes

$$Q = BJ_0(iY) = BJ_0\left(r\sqrt{R(c^2 + d^2)^{\frac{1}{2}}.\, e^{i\phi}}\right) \tag{26}$$

where $\phi = \frac{1}{2}\tan^{-1}\frac{b}{a}$, hence from equation (25)

$$u_2 = \frac{1}{c + id} + BJ_0\left\{r\sqrt{R(c^2 + d^2)}.\, e^{i\phi}\right\}$$

using boundary condition (10) in equation (26), we get

$$u_2 = \frac{1}{c+id}\left[1 - \frac{J_0\left\{r\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}{J_0\left\{c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}\right] \tag{27}$$

$$u_2 = \frac{c-id}{c^2+d^2}\left[1 - \frac{J_0\left\{r\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}{J_0\left\{c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}\right] \tag{28}$$

Further making use of the relations (22) and (28) into the relation (11), we obtain,

$$u = L_0\left[\frac{1}{M}\left\{1 - \frac{J_0(ir\sqrt{MR})}{J_0(ic_1\sqrt{MR})}\right\} + \lambda\frac{c-id}{c^2+d^2}\left\{1 - \frac{J_0\left\{r\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}{J_0\left\{c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}}.e^{i\phi}\right\}}\right\}e^{i\omega t}\right] \tag{29}$$

Now inserting equation (29) into the equation (7), we obtain

$$u = \frac{L_0}{\tau}\left[\frac{1}{M}\left\{1 - \frac{J_0(ir\sqrt{MR})}{J_0(ic_1\sqrt{MR})}\right\} + \lambda\frac{c-id}{c^2+d^2}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega t}\right]$$

$$v = L_0\left[\frac{1}{M}\left\{1 - \frac{J_0(ir\sqrt{MR})}{J_0(ic_1\sqrt{MR})}\right\} + \lambda\frac{c-id}{c^2+d^2}\frac{1}{(1+i\omega\tau)}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega t}\right] \quad (30)$$

Equations (29) and (30) denotes the velocity of the fluid and dust particle. If we take $\bar{u} = \frac{u}{L_0}$, $\bar{v} = \frac{v}{L_0}$
and u_m is defined as

$$u_m = \frac{1}{M}\left[1 - \frac{J_0(ir\sqrt{MR})}{J_0(ic_1\sqrt{MR})}\right] \quad (31)$$

Then the relations (29) and (30) reduces in the form

$$\bar{u} = u_m + \lambda\frac{c-id}{c^2+d^2}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega\tau} \quad (32)$$

and

$$\bar{v} = u_m + \lambda\frac{c-id}{(c^2+d^2)(1+i\omega\tau)}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega\tau} \quad (33)$$

4. RESULT AND DISCUSSION

The expressions for velocity of fluid and dust particles have been given in equations (32) and (33) and if the dust particles are assumed so small such that M = 0, then the expression for velocity distribution of the fluid particle is given by

$$\bar{u} = \left[\frac{R}{4}(1 - r^2) + \lambda\frac{c-id}{c^2+d^2}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega t}\right]$$

and the expression for velocity distribution of dust particle is given by

$$\bar{v} = \left[\frac{R}{4}(1 - r^2) + \lambda\frac{c-id}{(c^2+d^2)(1+i\omega\tau)}\left\{1 - \frac{J_0\left(r\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}{J_0\left(c_1\sqrt{R(c^2+d^2)^{\frac{1}{2}}.e^{i\phi}}\right)}\right\}e^{i\omega t}\right]$$

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